First Year B.Sc.

PHYSICS

Mechanics, Waves & Oscillations
First Year B.Sc.
PHYSICS
MECHANICS, WAVES & OSCILLATIONS

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APSCHE TEXTBOOK
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Foreword

The Andhra Pradesh State Council of Higher Education, in line with the directions of the Hon’ble Chief Minister of Andhra Pradesh, introduced English Medium in all Degree programmes from this academic year 2021-22. As part of empowering the students joining Degree programmes from vernacular medium, the AP State Council of Higher Education is bringing out podcasts, video casts and notably the bilingual text books. These bilingual books are written in English, with the gist of the content in Telugu to enable the students to comprehend the content in their mother tongue. The bilingual text book is envisioned on the core concept of Outcome Based Education, highlighting the learning outcomes for every chapter. These are better called as bilingual resources rather than textbooks, as the APSCHE has developed a template for the bilingual textbooks designing them with concepts and frameworks going beyond the usual reading material.

Furthering the detailed description of the topics, as per the common syllabus of the Redesigned Curricular Framework for Choice Based Credit System, the bilingual text book contains Glossary, where certain important terms which the student might be unfamiliar with are identified and explained in one or two sentences, which is not a mere dictionary meaning. Links to online videos or audios which will be useful for further reading and understanding of the topics are given under the Interactive links. To foster further reading, information on online resources, articles or another text book pertaining to the content are provided. To make the text book more of a resourceful book, Curricular Activities, wherein suggested activities that could be taken up in realization of the outcomes are provided for the benefit of students. To help the students to assess understanding of the content, Self Assessment instruments are provided. For Advanced Learners, caters to the needs of advanced learners providing them with additional material about the topics. Finally, for every chapter References are provided.
I sincerely appreciate the Authors and the Editors for taking pains in bringing out this bilingual text book in a record time, replete with knowledge which fosters the academic progression of students. I earnestly thank my Academic Officers, Dr. B. S. Selina, Sri. Srirangam Mathew, Dr. P. Anil Kumar for their coordinating activities and Prof. K. Rama Mohana Rao, the Vice-Chairman of APSCHE under whose guidance the publication is brought out.

K. Hemachandra Reddy
Chairman, APSCHE
Preface

Bilingual undergraduate textbook preparation is a path breaking initiative by the APSCHE in the bachelor degree education. It is our privilege to be a part of this prestigious project.

The textbook has been prepared keeping English as the primary language. Telugu translations have been made without translating the technical words as much as possible, which is the entire essence of this project. Hence for all purposes English version may be considered as standard text.

The textbook is provided with instructions to both students and faculty, which may be useful for the effective usage of the book. The book provides enough insights for both faculty and students to implement Bloom’s taxonomy for undergraduate physics in a smooth manner at teaching and learning levels. Implementation of it at evaluation level may need further improvements.

Every chapter is provided with outcomes specific to programs and also with wide range of applications which may bridge the gap between industry and academia to some extent.

We hope that the innovative attempt made here may reach the expectations and serve the needs of every stake holder associated with the project.

- Authors and Editor
# Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>INSTRUCTIONS TO STUDENTS</td>
<td>I</td>
</tr>
<tr>
<td>INSTRUCTIONS TO FACULTY</td>
<td>XIX</td>
</tr>
<tr>
<td>FUTURE PERSPECTIVES</td>
<td>XXXIII</td>
</tr>
<tr>
<td>LEARNING RESOURCES</td>
<td>XLIII</td>
</tr>
<tr>
<td>MECHANICS</td>
<td>1</td>
</tr>
<tr>
<td>INTRODUCTION TO MECHANICS</td>
<td>2</td>
</tr>
<tr>
<td>UNIT-I</td>
<td>25</td>
</tr>
<tr>
<td>1.  MECHANICS OF PARTICLES</td>
<td>25</td>
</tr>
<tr>
<td>1.1 INTRODUCTION</td>
<td>31</td>
</tr>
<tr>
<td>1.2 REVIEW OF NEWTON’S LAWS OF MOTION</td>
<td>33</td>
</tr>
<tr>
<td>1.3 MOTION OF VARIABLE MASS SYSTEM</td>
<td>47</td>
</tr>
<tr>
<td>1.4 ROCKET EQUATION</td>
<td>49</td>
</tr>
<tr>
<td>1.5 MULTISTAGE ROCKET</td>
<td>57</td>
</tr>
<tr>
<td>1.6 RUTHERFORD SCATTERING</td>
<td>61</td>
</tr>
<tr>
<td>1.7 FURTHER INSIGHTS</td>
<td>70</td>
</tr>
<tr>
<td>SOLVED PROBLEMS &amp; EXERCISES</td>
<td>71</td>
</tr>
<tr>
<td>GLOSSARY</td>
<td>82</td>
</tr>
<tr>
<td>2.  RIGID BODY DYNAMICS</td>
<td>85</td>
</tr>
<tr>
<td>2.1 INTRODUCTION</td>
<td>91</td>
</tr>
<tr>
<td>2.2 ROTATIONAL KINEMATIC RELATIONS</td>
<td>99</td>
</tr>
<tr>
<td>2.3 ANGULAR MOMENTUM AND MOMENT OF INERTIA TENSOR</td>
<td>105</td>
</tr>
<tr>
<td>2.4 EQUATION OF MOTION FOR A ROTATING BODY</td>
<td>117</td>
</tr>
<tr>
<td>2.5 EULER EQUATIONS</td>
<td>119</td>
</tr>
<tr>
<td>2.7 GYROSCOPE</td>
<td>125</td>
</tr>
<tr>
<td>2.8 PRECESSION OF ATOM AND NUCLEUS IN MAGNETIC FIELD</td>
<td>131</td>
</tr>
<tr>
<td>2.9 PRECESSION OF EQUINOXES</td>
<td>135</td>
</tr>
<tr>
<td>SOLVED PROBLEMS AND EXERCISES</td>
<td>141</td>
</tr>
<tr>
<td>GLOSSARY</td>
<td>159</td>
</tr>
<tr>
<td>UNIT-II</td>
<td>161</td>
</tr>
<tr>
<td>3.  MOTION IN CENTRAL FORCE FIELD</td>
<td>161</td>
</tr>
<tr>
<td>3.1 INTRODUCTION</td>
<td>167</td>
</tr>
<tr>
<td>3.2 CENTRAL FORCE CHARACTERISTICS &amp; EXAMPLES</td>
<td>173</td>
</tr>
<tr>
<td>3.3 EQUATION OF MOTION UNDER A CENTRAL FORCE</td>
<td>181</td>
</tr>
</tbody>
</table>
3.4 Kepler laws of planetary motion ............................................................... 187
3.5 Motion of Satellites .................................................................................. 199
3.6 Global Positioning System (GPS) ............................................................. 203
3.7 Weightlessness ......................................................................................... 209
3.8 Physiological effects of astronauts .......................................................... 213
Solved problems and exercises ........................................................................ 217
Glossary ........................................................................................................... 233

UNIT-III ............................................................................................................. 235

4. SPECIAL THEORY OF RELATIVITY ......................................................... 235
   4.1 Introduction ............................................................................................ 241
   4.2 Lorentz transformation .......................................................................... 247
   4.3 Michelson Morley experiment ............................................................... 257
   4.4 Postulates of special theory of relativity ............................................... 263
   4.5 Time dilation .......................................................................................... 263
   4.6 Length contraction ................................................................................ 267
   4.7 Relativistic mass .................................................................................... 269
   4.8 Relativistic kinetic energy, mass-energy relation .................................... 273
Solved problems and exercises ........................................................................ 279
Glossary ........................................................................................................... 290

WAVES AND OSCILLATIONS ........................................................................ 291

   Introduction to waves and oscillations ....................................................... 292

UNIT-IV ............................................................................................................. 303

5. UNDAMPED, DAMPED, FORCED OSCILLATIONS .................................... 303
   5.1 Introduction ............................................................................................ 309
   5.2 Solution of SHM differential equation ................................................... 325
   5.3 Damped harmonic oscillator .................................................................. 333
   5.4 Quality factor ........................................................................................ 339
   5.5 Logarithmic decrement ......................................................................... 343
   5.6 Forced oscillations ................................................................................ 345
Solved problems and exercises ........................................................................ 354
Glossary ........................................................................................................... 370

6. COUPLED OSCILLATIONS ........................................................................... 373
   6.1 Introduction ............................................................................................ 379
   6.2 Two coupled oscillators ....................................................................... 385
   6.3 N-coupled oscillator ............................................................................. 397
   6.4 Wave equation ....................................................................................... 401
   6.5 Applications ......................................................................................... 403
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOLVED PROBLEMS &amp; EXERCISE</td>
<td>.................................</td>
<td>404</td>
</tr>
<tr>
<td>GLOSSARY</td>
<td>................................</td>
<td>415</td>
</tr>
<tr>
<td>UNIT-V</td>
<td>................................</td>
<td>417</td>
</tr>
<tr>
<td>7. VIBRATING STRINGS</td>
<td>................................</td>
<td>417</td>
</tr>
<tr>
<td>7.1</td>
<td>INTRODUCTION</td>
<td>................................</td>
</tr>
<tr>
<td>7.2</td>
<td>TRANSVERSE WAVE PROPAGATION ALONG A STRETCHED STRING</td>
<td>..........</td>
</tr>
<tr>
<td>7.3</td>
<td>SOLUTION OF WAVE EQUATION</td>
<td>................................</td>
</tr>
<tr>
<td>7.4</td>
<td>MODES OF VIBRATION OF STRETCHED STRING CLAMPED AT ENDS</td>
<td>....................</td>
</tr>
<tr>
<td>7.5</td>
<td>OVERTONES AND HARMONICS</td>
<td>................................</td>
</tr>
<tr>
<td>7.6</td>
<td>MELDE’S EXPERIMENT</td>
<td>................................</td>
</tr>
<tr>
<td>7.6</td>
<td>APPLICATIONS OF VIBRATING STRINGS</td>
<td>................................</td>
</tr>
<tr>
<td>7.6</td>
<td>SOLVED PROBLEMS AND EXERCISES</td>
<td>................................</td>
</tr>
<tr>
<td>7.6</td>
<td>GLOSSARY</td>
<td>................................</td>
</tr>
<tr>
<td>8. ULTRASONICS</td>
<td>................................</td>
<td>455</td>
</tr>
<tr>
<td>8.1</td>
<td>INTRODUCTION</td>
<td>................................</td>
</tr>
<tr>
<td>8.2</td>
<td>PROPERTIES OF ULTRASONIC WAVES</td>
<td>................................</td>
</tr>
<tr>
<td>8.3</td>
<td>PRODUCTION OF ULTRASONICS</td>
<td>................................</td>
</tr>
<tr>
<td>8.4</td>
<td>MAGNETOSTRICTION METHOD</td>
<td>................................</td>
</tr>
<tr>
<td>8.5</td>
<td>PIEZOELECTRIC METHOD</td>
<td>................................</td>
</tr>
<tr>
<td>8.6</td>
<td>DETECTION OF ULTRASONIC WAVES</td>
<td>................................</td>
</tr>
<tr>
<td>8.7</td>
<td>APPLICATIONS OF ULTRASONICS</td>
<td>................................</td>
</tr>
<tr>
<td>8.8</td>
<td>SONAR</td>
<td>................................</td>
</tr>
<tr>
<td>8.8</td>
<td>SOLVED PROBLEMS AND EXERCISES</td>
<td>................................</td>
</tr>
<tr>
<td>8.8</td>
<td>GLOSSARY</td>
<td>................................</td>
</tr>
</tbody>
</table>
Instructions to Students

ఇంట్వాడు నిమిషాలు
సంగంమ విభాగము

సంగంమ విభాగము కొనసాగిన తమ్ముడు, వింతల, మనం పతిచేతినందున సంగంయం విభాగము కొనసాగిన తమ్ముడు చేసిన మన సంగంయం విభాగము కొనసాగిన తమ్ముడు కలిగి ఉంటుంది.

తండ్రి టేన్, తండ్రి, మనం సంగంయం విభాగము కొనసాగిన తమ్ముడు చేసిన మన సంగంయం విభాగము కొనసాగిన తమ్ముడు కలిగి ఉంటుంది.

అతి, అతి, అతి కొనసాగిన తమ్ముడు మన సంగంయం విభాగము కొనసాగిన తమ్ముడు కలిగి ఉంటుంది.
INSTRUCTIONS TO STUDENTS

About Physics

Physics is an everlasting wild quest into the secrets of nature. The branch of Physics evolved from natural philosophy, where people used to argue and discuss to arrive at some meaningful conclusions over the observations made on nature.

Scientific temper, i.e., a search for cause and effect relations in natural phenomenon was well established by Maharshi Kasyapa/Kanada in ancient India between 600BC to 200BC. His works were further consolidated and developed by Buddhist scholars like Dharmakirti (650AD), Dignāga (500AD) and others. Similar studies were begun independently in Greece by Aristotle (350BC), Archimedes (250BC) and several others. Major scientific developments were reported in India (in Sanskrit), Mesopotamia (modern Syria) (in Sumerian), Persia (Modern Iran) (in Persian) and Greece (in Greek). But all those were most rudimentary and most of the concepts were later found to be lacking in rigour. However, they laid the foundations for systematic development of thought in a scientific manner.

They were further developed in Arabian countries (in Arabic) by Alhazen (1000AD). He has introduced experimental verification of the concepts developed earlier and may for this reason be considered as the founder of experimental physics. Further, Madhava and Nilakantha of Sangamagrama (1400AD) established Kerala School of Astronomy and may be considered as the founder of analytical physics through his calculus. Speculations exist that his work was passed on to Europe through Jesuit scholars and further developed by Newton and Leibnitz. But Newtonian and Leibnitz calculus has turned the face of Physics into a more systematic science. The work of Newton’s principia is actually a consolidation of a compendium of empirical physics works of Copernicus, Galileo,
కసారోగ్యం 

ఈ ఆకాంక్షలనంతరం యమలకమంతకమబదఖన్నం, సమాధానాన్ని ప్రత్యేకంగా మాత్రమే ఆచరించాలని చెక్కాలేదు. మూడు సంవత్సరాల దినాల్లో ప్రత్యేకంగా మాత్రమే ఆచరించాలని చెక్కాలేండేదు. మూడు సంవత్సరాల అక్షరాదిత్వం ప్రత్యేకంగా మాత్రమే ఆచరించాలని చెక్కాలేండేదు. మూడు సంవత్సరాల అక్షరాదిత్వం ప్రత్యేకంగా మాత్రమే ఆచరించాలని చెక్కాలేండేదు.
Huygens and many others, with an added analysis on geometry and calculus. Due to his many contributions, which are fundamental in nature, Isaac Newton may very well be considered as the founder of Classical Physics.

Subsequently several developments have taken place in science and lead to industrialization in European countries in 1800s, thus, adding technology to science. Another golden era in science happened during 1890-1925 where modern Physics has evolved. Albert Einstein and Max Plank may be considered as the founders of relativistic mechanics and Quantum mechanics respectively. In relativistic mechanics, speed of the objects is comparable to the speed of light, while in quantum mechanics, the size of the objects is comparable to angstrom. Finally after the development of standard model in 1970, Quantum field theory has evolved to deal with high speed interactions at atomic level in terms of fields.

Fig: Branches of Modern Physics. (Source: Wikipedia commons)

Second world war in 1940s has paved the way to the invention of computer, that in a sense hindered the flourishing research in theoretical physics for quite some time, as majority of the researchers have turned towards numerical methods instead of analytical methods of problem solving. With the introduction of Artificial Intelligence, computers also have attained speeds and thinking abilities that surpass human abilities. Further focus on fundamental theoretical research took a new turn with supported experimental research in Nanoscience, Cosmology and Particle Physics.
అధికరణ అనుమతి పొందిన అధ్యాపకుడు అధ్యాపకుడు అధికరణానికి ప్రతి "కంద్రగాలు". అంతా, ప్రత్యేకించిన నిర్ధారించానికి పని తొడిసినందువలూ అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు. అంటే నిర్ధారించిన నిర్ధారించానికి పని తొడిసినందువలూ అధ్యాపకుడు అధ్యాపకుడు.

సాధనాలు, అధ్యాపకుడు అధ్యాపకుడు కంటే అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు. అంటే అధ్యాపకుడు వాడానికి అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు. అంటే అధ్యాపకుడు వాడానికి అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు. అంటే అధ్యాపకుడు వాడానికి అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు. అంటే అధ్యాపకుడు వాడానికి అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు. 

పినికిందా, అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు. 

ఎండు సమయం అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు. 

పినికిందా, అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు అధ్యాపకుడు. 

$$ T = 2\pi \frac{l}{\sqrt{g}} $$
Study of Physics

In this modern era, the study of Physics may be majorly classified as energy conversions and light matter interactions. The classical physics deals with conservation laws for energy, momentum and angular momentum in various physical phenomena.

The basic methodology followed to study Physics is the scientific method. Here any natural phenomenon observed is questioned first at the fundamental level. A sequence of such questions may be considered as a research area/topic. A hypothesis will be made to explain the phenomenon. Subsequently, the hypothesis may be put to test through experiments or through computer simulations. The data thus received will be analysed to draw conclusions. The conclusions may lead to further questions on the observed phenomenon and the process continues.

Fig: Scientific method.

A basic thumb rule to study the contents of the book are as follows.

Concepts:

1. Know the fundamental rules/conservation laws precisely.
2. Identify and verify them in each and every phenomenon, equation and derivation.
3. Verify the units and dimensions of physical quantities during the flow of derivation. Remember that terms or product of terms with same dimensions only can be added or subtracted.
ప్రశ్నలు ప్రస్తుతించండి:

1. సకర్ణంం నంతర నియంత్రణ విధానం చేయడానికి అంతర్భువ పనిగా, అంధకారం, రేటు, థి లేదా కంప్యూటర్ సిమ్యులేషన్ ఉపయోగం చేస్తారు.

2. సకర్ణంం పనిచేస్తున్న విద్యార్థుల లోపల పరిస్థితి అవసానం అంధకారం ఉపయోగం చేస్తారు.

3. అనేక మేములోస్సు ఇతర గురించి (e, లా రుండా సుమారు అంషాలు సంచే), నేటికి ప్రశ్నలు (డా, నాలుగు ప్రశ్నలు) మాత్రమే క్రియలు లేదా (l, T) అంధకారం.

4. LHS విధానం, RHS అంధకారం ఉపయోగం చేస్తుంది. వాటిల్లి ప్రశ్నలు అంధకారం చేస్తారు. 

పరిస్థితి సంస్థలు అవసరం అవసరం:

ADDIE విధానం అంధకారం ముఖ్యంగా, ADDIE ముందు వరిచి, కాలి, అంధకారం, అంధకారం మాత్రమే విశేషాలు ఉపయోగం చేస్తారు. అంధకారం మాత్రమే విశేషాలు అంధకారం సిమ్యులేషన్ ఉపయోగం చేస్తారు.

పరిస్థితి సంస్థలు అవసరం అవసరం విభాగం ఉదాహరణలు ఉత్పత్తి చేయడానికి అవసరం అవసరం పరిస్థితి సంస్థలు అనేక మేములోస్సు ఉదాహరణలు ఉత్పత్తి చేయడానికి అవసరం అవసరం.

1. పరిస్థితి సంస్థలు అవసరం అవసరం విశేషాలు ఉపయోగం చేయడానికి అవసరం అవసరం.

2. పరిస్థితి సంస్థలు అవసరం అవసరం విశేషాలు ఉపయోగం చేయడానికి అవసరం అవసరం.

\[ g = 4\pi^2 l/T^2 \]
5. There may be discrepancy with units on either side of an equation like the one given below. But the rule to follow is that dimensions on either side of equation must always match.

6. Try to put every equation into words.

Equations:
For example, consider the simple pendulum equation.

\[ T = 2\pi \sqrt{\frac{l}{g}} \]

1. Identify the approximations that were made while deriving the equation, like, the angle, \( \theta \) must be very small etc.

2. Identify the conservation laws considered while deriving the equation.

3. Identify local constants \((g, a\) constant as long as we are on earth\), Global constants \((2\pi, a\) universal constant\) and variables \((l, T)\) in the given equation.

4. Identify the dependence of the terms on LHS, on the variables on RHS. Remember that one can not change the global constants at all and also can not change the local constants in a given situation.

Designing Experiments

Experiments are designed by using ADDIE model. ADDIE stands for Analysis, Design, Development, Implementation and Evaluation. For example consider the familiar simple pendulum equation.

**Fig: ADDIE Model.**
Designing Experiments

3. The experiment is carried out in the following manner: Once the experiment is conducted, the results are recorded and analyzed. The variables are measured and recorded in the form of graphs or tables.

4. Upon completion, the experiment is repeated with different variables to ensure accuracy. The data obtained from the experiment is then analyzed to determine the effect of each variable on the outcome.

5. The outcome of the experiment is then interpreted and conclusions are drawn. These conclusions are then used to improve future experiments.
For the experimental verification of the simple pendulum formula, or the calculation of $g$ using simple pendulum.

1. One has to analyse the equation first to identify the variables and constants. The local constants may sometimes be the material dependent constants like mass, area, volume, radius, size etc.

2. Identify the dependent and independent variables. Here $l$ is the independent variable and $T$ is the dependent variable.

$$g = 4\pi^2 \frac{l}{T^2}$$

3. Identify the approximations taken while deriving the equation to implement the same during the experiment, such as the small angle approximation, which assumes $\theta$ to be very small.

4. Design of experiment includes identification of the possible errors and possible ways out in terms of repetition and averaging.

5. If we have perfectly accurate and precise instruments, the result can be calculated with a single measurement of single swing of the pendulum. Still we repeat the experiment for various lengths of the pendulum to avoid error in measurement of length of pendulum (from point of suspension to center of the bob). Counting time for 10 (say) oscillations and dividing by 10 reduces the errors in precision of the clock. Suppose the exact time period is 1.2 sec. for one oscillation. But as the clock does not have decimal point measurement, we then count the time for 10 oscillations. Then if our measurement is correct, we must get 12 sec. Thus, increasing the count 10 times, increases the precision by 1 decimal place. Since after counting 10, there may be some delay in stopping the clock, one may count 11 sec. or 13 sec. any higher value as per convenience. To avoid the error due to human mistakes, the 10 oscillation measurement for a given $l$ will be repeated at least two or three times and then averaged.

6. Development part deals with precise instrumentation and specifications of rest of the components to be used in the experiment. For example, the thread type used, quality of the suspension point, etc.

7. Implementation part will be straightforward, if design and development parts are well taken care of.
Designing Experiments

కుమారి ప్రతిష్ఠ:

ఇంటిలో కాదు కూడా లోపిలేస్తారని, 1Vs. T^2 వైమానం గురించి ప్రతి సమాచారం మాత్రం సాధారణంపను లేదా ప్రతి సమాచారం మాత్రం సాధారణంపను లేదా మరొక సమస్యలు ఉండవచ్చు. చాలా నిర్ణయించడం చేసి, సమయం నిర్ణయించడం అంశం అంధక పదార్థాల యొక్క నిర్ణయానికం సాధారణం. సమయం నిర్ణయించడం అంధక పదార్థాల యొక్క నిర్ణయానికం చాలా సమయం నిర్ణయించడం అంధక పదార్థాల యొక్క నిర్ణయానికంపై పెట్టినాం అంధక పదార్థాల యొక్క నిర్ణయానికం చాలా సమయం నిర్ణయించడం అంధ పదార్థాల యొక్క నిర్ణయానికం చాలా సమయం నిర్ణయించడం అంధ పదార్థాల యొక్క నిర్ణయానికం.
8. Once the experiment is implemented, the collected data will be analysed. Graphical analysis is the most accurate and precise analysis to understand the source of errors.

**Graphical analysis:**
If our measurements are accurate, every point on the graph of $l$ Vs. $T^2$ should fall on a straight line that passes through origin. But one may observe some patterns of deviations from the actual result. They can be understood as follows.

![Graphical analysis of error](image)

**Fig: Simple pendulum graphical analysis of error.**

Suppose the time period is measured accurately and length is overestimated. i.e.; if the actual length is 50cm, it is noted as 51cm, say. If the length is always measured up to the bottom of the bob, then for every reading, the measured point is plotted at one point shifted to longer lengths. Thus the entire line shifts right/bottom without passing through origin.

If the length of the pendulum is measured up to the top end of the bob, then instead of 50cm, we would note down 49cm. Then the entire graph shifts towards left/upwards.

If the graph passes through origin, that implies the length measurements are accurate.

If the starting count of the oscillations is counted as 1 instead of zero, then one counts 11 instead of 10 oscillations. Thus $T^2$ curve rises up though it passes through origin. If 9 oscillations are counted always instead of 10, then the slope of $T^2$ curve falls.
Designing Experiments

In this talk, we discuss the design of experiments. A total of 10 subjects were divided into two groups. One group underwent 10 experiments, while the other group underwent 20 experiments. The subjects were randomly assigned to the groups. It is important to note that each subject underwent only one experiment per day. The data was analyzed using statistical software. The results showed that the two groups were significantly different in terms of response variables. The findings were further validated using additional experiments. Overall, the study provides valuable insights into the design of experiments.
Thus though the curve looks linear, the nature of errors in measurement can be estimated from their graph.

Finally, remember that the equations or derivations or experiments that you have come across you may never encounter them in your future life. So the aim of all these exercises is to train you for a concept, or for a method of solving equations, or for a method of designing experiments. People have developed atomic model inspired by the planetary model. In the same manner you should get some inspiration, from the lessons in this book, for your future career and not just a mere memory based activity.

In this fast developing era of science and technology, the concepts learned 10 years back may not be existing now and in the same manner the topics that we learn today may not be useful after 10 years. The important message to take home is that one should prepare for the change and develop theoretical, experimental and computational tools to conquer the unknown future.

In this book, every chapter starts with the learning objectives, learning outcomes, outcomes specific to program and future perspectives. Here objectives are the specific plan of action of the teacher. Outcomes are divided into 6 levels of learning as suggested in Bloom’s taxonomy. (Details are contained in the instructions to faculty). Future perspectives and specific outcomes link the topics in the respective chapter with the program, which the student is a part of. This is most essential to begin any chapter with, as not connecting a given topic with future perspectives, leads to forgetfulness. Also at the end of 5th semester you are expected to be ready to approach any industry for one semester internship. Hence learn the concepts with some goal. Don’t just enter the class room without proper objective, outcome and future perspective.

Remember that brain attaches an expiry date to the information that it stores. If you set the expiry date as end of exams or start of new course, it forgets immediately after the end of exams or at the joining of new course. So try to attach a longer standing time stamp for the information you wish to store in your brain. Otherwise all your 15-20 years of effort in academics will lead you to nothing. Finally one may have to take a short term training to join a small job
Designing Experiments

https://en.wikipedia.org/wiki/List_of_people_considered_father_or_mother_of_a_scientific_field
https://en.wikipedia.org/wiki/History_of_science_and_technology_in_the_Indian_subcontinent
https://en.wikipedia.org/wiki/Vai%C5%9Be%E1%B9%A3ika_S%C5%ABtra
https://en.wikipedia.org/wiki/Atomism
https://en.wikipedia.org/wiki/Nilakantha_Somayaji
https://en.wikipedia.org/wiki/Madhava_of_Sangamagrama
https://en.wikipedia.org/wiki/Modern_physics
leaving behind years of effort aimlessly. In one sentence, “Learn for life” not just for a certificate. Map every information that you learn with a future job or career or life. Even Diamond, if not cut without proper planning, looks like an ordinary stone. It is your choice to shape your brain with proper objectives and outcomes or just use it as a mere random information dump yard.

Wishing you happy learning

- Authors
Instructions to faculty
ఉలచన
యుక్తి గుడ్రండా ఉంటుందని

ప్రప్ణం రచయిత రామారాధానం బిందువు తణాంగంగా కాక మరియు మరియు అయితే రామారాధానం నిండూ ఉంది, మిశ్రంతు పృథ్వీ రామారాధానం వీటించడం జరిగింది, అడుగు మాత్రమే సంస్థాంత్రికులు చేసాంది. ఇందులో కొనసాగించడానికి నాటికి అప్పుడే కొనసాగించడానికి కొనసాగించడానికి నాటికి అప్పుడే కొనసాగించడానికి

1. కాలచారంను ఎందుకంటుందని:

ప్రత్యేకంగా మూడు సంస్థావనము సమాచరం చేయడానికి, కాండామానం తెలిపిని తాత్కాలిక సమాచారం చేయడానికి. అందుకే, ప్రత్యేకంగా మూడు సంస్థావనము సమాచరం చేయడానికి, కాండామానం తెలిపిని తాత్కాలిక సమాచారం చేయడానికి. ఇది ఉపయోగించబడింది, మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి. ఇది ఉపయోగించబడింది, మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి. ఇది ఉపయోగించబడింది, మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి మరింత మనం సమాచారం చేయడానికి.
About Teaching Physics

Teaching Physics is the most exciting ride for physics faculty as one can experience the joy of unveiling the secrets of nature in a systematic manner. Though every teacher may have her/his own way of establishing facts and presenting content, the following instructions provide a more general sequence of events/guidelines that the teachers may find useful to implement for a smooth journey.

1. **Teacher is a facilitator:**
   In this digital era of information explosion and learning, teacher has to play the role of facilitator. i.e.; every information available with the student should be well filtered and guided by teacher in the classroom. In order to fulfil this, one has to work on possible sources of information available to the common reach of student and prepare proper filters to train them on clearly differentiating the right from wrong.

2. **Flipped classroom:**
   As told by Einstein, “The value of college education is not the learning of many facts but the training of mind to think.”
Bloom’s taxonomy

3. Understand:

The formula for the path of a ray is given by

\[ \mu_2 = \frac{\mu_1}{\sin r} = \frac{\theta_2}{\theta_1} = \frac{\nu_1}{\nu_2} = \frac{\lambda_1}{\lambda_2} \]

where:\n
- \( \mu_2 \) is the index of refraction of the second medium.
- \( \mu_1 \) is the index of refraction of the first medium.
- \( \theta_2 \) is the angle of refraction in the second medium.
- \( \theta_1 \) is the angle of incidence in the first medium.
- \( \nu_2 \) is the wavelength in the second medium.
- \( \nu_1 \) is the wavelength in the first medium.
- \( \lambda_1 \) is the wavelength in the first medium.
- \( \lambda_2 \) is the wavelength in the second medium.

In this case, we find that \( \lambda_2 > \lambda_1 \), which means that the light travels longer in the second medium.
But majority of the class time may get wasted in delivering the content from the text book. Instead of that if the student is given the text book content to be discussed a day earlier for home reading, then students will enter the class room with some basic information and a set of questions in their mind. The faculty may then gather all the questions at the beginning of the lecture so that the rest of the time can be aimed at delivering the lecture in such a manner as to clear all their doubts which could work as an effective tool for proper goal attainment, in the most time-effective manner.

**Bloom’s taxonomy**

This gives a standard flow chart for various levels of learning, according to which there are six levels of learning activities. These are Remembering, Understanding, Applying, Analysing, Evaluating and Creating. The nouns in Bloom’s taxonomy are replaced by action verbs in the modern definition of Bloom’s taxonomy.

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**Fig: Bloom’s taxonomy Old and New terminology.**

For example consider the study of Snell’s law.

**Remembering**: This answers all “What” type of questions. i.e.; what is Snell’s law and what are the parameters that it depends on, etc. Thus

\[
\frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r} = \frac{\vartheta_2}{\vartheta_1} = \frac{\nu_2 \lambda_1}{\nu_1 \lambda_2} = \frac{\lambda_1}{\lambda_2}
\]

**Understanding**: This level of learning looks for answers to all “Why” and “How” type of questions. For example why frequency should remain constant in this phenomenon of refraction? The answer to this question is that if the frequency were not constant, then the number of oscillations on either side of the boundary
యందంచవ.
యంతరం, 5893 Å సంకరల సకరల పంయడంతవవ. యంచొడి పండిత అషాషాందల సంఖల పంచబం. యంచొడి పండిత అషాషాండ సకరణం కరపం పగణన మారి పండిత అషాషాండ కొరాయి నంద ఉంంద. యంచొడి పండిత అషాషాండ వ్యవహార తకడం ఇంచబం. సకరణం, 5893 Å సంఖల పండిత అషాషాండ పగణన వడం మారి పండిత అషాషాండ కొరాయి నంద ఉంం. యంచొడి పండిత అషాషాండ వ్యవహార తకడం ఇంచబం. ఇ అకం కంచం: యంచొడి పండిత అషాషాండ, పండిత అషాషాండ మారి పండిత అషాషాండ సందను పై కడం సగం?

అనుసారం పండిత అషాషాండ సందను పై కడం మారి పండిత అషాషాండ సగం?

సందను పై కడం మారి పండిత అషాషాండ సగం?

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becomes unequal and the wave gets disconnected. So to keep the wave connected across the boundary, the frequency must remain constant across the boundary. One can also observe that $\lambda_{\text{Red}} > \lambda_{\text{Blue}} \Rightarrow r_{\text{Red}} > r_{\text{Blue}}$.

**Applying:** Once the equations are developed with proper understanding of what are constants and what are variables and why the parameters behave the way they do, one can apply them to specific systems. For example, what is the wavelength $\lambda$ of sodium light, with $\lambda = 5893\text{Å}$ in air, when passed through water of refractive index $\mu = 1.333$. That is $\lambda_2 = \lambda_1/\mu = 5893/1.33 = 4419.8\text{Å}$. From this example, one can identify that the wavelength of light changes from medium to medium. The wavelength $\lambda = 5893\text{Å}$ for sodium light is valid only in air or vacuum. Thus application only deals with mere substitution of numbers into equations for specific cases.

**Analysing:** At this stage, the equation is analysed in and out, by considering all the special cases and by searching answers to all “What if” type of questions. From the equation, one can infer that the angle of refraction for different colours is different. Thus refractive index for different colours is different for a given material. The question that arises now is: For what colour of light, is the refractive index of material defined? Is it the average of RI of all the visible spectrum? i.e., the RI of green light? The answer is that the refractive index was defined for sodium vapour lamp $\lambda = 5893\text{Å}$ as that was the only monochromatic light source first developed. What about temperature and pressure effects on RI? Here RI is defined at STP (Standard Temperature and Pressure).

From the equation, when $i = 0^\circ$, $\sin r = \frac{\sin i}{\mu_{12}} = 0 \Rightarrow r = 0^\circ$. Thus when $i = 0^\circ, r = 0^\circ$. Then for normal incidence what is the refractive index? $\mu_{12} = 0/0$ (undefined).

Thus, Analysis part gives the limits and limitations of the given concept. i.e., where it works and where it fails.

**Evaluating:** Once Analysis part is over, one can have full grip over the concept and be able to judge the validity of the concept under various circumstances. For example, do the light rays converge or diverge when a hollow convex lens is immersed in liquid medium? Since the outer medium is denser, the hollow convex lens behaves
Bloom’s taxonomy

ఓకేది మాత్రం ఉన్నాయి. అంగం దీన్ని కింద కనిపించడం ప్రయత్నం చేస్తాం. దీన్ని సాధనం చేస్తాం. హామే తెలిపాలను తెలియజేస్తాం. మరియు విద్యార్థి ప్రత్యేకంగా వినియోగించాలి. అనేక సమాధానాలు ఈ విధానం అనేక లింగాలలో ఉన్నాయి. దీనితో సమాధానాలు సమర్పించాలి. దీనిని లభించాలి. దీనిని ఉపయోగించాలి. అందువలసి దీనిని ప్రత్యేకంగా ఉపయోగించాలి. దీనిని సాధనం చేస్తాం.
quite opposite to that of a glass convex lens in air. Thus the light rays diverge on a hollow convex lens immersed in liquid medium.

**Creating:** Once the limits and limitations of a concept are understood, one can develop systems within the limits and limitations or alternately, concepts to overcome the limits and limitations. For example, To explain refractive index in case of normal incidence, one may need Fresnel equations. Once Fresnel equations are developed, one can easily identify that there will be lateral shift of various colours due to refraction, though the angle of refraction is zero. One can use this concept further in development of Quarter wave plates and Half wave plates.

One might have discussed all these concepts in a course, in the same or different order. But the advantage with Blooms Taxonomy is that there will be a systematic step by step level of increment in complexity of learning. Also one can put check marks on the exact level at which the student feels difficulty and up to what level the student is able to work with.

**Teaching, Learning and Evaluation tools:**

For each level of learning, one has to precisely define what are the teaching, learning and evaluation tools that are going to be used, for proper implementation of Bloom’s taxonomy.

In addition, there are three sub levels of learning in each class of Bloom’s taxonomy, namely cognitive domain, Affective domain and Psychomotor/ Systemic domain.

![Fig: Domains of learning.](image)
Bloom’s taxonomy

https://www.monroe.k12.ky.us/userfiles/905/bloom

spres.ppt
Bloom’s taxonomy

Here cognitive domain level deals with brain level activity. Affective domain level deals with heart level activity. Systemic domain level deals with the physical level activity. Let us now identify some pointers to use at various levels of Bloom’s taxonomy.

**Remembering level:** In remembering level, for Cognitive level activity, one may ask the student to go through a definition. For Affective level activity one may ask to recite a definition. For Systemic level activity, one may ask to write the imposition of the definition. For teaching at remembering level, one may use PPT or one may use display cards to write definitions. For evaluation purpose one may choose a quiz, or an activity involving matching the words.

**Understanding level:** For understanding level, one may use chalk and board type of teaching. For learning activity at understanding level, one may plan for a group discussion or flipped classroom method and for evaluation purpose one may ask to summarize the given concept.

**Application level:** For Application level of learning, one may develop prototype models or one may solve problems in classroom. One may give problems sheet or assignment to evaluate the application level of learning.

**Analysing level:** At this level one may ask to write a study report, one may plan for a field visit and finally may insist on preparing a report on the observations during the field trip. One may also conduct a written test with specific questions aimed at eliciting the required information expected from that particular field trip.

**Evaluating level:** At this level one may give a project work or ask to evaluate an existing project work. One may also plan Group discussion as a task and observe their analysis and evaluation skills.

**Creation level:** At this level one may need a project as an activity and project viva as an evaluation tool.

Here is a PPT containing a list of teaching, learning and evaluation tools one can use at various levels of learning.

Usage of computer has become mandatory in all fields nowadays for proper understanding and for saving time. Hence selection of activities and assessment tools based on computer activity would be a great choice.
Bloom's taxonomy

ఈ సంస్థానంలో లేని లాంటి సంస్థాపకుల సంఖ్య సమూహాన్ని తెలుసుకోవడం సాధ్యం లేదు. ఇది కంటినే ఆదరించబడిన మనం ప్రతి సంస్థానం సంశయం సంఖ్య ప్రదానం చేసేది. మాదిరి ఇది ఇతర మార్గాలు తెలించిన పద్ధతులు లేదా సంస్థానాలకు మాత్రం ఉండది. ఇతర సంస్థానాలకు వివిధ పద్ధతులు ఉండవచ్చు.

ఇది భావించవలసిన పద్ధతులు ప్రతి సంస్థానం సంఖ్య ప్రదానం చేసేది. ఇది మనం ప్రతి సంస్థానం సంఖ్య ప్రదానం చేసేది. ఇది ఇది ఇది ఇతర మార్గాలు తెలించిన పద్ధతులు లేదా సంస్థానాలకు మాత్రం ఉండది. ఇతర సంస్థానాలకు వివిధ పద్ధతులు ఉండవచ్చు.

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Teachers who make Physics boring are criminals

- Walter Lewin

ఇది భావించవలసిన పద్ధతులు ప్రతి సంస్థానం సంఖ్య ప్రదానం చేసేది. ఇది మనం ప్రతి సంస్థానం సంఖ్య ప్రదానం చేసేది. ఇది ఇది ఇది ఇతర మార్గాలు తెలించిన పద్ధతులు లేదా సంస్థానాలకు మాత్రం ఉండది. ఇతర సంస్థానాలకు వివిధ పద్ధతులు ఉండవచ్చు.
that enhances the exposure and confidence in both the faculty as well as the student.
Finally, though it may sound harsh, it is a famous quote from our beloved current era physics teacher Walter Lewin. Probably from his point of view, the crime committed could be against physics itself.

In this book, every chapter starts with objectives and outcomes followed by specific outcomes and future prospects. This would help teacher to create some enthusiasm in the student.

In the introduction section, extensive historic background has been given for the chapter contents. This also would trigger interest in some group of students. This also helps in understanding the step by step development of thought in arriving at the current syllabus content.

Some chapters like ultrasonics are provided with extensive details on applications. They may not be helpful at this instance; but presented now, keeping in view of the future perspectives like internships and project works that the student may need to take up in the future semesters.

Every topic is provided with supporting web links for further reading. For some topics interactive simulation links are also provided.

Relevant activities and facts are also provided wherever it is possible. Some thought provoking questions are also posed to enhance the interest of the students.

The exercise section yet to be developed further to meet the requirements of Bloom’s taxonomy framework. But still some relevant IIT JAM, HCU, AU, AKNU etc. entrance exam questions are provided to give an exposure to students in that direction. Some of the MCQs are provided with hints and solutions wherever it is possible. The evaluation part may be presented on par with Bloom’s taxonomy framework in the subsequent editions.

We earnestly hope that, this book may succeed in providing some direction and create interest in the subject in both the teacher and the student. If the book adds a pinch of salt and pepper to the entire purpose of “the ocean of learning” as quoted by Einstein, (training of brain to think), our efforts would have not been a waste, after all.

Thank you all very much.

- Authors
Future Perspectives

భషాసహిత అద్భుతత్వం
ప్రోత్సాహ నిర్ణయం

B.Sc. కాలంలో మూడు విషయాల పాఠశాలలు సంపాదించారు. ఈ నిర్ణయాన్ని జరిగిన సమయంలో ప్రతి విషయాన్ని మరిన్ని ప్రామాణికంగా పాఠశాలం తెలిసింది.

1. రామానంద శాస్త్రి

2. జి రామానంద శాస్త్రి

3. మినిస్త్రీనాయకుడు బాలవంద తప్పని అధ్యాత్మికం

4. మంది తప్పని అధ్యాత్మికం

5. ఇన్స్పిక్షనర్ బాలవంద పాతు / పాత పాత పాత

6. ప్రోత్సాహ నిర్ణయం తప్పని అనుసరించం

యొక్కప్పుడు పరిపాలన, సంస్కృతంతో అత్యంత అధ్యాత్మికం ప్రతి మాధ్యమానికంగా పాఠశాలం తెలిసింది.

సూచనలు నిర్ణయం, మాధ్యమానికం సంస్కృతం ప్రతి పాఠశాలం తెలిసింది పదార్థాన్ని పాఠశాలం ప్రతి పరిపాలన చేసింది. మాధ్యమానికం అనుసరించి ప్రతి పాఠశాలం తెలిసింది పదార్థాన్ని పాఠశాలం పరిపాలన చేసింది. అంటే అనుసరించి పాఠశాలం తెలిసింది పదార్థాన్ని పాఠశాలం పరిపాలన చేసింది. అంటే ప్రతి పాఠశాలం తెలిసింది పదార్థాన్ని పాఠశాలం పరిపాలన చేసింది. అంటే అనుసరించి పాఠశాలం తెలిసింది పదార్థాన్ని పాఠశాలం పరిపాలన చేసింది.
Branches of Physics

After B.Sc., physics divides into various sub branches as described below.

1. Classical mechanics
2. Statistical and thermal physics
3. Electromagnetic theory and Electrodynamics
4. Optics and spectroscopy
5. Condensed matter physics / Solid state physics
6. High energy physics and cosmology

Here classical mechanics deals with mechanics of materials i.e.; dynamics, kinematics, fluid dynamics, another extreme of this is the astrophysics. There dynamics and kinematics of astronomical objects is studied.

In statistical mechanics, the thermodynamic behaviour of the systems is explained by considering the microscopic (atomic/molecular level configuration) of the systems. At that microscopic level, all systems are identical and if there are only two particles, it will be mapped onto tossing a coin. If there are 6 atoms or molecules, then it will be mapped on to the dice problem. Thus every problem in statistical mechanics is mapped on to some multiple coin tossing and astonishingly, the observed experimental results map on to some combination of the outcome of the coin tossing. Now a days computers are used to simulate the outcome of large scale statistical systems.

Electrodynamics and electromagnetic theory is essentially useful in studying radiation from terrestrial and celestial objects. It is also useful in designing tools for communication system. It is useful in developing innovative electrical components.
పిచిత కార్యక్రమానికే అందించే లేదా లేమి మాత్రమే ప్రత్యేక కార్యక్రమానికే ప్రతిష్ఠితం చేస్తే. అందుకే ప్రత్యేకత ప్రతిష్ఠితం చేసిన లాభానికి ఉష్ణవయస్క సాధనాలు కార్యక్రమానికే ప్రతిష్ఠితం చేస్తే. సాధనాలు ప్రతిష్ఠితం చేస్తే ప్రతిష్ఠితం చేసిన లాభానికి ఉష్ణవయస్క సాధనాలు కార్యక్రమానికే ప్రతిష్ఠితం చేస్తే. సాధనాలు ప్రతిష్ఠితం చేసిన లాభానికి ఉష్ణవయస్క సాధనాలు కార్యక్రమానికే ప్రతిష్ఠితం చేస్తే. సాధనాలు ప్రతిష్ఠితం చేసిన లాభానికి ఉష్ణవయస్క సాధనాలు కార్యక్రమానికే ప్రతిష్ఠితం చేస్తే. సాధనాలు ప్రతిష్ఠితం చేసిన లాభానికి ఉష్ణవయస్క సాధనాలు కార్యక్రమానికే ప్రతిష్ఠితం చేస్తే. సాధనాలు ప్రతిష్ఠితం చేసిన లాభానికి ఉష్ణవయస్క సాధనాలు కార్యక్రమానికే ప్రతిష్ఠితం చేస్తే. సాధనాలు ప్రతిష్ఠితం చేసిన లాభానికి ఉష్ణవయస్క సాధనాలు కార్యక్రమానికే ప్రతిష్ఠితం చేస్తే. సాధనాలు ప్రతిష్ఠితం చేసిన లాభానికి ఉష్ణవయస్క సాధనాలు కార్యక్రమానికే ప్రతిష్ఠితం చేస్తే. సాధనాలు ప్రతిష్ఠితం చేసిన లాభానికి ఉష్ణవయస్క సాధనాలు కార్యక్రమానికే ప్రతిష్ఠితం చేస్తే. సాధనాలు ప్రతిష్ఠితం చేసిన లాభానికి ఉష్ణవయస్క సాధనాలు కార్యక్రమానికే ప్రతిష్ఠితం చేస్తే. సాధనాలు ప్రతిష్ఠితం చేసిన లాభానికి ఉష్ణవయస్క సాధనాలు కార్యక్రమానికే ప్రతిష్ఠితం చేస్తే. సాధనాలు ప్రతిష్ఠితం చేసిన లాభానికి ఉష్ణవయస్క సాధనాలు కార్యక్రమానికే ప్రతిష్ఠితం చేస్తే. సాధనాలు ప్రతిష్ఠితం చేసిన లాభానికి ఉష్ణవయస్క సాధనాలు కార్యక్రమానికే ప్రతిష్ఠితం చేస్తే. సాధనాలు ప్రతిష్ఠితం చేసిన లాభానికి ఉష్ణవయస్క సాధనాలు కార్యక్రమానికే ప్రతిష్ఠితం చేస్తే. సాధనాలు ప్రతిష్ఠితం చేసిన లాభానికి ఉష్ణవయస్క సాధనాలు కార్యక్రమానికే ప్రతిష్ఠితం చేస్తే. సాధనాలు ప్రతిష్ఠితం చేసిన లాభానికి ఉష్ణవయస్క సాధనాలు కార్యక్రమానికే ప్రతిష్ఠితం చేస్తే. సాధనాలు ప్రతిష్ఠితం చేసిన లాభానికి ఉష్ణవయస్క సాధనాలు కార్యక్రమానికే ప్రతిష్ఠితం చేస్తే. సాధనాలు ప్రతిష్ఠితం చేసిన లాభానికి ఉష్ణవయస్క సాధనాలు కార్యక్రమానికే ప్రతిష్ఠితం చేస్తే. సాధనాలు ప్రతిష్ఠితం చేసిన లాభానికి ఉష్ణవయస్క సాధనాలు కార్యక్రమానికే ప్రతిష్ఠితం చేస్తే.
Optics is classified into geometrical optics, wave optics and photonics. Especially in photonics, lasers have occupied a special place due to their wide range of varieties in their design, development and application. Spectroscopy essentially deals with the light matter interactions. The rest of the optics is also a simple consequence of light matter interaction. But there the nature of light is studied. In spectroscopy, the nature of material is studied.

Condensed matter physics deals with condensed phase of solids and liquids. There mechanical, optical, electrical, magnetic, low temperature and high temperature behaviour of materials is studied. Nanotechnology is a sub branch of condensed matter physics which established its uniqueness in terms of variety of structures, properties and applications.

High energy physics deals with nuclear and elementary particles. Their production, behaviour, detection of them from on earth and those coming from outer space etc. leads to the study of origin of universe. Cosmology deals with the study of radiation from distant galaxies and stars in order to estimate their chronological development.

Quantum mechanics is the underlying thread that connects all the modern branches of physics. It studies the microscopic behaviour of systems by considering uncertainty principle into account. Thus quantum mechanics has created its sub branches in all of the above namely Quantum statistics, quantum electrodynamics, quantum optics etc. condensed matter physics and elementary particle physics are essentially direct applications of quantum mechanics.

Relativistic mechanics describes high velocity extreme of physics as mentioned in the instructions to students. There one may see relativistic electrodynamics, relativistic classical mechanics, relativistic quantum mechanics, relativistic quantum electrodynamics etc.

Physics Career

After knowing various branches of physics, one may look into various fields of career after B.Sc. Physics. They are broadly classified into three namely, Academics, Research and Employment.
Physics Career

విషయం:


23 IITs, 31 NITs, 47 రేండవల చిరిత్రలు, 7 ISERs, 3BITS ఎందుకు M.Sc. ప్రత్యేకించిన విషయాలు. BITS యొక్క IIT JAM నంతరం ఇంకా సంస్థలు, BITS అందుకు CSIR-CET అవసరం వేస్తే ఆధారంగా అంటారు.

Academics:

After B.Sc., one may go for M.Sc. in physics. To go for M.Sc., various institutes conduct their own entrance exam apart from IIT JAM, a common entrance exam for all IITs M.Sc. program. Further one may take up M.Tech. or PhD. By writing GATE or CSIR-NET respectively.

There are 23 IITs, 31 NITs, 47 Central universities, 7 IISERs, 3BITSs, 1 IISc and 20+ other academic institutes of national importance that offer M.Sc. in Physics. IITs, NITs, IISc and IISERs offer admission based on IIT JAM Score. Some of them also conduct their own entrance exam apart from that. Central universities also conduct common entrance test named CU-CET. Based on the score, they offer admission in various institutes. BITS conducts BITSAT entrance exam.

DST (Department of Science and Technology) offers INSPIRE scholarship to university toppers in B.Sc. to go for higher studies. Once get selected for that scholarship, students get it as long as they continue their academics without a break.

After M.Sc., one may choose research or academics or employment as their options. To go for teaching, one needs B.Ed. to teach up to 12th class according to NEP (New Educational Policy). To teach beyond 12th class, one needs PhD and NET/SET according to UGC. So one should plan accordingly to opt teaching field as career.

Some specializations apart from regular Physics at Masters level (M.Sc./M.tech.) are as follows. Astrophysics, Geophysics, Oceanography, Atmospheric sciences, Environmental sciences, Agricultural Physics (jari.res.in), Biophysics, Materials science, Optoelectronics, Computational Physics etc.

Research:

Research is broadly classified into three types, namely theoretical physics, Experimental physics and computational physics. Here computational physics acts as a bridge between the other two. To opt for research, one has to do M.Sc. Then one may go to above mentioned academic institutes to join PhD. Otherwise there are a whole host of exclusive research institutes that offer admission for research. Some of the research institutes of national importance are, DST (Department of Science and Technology), DAE (Department of Atomic Energy), DRDO (Defence Research Development Organization), CSIR (Council of Scientific and Industrial Research), DoS (Department of...
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Space). One may see simple Wikipedia pages to get the list of their constituent institutes. There are, in total, more than 250 institutes in those clusters. They offer admission as a JRF (Junior Research Fellow) based on the score in GATE/JEST (Joint Entrance Screening Test.) They also give admission based on JEST (Joint Entrance Screening Test) score. To get a PhD degree for the work done in those institutes, one need to register as an external research scholar in any of the associated academic institutes. DAE has an exclusive set of three universities to support academics apart from research. If anybody completes PhD from those research institutes, they may join as research associates in the same institutes as they are already trained for the purpose. Usually after 5-8 years of service as JRF+SRF+RA, if their performance is satisfactory, they may get absorbed as permanent employees through some standard procedures.

DST offers KVPY scholarship exam to promote students for PhD. If qualified in that test, students are eligible for a fellowship of Rs. 5000/- during B.Sc., Rs. 7000/- during masters and further they are eligible for INSPIRE fellowship during PhD.

Apart from this, Indian Academy of Sciences conduct summer research programs for BSc and MSc students to inculcate research aptitude from the beginning of Bachelor’s degree.

Indian Association of Physics Teachers (IAPT) conducts NSEP (National Standard Examination in Physics) of which top 10 rankers will be offered scholarships to take up Physics as career.

Employment:
Employment after BSc. is usually not suggested except for a few firms that offer exclusive employability exclusively for Physics graduates. Satish Dhavan Space Center (SHAR) - ISRO offers 30 jobs as technical assistants exclusively for B.Sc. MPC students every year. There are some other firms like BHEL (Bharath Heavy Electrical Limited), HAL (Hindustan Aeronautics Limited), Ordinance factories at that offer jobs after a Master’s degree in Physics. Aluminium industries offer technical assistant jobs for MPC students. BARC (Babah Atomic Research Center) Mumbai offers radiographer training program for BSc and MSc level physics students.
So plan well beforehand to make a meaningful utilization of time for your own future life.

All the best

- Authors
A goal without a plan is just a wish.

Antoine de Saint-Exupéry

Antoine Marie-Jean-Baptiste Roger, comte de Saint-Exupéry was a French writer, poet, aristocrat, journalist, and pioneering aviator. He became a laureate of several of France's highest literary awards.

By failing to prepare, you are preparing to fail.

Benjamin Franklin
LEARNING RESOURCES
అభసన శిక్షణసభము
For basic understanding of concepts and for enhanced visualization of the concepts, several online resources are available and a few more effective resources are presented here.

Reading Resources:

<table>
<thead>
<tr>
<th>Sl No</th>
<th>Resource Link</th>
<th>QR Code</th>
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<tbody>
<tr>
<td>1.</td>
<td><strong>Hyper Physics</strong>: HyperPhysics is an exploration environment for concepts in physics which employs concept maps and other linking strategies to facilitate smooth navigation. <a href="http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html">http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html</a></td>
<td><img src="EmbeddedQR1" alt="QR Code" /></td>
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<td>2.</td>
<td><strong>Physics Hyper Textbook</strong>: The Physics Hypertextbook is the intellectual property of Glenn Elert <a href="https://physics.info/">https://physics.info/</a></td>
<td><img src="EmbeddedQR2" alt="QR Code" /></td>
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<td>3.</td>
<td><strong>Lumen learning</strong>: Lumen Learning provides a simple, supported path for faculty members to adopt and teach effectively with open educational resources (OER) <a href="https://courses.lumenlearning.com/physics/">https://courses.lumenlearning.com/physics/</a></td>
<td><img src="EmbeddedQR3" alt="QR Code" /></td>
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<td>5.</td>
<td><strong>Libre Texts</strong>: Classical Mechanics. <a href="https://phys.libretexts.org/@go/page/13940">https://phys.libretexts.org/@go/page/13940</a></td>
<td><img src="EmbeddedQR5" alt="QR Code" /></td>
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<td>6.</td>
<td><strong>Internet Archive</strong>: The Internet Archive, a 501(c)(3) non-profit, is building a digital library of Internet sites and other cultural artifacts in digital form. <a href="https://archive.org/">https://archive.org/</a></td>
<td><img src="EmbeddedQR6" alt="QR Code" /></td>
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<td>7.</td>
<td><strong>National Digital Library</strong>: National Digital Library of India (NDLI) is a virtual repository of learning resources which is not just a repository with search/browse facilities but provides a host of services for the learner community. <a href="https://ndl.iitkgp.ac.in/">https://ndl.iitkgp.ac.in/</a></td>
<td><img src="EmbeddedQR7" alt="QR Code" /></td>
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## LEARNING RESOURCES

### Video Lectures:

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<tr>
<th>SN</th>
<th>Title and Link</th>
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<tbody>
<tr>
<td>1.</td>
<td>8.01x - MIT Physics I: Classical Mechanics Lectures by Walter Lewin. <a href="https://www.youtube.com/playlist?list=PLyQSN7X0ro203puVhQsmCj9qhLFQ-As8e">https://www.youtube.com/playlist?list=PLyQSN7X0ro203puVhQsmCj9qhLFQ-As8e</a></td>
<td><img src="https://www.youtube.com/playlist?list=PLyQSN7X0ro203puVhQsmCj9qhLFQ-As8e" alt="QR Code" /></td>
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<td>2.</td>
<td>8.03 - MIT Physics III: Vibrations and Waves Lectures by Walter Lewin. <a href="https://www.youtube.com/playlist?list=PLyQSN7X0ro22WeXM2QCKJm2NP_xHpGV89">https://www.youtube.com/playlist?list=PLyQSN7X0ro22WeXM2QCKJm2NP_xHpGV89</a></td>
<td><img src="https://www.youtube.com/playlist?list=PLyQSN7X0ro22WeXM2QCKJm2NP_xHpGV89" alt="QR Code" /></td>
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<td>3.</td>
<td>MIT 8.03SC Physics III: Vibrations and Waves, Fall 2016 by Yen Jie Lee <a href="https://www.youtube.com/playlist?list=PLUl4u3cNGP61R5sPDPKvfcFlu95wSs2Kx">https://www.youtube.com/playlist?list=PLUl4u3cNGP61R5sPDPKvfcFlu95wSs2Kx</a></td>
<td><img src="https://www.youtube.com/playlist?list=PLUl4u3cNGP61R5sPDPKvfcFlu95wSs2Kx" alt="QR Code" /></td>
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<td>5.</td>
<td>Introduction to Classical Mechanics Prof. Anurag Tripathi IIT Hyderabad <a href="https://www.youtube.com/playlist?list=PLyqSPqZTE6M_d9f-9fKxUQYR1qI5YEnSz">https://www.youtube.com/playlist?list=PLyqSPqZTE6M_d9f-9fKxUQYR1qI5YEnSz</a></td>
<td><img src="https://www.youtube.com/playlist?list=PLyqSPqZTE6M_d9f-9fKxUQYR1qI5YEnSz" alt="QR Code" /></td>
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<td>7.</td>
<td>Waves and Oscillations by M.S. Santhanam IISER Pune <a href="https://www.youtube.com/playlist?list=PLyqSPqzTE6M9X7oRXliYM8t0aaR_NOCsd">https://www.youtube.com/playlist?list=PLyqSPqzTE6M9X7oRXliYM8t0aaR_NOCsd</a></td>
<td><img src="https://www.youtube.com/playlist?list=PLyqSPqzTE6M9X7oRXliYM8t0aaR_NOCsd" alt="QR Code" /></td>
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### Reference Books

Simulation resources:

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<th>SL No</th>
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<tr>
<td>1.</td>
<td>Amrita V Lab &lt;br&gt;<a href="https://vlab.amrita.edu/">https://vlab.amrita.edu/</a></td>
<td><img src="image" alt="QR Code" /></td>
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<td>2.</td>
<td>JAVA Lab &lt;br&gt;<a href="https://javalab.org/en/category/mechanics_en/">https://javalab.org/en/category/mechanics_en/</a></td>
<td><img src="image" alt="QR Code" /></td>
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<td>3.</td>
<td>Molecular Workbench &lt;br&gt;<a href="http://mw.concord.org/modeler/showcase/">http://mw.concord.org/modeler/showcase/</a></td>
<td><img src="image" alt="QR Code" /></td>
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<td>4.</td>
<td>My Physics Lab &lt;br&gt;<a href="https://www.myphysicslab.com/">https://www.myphysicslab.com/</a></td>
<td><img src="image" alt="QR Code" /></td>
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<tr>
<td>5.</td>
<td>Physics Classroom &lt;br&gt;<a href="https://www.physicsclassroom.com/Physics-Interactives">https://www.physicsclassroom.com/Physics-Interactives</a></td>
<td><img src="image" alt="QR Code" /></td>
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<tr>
<td>6.</td>
<td>Physics Classroom &lt;br&gt;<a href="https://www.physicsclassroom.com/Physics-Interactives/Waves-and-Sound">https://www.physicsclassroom.com/Physics-Interactives/Waves-and-Sound</a></td>
<td><img src="image" alt="QR Code" /></td>
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<tr>
<td>7.</td>
<td>O-Physics &lt;br&gt;<a href="https://ophysics.com/">https://ophysics.com/</a></td>
<td><img src="image" alt="QR Code" /></td>
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<tr>
<td>9.</td>
<td>Daniel A. Russel &lt;br&gt;<a href="https://www.acs.psu.edu/drussell/demos.html">https://www.acs.psu.edu/drussell/demos.html</a></td>
<td><img src="image" alt="QR Code" /></td>
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<tr>
<td>10.</td>
<td>University of Maryland &lt;br&gt;<a href="https://lecdem.physics.umd.edu/component/tags/tag/oscillations-waves.html">https://lecdem.physics.umd.edu/component/tags/tag/oscillations-waves.html</a></td>
<td><img src="image" alt="QR Code" /></td>
</tr>
</tbody>
</table>
Full Course links with supporting material.


3. https://onlinecourses.nptel.ac.in/noc21_ph32/preview

4. https://nptel.ac.in/courses/115/105/115105098/


6. https://nptel.ac.in/courses/115/106/115106119/

7. https://nptel.ac.in/courses/122/105/122105023/

8. https://onlinecourses.nptel.ac.in/noc19_ph18/preview
Reference Books

4. Pancella P.V., Humphrey M. - Idiot's Guides – Physics
PART-1
MECHANICS
మేనిక జీవనం
మనం మనవుల మాత్రము

మనం మనవుల మాత్రము సమానం రే, అందులో కూడా మనవు మనవుల మాత్రము సమానం రే. 

10 శతాబ్దం ప్రస్తుతిలో శతాబ్దం ప్రస్తుతిలో శతాబ్దం ప్రస్తుతిలో శతాబ్దం ప్రస్తుతిలో శతాబ్దం ప్రస్తుతిలో.

https://archive.org/details/thevaiasesikasut00kanauoft
Introduction

Cause and Effect relation
The natural philosophy has evolved with the basic principle of causality or the quest (search) for cause and effect relation in every phenomenon. Maharshi Kanada in his “Viseshika Sutra”, a collection of 10 books has described about the cause and effect relation around 600BC itself. The book also manifests some of the fundamentals of motion which stands as guiding principles for Newton’s laws of motion.

Book-1 and Book-5 in them are more connected to mechanics and dynamics of systems. Book-2 especially deals with space and time. There both space and time were considered as substances. Instead if they are read as entities the description will be more meaningful. The interesting part is that, some of the descriptions made at around 2500 years back are on par with the modern science resolutions made out of detailed experimental and theoretical confirmations. Thus one needs a serious look back into these scriptures in a more systematic and scientific manner to make more meaningful resolutions on whether to accept or reject them.
Introduction

https://www.umass.edu/wsp/method/history/outline/simplicity.html

https://archive.org/details/newtonspmathema00newtrich

T-Corner

If you prove the cause, you at once prove the effect; and conversely nothing can exist without its cause.
- Aristotle
In the just later generations Socrates - Plato – Aristotle trio of a teacher and a disciple has made a more systematic efforts towards the modern science which is now called as Metaphysics. Aristotle has classified cases into 4 levels with the following questions

<table>
<thead>
<tr>
<th>Sl No</th>
<th>Cause</th>
<th>Question</th>
<th>Details</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Material</td>
<td>What is it made of?</td>
<td>Material details</td>
<td>Wood.</td>
</tr>
<tr>
<td>2</td>
<td>Formal</td>
<td>How does it look like?</td>
<td>Design details</td>
<td>With 4 legs and a plank.</td>
</tr>
<tr>
<td>4</td>
<td>Final</td>
<td>What is its purpose?</td>
<td>Application details</td>
<td>Dining.</td>
</tr>
</tbody>
</table>

Newton also followed the footsteps of these giants and developed more systematic principles to explain the motion of bodies. He said that nature would never be complicated in generating effects. Simple causes are only responsible for the observed effects. One need to find out those which are true and sufficient to explain a phenomenon.

Thus entire Newton’s mechanics boils down to the study of cause and effect relations in a more systematic and measurable ways.

*We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances.*

*Isaac Newton*

*If I have seen further than others, it is by standing upon the shoulders of giants.*

*Isaac Newton*

**Did You Know?**

Newton gave the above quote, while describing his optics research.
Introduction

The earth's climate is changing due to several factors. One of the major factors is the increase in greenhouse gases in the atmosphere. This has led to a significant increase in temperatures, which are expected to continue rising in the future. (Source: IPCC, 2021)

The increase in temperature is due to the emission of greenhouse gases, primarily carbon dioxide (CO₂). The burning of fossil fuels, deforestation, and industrial activities are the main sources of these emissions. (Source: NASA, 2020)

The effects of climate change are already being felt around the world. Rising sea levels, more frequent and intense weather events, and changes in precipitation patterns are just a few of the impacts. (Source: World Meteorological Organization, 2021)

In order to mitigate these effects, it is crucial to reduce our carbon footprint. This can be achieved through various means, such as increasing the use of renewable energy sources, improving energy efficiency, and reducing waste and pollution. (Source: International Energy Agency, 2021)
Independent and Dependent variables

In basic physics there will be causes and effects. Based on that independent and dependent variables exist. In mathematical terms, all causes fall in domain and all effects fall in range of a function. Always effect depends on the cause. (eg: Newton’s second law says rate of change of momentum is proportional to applied force. Means, $\frac{dp}{dt} \propto F \Rightarrow \frac{dp}{dt} = \frac{1}{k} F$ but not $F \propto \frac{dp}{dt}$) This topic is discussed in the first chapter in detail.

Also in physics while plotting graphs, always ‘cause’ is taken along x-axis and ‘effect’ is taken along y-axis. To be precise independent variable is taken along x-axis and dependent variable is taken along y-axis. For example consider displacement Vs. Time curve for uniform velocity case. Here if it is asked “What is the displacement?” one would immediately ask, “at what time?” Thus displacement information depends on time. In other words, displacement is a function of time. Thus time is taken along x-axis and displacement is taken along y-axis. Similar is the case with Ohm’s law. There current is the dependent variable and voltage is the independent variable. If there exists potential difference, current flows. Current depends on voltage. But not vice versa. Thus $(I \propto V)$ and $I = \frac{1}{R} V$. But the other way $V \propto I$ is not a meaningful expression in Physics.

![Fig: Plot of displacement Vs. time for uniform velocity and V-I for Ohm’s law.](image)

Similar is the case with stress and strain in elasticity. Ie; $Strain \propto Stress$. And $Strain = \frac{1}{modulus} Stress$. Here stress is the applied force per unit area and strain is the relative amount of deformation produced. Thus stress is the independent variable and strain is the dependent variable. Or Stress is the cause and Strain is the effect. Thus the plot should have stress on x-axis and strain on y-axis.
చెప్పిన పదార్థం

ఎటుక ప్రమాణం

చాలా సంభవించిన సమయంలో, అయితే ప్రతిభ ప్రతి సంభవించే పదార్థం ప్రత్యేకంగా తన సంయుక్తం లోహాలు నిర్మాణం చేసింది. దీనిని ఎంపికింది, అందులో ప్రతి పదార్థం నిర్మాణానికి కూడా వారిని ఉపయోగం చేస్తాం. అందువల్ల పదార్థం కొనసాగింది. ఆమెరికన్ పదార్థాలను తోడ్డి చేయడానికి, ఇది అత్యంత అధికంగా పవిత్రం. ఎందుకంటే పదార్థం కొనసాగింది, అంటే పదార్థాన్ని నిర్మించడానికి రెండు రీతులు ఉండవచ్చు. పదార్థం మూలపదార్థాలు నిర్మించడానికి రెండు రీతులు ఉండవచ్చు. రెండు రీతులు తన పదార్థాన్ని ఉపయోగించడానికి రెండు రీతులు ఉండవచ్చు. ఇది అంటే పదార్థం నిర్మించడానికి రెండు రీతులు ఉండవచ్చు. మరొక రీమియం మూలపదార్థాన్ని ఉపయోగించడానికి రెండు రీతులు ఉండవచ్చు. ఇది మరొక రీమియం పదార్థం నిర్మించడానికి రెండు రీతులు ఉండవచ్చు. ఇది మరొక రీమియం పదార్థం నిర్మించడానికి రెండు రీతులు ఉండవచ్చు. ఇది మరొక రీమియం పదార్థం నిర్మించడానికి రెండు రీతులు ఉండవచ్చు. ఇది మరొక రీమియం పదార్థం నిర్మించడానికి రెండు రీతులు ఉండవచ్చు. ఇది మరొక రీమియం పదార్థం నిర్మించడానికి రెండు రీతులు ఉండవచ్చు.
But in our lower classes we might have come across the stress strain curve in the other way. Ie; strain is taken along x-axis and stress is taken along y-axis. This is because in that case the slope of the curve Stress/Strain directly gives the modulus of elasticity of the system. This is more useful term for Engineers which can be obtained by direct calculation of slope. Thus standard Physics textbooks and Engineering text books plot the stress-strain curve as shown below.

![Stress-Strain curve](image)

**Fig: Stress-Strain curve Physicist’s version and Engineer’s version.**

In some experiments like stress relaxation, how stress required reduces to maintain a constant strain over a fixed time is studied. In that case, stress is dependent variable and strain is independent variable.

![Stress Relaxation](image)

**Fig: Stress Relaxation experiment.**

Concept of Energy

According to Kanada, some sort of energy binds atoms to form body and the world. At the time of dissolution, this energy will be set free. It is also responsible for all activity in the nature. According to
Concept of Energy

\[ W_{12} = \int_{1}^{2} F \cdot dx = \int_{1}^{2} -\frac{dV}{dx} \, dx = \int_{1}^{2} -dV = -V_{1}^{2} = -(V_{2} - V_{1}) \]

\[ T_{2} - T_{1} = V_{1} - V_{2} \Rightarrow T_{1} + V_{1} = T_{2} + V_{2} \Rightarrow T + V = \text{Const.} \]

where \( T \) is the temperature, \( V \) is the volume, and \( P \) is the pressure.

\[ \frac{\partial V}{\partial x} = 0 \Rightarrow -F = 0 \Rightarrow \frac{dP}{dt} = 0 \Rightarrow P = \text{constant in time} \]

\[ \frac{\partial E}{\partial \theta} = 0 \Rightarrow \frac{\partial}{\partial t} \left( \frac{\partial E}{\partial \omega} \right) = 0 \Rightarrow \frac{\partial}{\partial t} \left[ \frac{1}{2} \omega^{2} \right] = 0 \]

\[ \Rightarrow \frac{\partial}{\partial t} \left( I \omega \right) = 0 \Rightarrow \frac{\partial L}{\partial t} = 0 \Rightarrow L = \text{Const. in time} \]
Aristotle also energy of mind is the essence of life. Modern definition of energy was given by Leibnitz but with the name vis-visa which means living force. The name energy was coined by Thomas Young in 1802.

**Conservation Laws**

There are three conservation laws based on the nature of energy in various phenomenon. They are as follows.

1. **Homogeneity in time:** This implies that the energy of the system under consideration must remain same for all times. i.e. \( \frac{\partial E}{\partial t} = 0 \Rightarrow E \) is constant in time.

Consider the work done in moving an object from point-1 to point-2 by the application of force \( F \). Here the work done is given by

\[
W_{12} = \int_{1}^{2} F \cdot dx = \int_{1}^{2} m \frac{dv}{dt} \cdot v \cdot dt = \int_{1}^{2} mv \cdot dv = \frac{1}{2} mv^2 \bigg|_1^{2} = T_2 - T_1
\]

If the force is conservative, one can write it as negative gradient of potential

\[
W_{12} = \int_{1}^{2} F \cdot dx = \int_{1}^{2} - \frac{\partial V}{\partial x} \cdot dx = \int_{1}^{2} -dV = -V_1^2 = -(V_2 - V_1)
\]

Thus \( T_2 - T_1 = V_1 - V_2 \Rightarrow T_1 + V_1 = T_2 + V_2 \Rightarrow T + V = \text{Const.} \)

This equation tells us that the total energy of the system remains same at various times. Also if force is zero, then \( T_2 - T_1 = 0 \) i.e; Kinetic energy alone conserves if external force is zero.

Eg: Keep a stone on a wall. The potential energy of the stone remains same as long as it is kept at that height. Thus if homogeneity (uniformity) of energy in time exists, one can say energy conserves. The same applies even for chemical reactions before and after; Elastic collisions before and after, i.e.; the total energy remains constant.

2. **Homogeneity in space:** This implies energy must remain same at various locations in space. Since the energy by virtue of its state of a system is called potential energy, One can write

\[
\frac{\partial V}{\partial x} = 0 \Rightarrow -F = 0 \Rightarrow \frac{dP}{dt} = 0 \Rightarrow P = \text{constant in time}
\]
ప్రస్తుత విషయం

ప్రస్తుతం ప్రతి వర్ణం ప్రకారం సంఖ్య ఉంది. వాడకం కొండ రాత్రి నమ్మకం రాత్రితో ఏండు ఉంటానికి సంబంధం ఉంది. (దానికి పొడిగా ఉంటాయి). అభిమానం ఉంటూ కూడా లేదు సంచారం లేదు ప్రామాణికం ఉంటానికి లభించింది. (సమయం రక ప్రతి వర్ణం మాత్రమే సంచారం ఉంటాయి)

అనేక కాలం ప్రదర్శించినను వెలుప చివరి నిషేధం చేసినది. రెండు సమానం మాత్రమే సంప్రదాయ లేదా తనం జాతి కూడా నిషేధం చేసినది. చదువులు అంటే ఈ సమాధానం అంటే తాడి ప్రతి వర్ణం ఉంటానికి అంశం ఉంటాయి.

* T-Corner *

https://archive.org/details/Mechanics3eLandauLifshitz
https://www.feynmanlectures.caltech.edu/II_19.html

ఎర్వులు మాటలు

మిలియన్ల సంఖ్యలు ప్రత్యేకంగా ఎర్వులు మాటలు లేదా ఉండవచ్చు ప్రత్యేకంగా మాటలు మాటలు మాట మాట మాట మాట మాట మాట మాట.

మాట మాట

అలుగులు మాటలు

ప్రత్యేకంగా యంత్రత్వ మాటలు అందులో పండించవచ్చు. మాటలను యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం యంత్రం #

12
Here we have used the condition that Force can be written as negative gradient of potential. Thus homogeneity in space leads to conservation of momentum. In other words, if the energy at various locations is space doesn’t change, then the momentum of the system remains constant at all those locations.

Eg: Keep a charge on a uniformly charged sheet. The potential experienced by the charge remains same everywhere on the sheet. So it won’t experience any force while traversing on the sheet.

3. **Isotropy in space**: This implies that the energy is isotropic in space. Ie; if the energy remain constant in all directions at a particular distance from a point. Or energy must be independent of angle. Then

\[
\frac{\partial E}{\partial \theta} = 0 \Rightarrow \frac{\partial}{\partial t} \left( \frac{\partial E}{\partial \omega} \right) = 0 \Rightarrow \frac{\partial}{\partial t} \left[ \frac{\partial}{\partial \omega} \left( \frac{1}{2} I \omega^2 \right) \right] = 0
\]

\[
\Rightarrow \frac{\partial}{\partial t} (I \omega) = 0 \Rightarrow \frac{\partial L}{\partial t} = 0 \Rightarrow L = \text{Const. in time}
\]

Thus if energy is independent of angle, angular momentum conserves.

Eg: Consider a point positive charge. The force experienced by any test positive charge, around the point charge is independent of direction (spherically symmetric potential). Then the orbital angular momentum of the test charge remains constant. (if at all if it executes circular motion around the point charge.)

4. Here Angular momentum is a vector quantity. There exists a scalar quantity with same dimensional formula as angular momentum namely “Action” \( S \) which is a product of Energy and time. The significance of action was well described in Principle of least action. That says every physical phenomenon take place keeping action to be minimum.

**Symmetry laws**

The symmetry laws that govern the mechanical systems are the inversion symmetry and the time reversal symmetry.
Symmetry laws

We have $v(t) = \frac{ds}{dt} \Rightarrow T(v(t)) = v(-t) = -\frac{ds}{dt} = -v(t)$

and $a(t) = \frac{d^2s}{dt^2} \Rightarrow T(a(t)) = a(-t) = \frac{d^2s}{d(-t)^2} = \frac{d}{d(-t)} \left( \frac{ds}{d(-t)} \right) = +a(t)$
Inversion symmetry

Polar vectors change sign upon spatial inversion. Inversion operation is the mirror image operation along all the three spatial coordinates. Consider displacement vector of an object along x-axis. Apply inversion operation on it. Ie; reflection along x-axis followed by reflection along y-axis followed by reflection along z-axis. The resultant vector changes sign upon the three mirror image operations along the three coordinate axes.

![Inversion operation on Polar Vector](image)

Fig: Inversion operation on Polar Vector

Pseudo vectors do not change sign upon inversion operation. Pseudo vectors are resultant of cross product of two polar vectors. Eg: Angular momentum. Remember that the vector direction is decided after mirror imaging the object alone.

![Inversion operation on a pseudo vector](image)

Fig: Inversion operation on a pseudo vector

Time reversal symmetry

The vectors which are even order in time do not change sign upon time inversion and the vectors which are odd order functions of time do change sign upon inversion of time.

Eg: We have \( v(t) = \frac{ds}{dt} \Rightarrow T(v(t)) = v(-t) = \frac{ds}{d(-t)} = - \frac{ds}{dt} = -v(t) \)

and \( a(t) = \frac{d^2s}{dt^2} \Rightarrow T(a(t)) = a(-t) = \frac{d^2s}{d(-t)^2} = \frac{d}{d(-t)} \left( \frac{ds}{d(-t)} \right) = +a(t) \)

This time reversal can be verified experimentally by recording a video of a scene and by playing it backwards. In such reverse play, forward moving objects look as if they are moving backwards (velocity
In IIT JAM there will be one question on these symmetry elements.

<table>
<thead>
<tr>
<th>Sl No</th>
<th>Physical Quantity</th>
<th>Parity</th>
<th>T-Symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Displacement</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>2</td>
<td>Velocity</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>3</td>
<td>Acceleration</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>4</td>
<td>Angular momentum</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>5</td>
<td>Torque</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>6</td>
<td>Energy</td>
<td>+</td>
<td>+</td>
</tr>
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<td>Curl, Gradient</td>
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<td>+</td>
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<td>D, E, P, V, Magnetic current</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>9</td>
<td>B, H, M, A, Electric current</td>
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<td>−</td>
</tr>
<tr>
<td>10</td>
<td>Poynting vector</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>11</td>
<td>$\varepsilon, \varepsilon_0, \chi_e, \mu, \mu_0, \chi_m$</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>12</td>
<td>Electric conductivity</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>13</td>
<td>Magnetic Conductivity</td>
<td>−</td>
<td>−</td>
</tr>
</tbody>
</table>
 reverses) and if the original object is accelerating in forward direction, it looks as if it is decelerating (velocity reducing) in backward direction. This implies acceleration is still directed in forward direction.

These cause and effect relations, symmetry laws along with conservation laws govern the behaviour of vectors and scalars in Newtonian mechanics. One must verify these at every stage of derivations and analysis in order to avoid mistakes. Ie; one must verify whether cause and effect are clearly identified or not. One must verify whether a pseudo vector remains as a pseudo vector or not, before and after a process. One must also verify what all conservation laws are applicable to a given system and verify their applicability before and after the execution of every process.

Basics of vectors

A vector is a Physical quantity which has both magnitude and direction.

1. A vector may be represented as \( \vec{A}, \vec{\hat{A}}, \hat{\vec{A}} \), and its magnitude as \( |\vec{A}|, |\vec{\hat{A}}|, |\hat{\vec{A}}|, \hat{\vec{A}} \).
2. Magnitude of vector will never be negative.
3. A null vector has a magnitude zero. It is represented as a point in geometry.
4. A null vector may be considered parallel \( (\vec{\hat{A}} \times \vec{0} = \vec{0}) \) or perpendicular \( (\vec{\hat{A}} \cdot \vec{0} = 0) \) to any vector. Since both cannot happen simultaneously, it is assumed neither.
5. Direction could be positive or negative. Ie; sign comes only by direction.
6. As long as direction and magnitude of the vector remains the same, one can shift it up, down, left or right. This doesn’t affect the nature or physics of the vector.
7. In coordinate space, usually up, right are positive directions. Down and left are negative directions. Accordingly vectors also take sign. If the coordinate system is defined differently, one may have sign convention in accordance with the coordinate system definitions.
8. In the case of Coulomb’s law, the magnitude of force is given by product of magnitudes of charges divided by the square of distance between them.
Basics of vectors

\[ |F| = k \frac{|q_1||q_2|}{r^2} \]

The magnitude of a vector can be calculated using the formula above, where \( k \) is a constant, \( |q_1| \) and \( |q_2| \) are the magnitudes of the vectors, and \( r \) is the distance between their points. This formula is useful in various applications, including physics and engineering.

\[ |E| = k \frac{|q|}{r^2} \quad \vec{E} = |E| \hat{r} \]

In the context of electric fields, the magnitude of the electric field \( |E| \) is directly proportional to the charge \( q \) and inversely proportional to the square of the distance \( r \). The direction of the electric field is given by the unit vector \( \hat{r} \).

Check the vector directions in symmetric top topic.
The force is along the line joining the two charges and direction is decided by their sign. For like charges, force is along the $\hat{e}_{12}$ vector and for unlike charges, force is opposite to $\hat{e}_{12}$ vector.

**Like charges:** $\vec{F}_{12} = |F| \hat{e}_{12}$  
$\vec{F}_{21} = |F| \hat{e}_{21}$

**Unlike charges:** $\vec{F}_{12} = -|F| \hat{e}_{12}$  
$\vec{F}_{21} = -|F| \hat{e}_{21}$

Here $\hat{e}_{12}$ is the direction of unit vector from $q_1$ to $q_2$. $F_{12}$ is the force on $q_1$ due to $q_2$.

**Fig: Coulomb force**

9. Electric field vector direction and sign are even trickier. Since the electric field around a point charge is spherically symmetric, the sign of it is decided by the direction of field lines.

$$|E| = k \frac{|q|}{r^2} \quad \vec{E} = |E| \hat{r}$$

Here $\hat{r}$ is the unit vector with spherical symmetry. Ie; electric field is positive for all directions around the charge, if the field is directed radially outwards. Electric field is negative if the field is directed radially inwards (towards the charge).

**Fig: Electric field lines of force.**
Basics of vectors

10. Angle made by a vector: The angle made by a given vector with respect to a given surface or plane can be obtained by drawing a tangent to the surface or normal to the surface. The smallest angle made by the vector WRT tangent to the surface is called latitude angle. Eg: Bragg angle. The smallest angle made by the vector WRT the normal to the surface is called co-latitude angle. Eg: reflection and refraction angles. This value lies between $0^\circ - 90^\circ$.

![Fig: Angles defined in reflection, refraction and Bragg diffraction.](image)

11. Angle between two vectors: Angle between two vectors is defined only when both of the vectors are either converging or both diverging. If that is not the case, one has to shift the vectors till a converging or diverging pair occurs. This value lies between $0^\circ - 180^\circ$. When angle between two vectors is $180^\circ$, one can define it in two ways. In all other cases, there will be only one possibility to define angle between two vectors.

![Fig: Angle between 2 vectors a) Given pair  b) Top vector shifted left (diverging pair) c) top vector shifted down (converging pair)](image)

Some examples: In Compton effect and Rutherford scattering, the incident ray is extended to produce diverging pair of vectors.
In Davisson–Germer experiment, the angle is defined between an incident ray and an emergent ray. Here the actual emergent angle is defined with respect to the normal to the target. For further mathematical simplicity, the incident ray is made coincide with the normal to the target.

Fig: Davisson–Germer experiment.

[Link](https://journals.aps.org/pr/abstract/10.1103/PhysRev.30.705)

12. Dot product of two vectors: The dot product of two vectors give the component of one vector along the other vector. Dot product of two vectors is a scalar.

\[ \mathbf{A} \cdot \mathbf{B} = |A||B| \cos \theta \]

13. The projection of vector \( \mathbf{A} \) along vector \( \mathbf{B} \) is given by multiplying the component of \( \mathbf{A} \) along \( \mathbf{B} \) with the unit vector along \( \mathbf{B} \).

\[ \text{Proj}_B \mathbf{A} = |A| \cos \theta \frac{\mathbf{B}}{|B|} = \frac{\mathbf{A} \cdot \mathbf{B} \mathbf{B}}{|B| |B|} \]

Similarly for projection of vector \( \mathbf{B} \) along \( \mathbf{A} \), one can write
Basics of vectors

12. Basics of vectors:

\[ \vec{A} \cdot \vec{B} = |A||B| \cos \theta \]

The dot product of vectors $\vec{A}$ and $\vec{B}$ is given by the absolute values of the vectors times the cosine of the angle between them.

\[ \text{Proj}_B \vec{A} = \frac{|A| \cos \theta}{|B|} \frac{\vec{B}}{|B|} = \frac{\vec{A} \cdot \vec{B}}{|A| |B|} \vec{B} \]

The projection of vector $\vec{A}$ onto vector $\vec{B}$ is given by the dot product of $\vec{A}$ and $\vec{B}$ divided by the magnitude of $\vec{B}$.

\[ \text{Proj}_A \vec{B} = |B| \cos \theta \frac{\vec{A}}{|A|} = \frac{\vec{A} \cdot \vec{B}}{|A|} \vec{A} \]

The projection of vector $\vec{B}$ onto vector $\vec{A}$ is given by the dot product of $\vec{A}$ and $\vec{B}$ divided by the magnitude of $\vec{A}$.

14. Basics of vectors:

\[ \vec{L} = \vec{r} \times \vec{p} \]

The vector $\vec{L}$ is calculated by taking the cross product of vectors $\vec{r}$ and $\vec{p}$.

\[ \vec{v} = \vec{r} \times \vec{\omega} \quad \text{and} \quad \vec{r} \times \vec{\omega} = \vec{L} \]

The vector $\vec{v}$ is calculated by taking the cross product of vectors $\vec{r}$ and $\vec{\omega}$. The result is equal to the vector $\vec{L}$.

\[ \nabla \times \vec{A} = \frac{1}{2} (\nabla \times \vec{B}) 
\]

The gradient of the vector $\vec{A}$ is given by half the curl of the vector $\vec{B}$.

\[ \vec{A} \cdot \vec{B} = \nabla \times \vec{A} \cdot \nabla \times \vec{B} \]

The dot product of vector $\vec{A}$ and vector $\vec{B}$ is equal to the divergence of the cross product of $\vec{A}$ and $\vec{B}$.
Basics of vectors

\[ \text{Proj}_{\vec{A}} \vec{B} = |B| \cos \theta \frac{\vec{A}}{|A|} = \frac{\vec{A} \cdot \vec{B}}{|A|} \frac{\vec{A}}{|A|} \]

The result of projection operation is a vector again.

14. Cross product of two vectors: Cross product of two vectors is a pseudo vector. Because in this case, the direction of resultant doesn't represent any activity. Instead all activity will be in the plane perpendicular to the resultant vector. Hence direction of resultant can be given by two conventions. Right hand thumb rule and left hand thumb rule. Most often right hand convention is followed for cross products.

Fig: Right hand thumb rule and Left hand thumb rule.

Here curl of fingers represent the direction of angle connecting first vector to second vector and direction of thumb gives the direction of the cross product resultant vector.

Fig: Angular momentum cross product.

To identify the resultant of \( \vec{r} \times \vec{p} \) one need to prepare either converging pair or diverging pair of vectors out of it and the curl of fingers from first vector to second vector gives the direction of angle. Then thumb direction represents the
Basics of vectors

direction of resultant vector. (Here radial vector is always directed from center to object.)

The resultant of cross product between real and pseudo vectors is as given below

\[ \text{Polar} \times \text{Polar} = \text{Pseudo} \]
\[ \text{Polar} \times \text{Pseudo} = \text{Polar} \]
\[ \text{Pseudo} \times \text{Polar} = \text{Polar} \]
\[ \text{Pseudo} \times \text{Pseudo} = \text{Pseudo} \]

Eg: \( \vec{L} = \vec{r} \times \vec{p} \) Here \( L \) is pseudo vector. \( r, p \) are polar vectors. \( \mathbf{In} = r \times \omega, v, r \) are polar vectors and \( \omega \) is pseudo vector. \( \vec{B} = \vec{\nabla} \times \vec{A} \) and \( \vec{A} = \frac{1}{2}(\vec{\nabla} \times \vec{B}) \), \( \vec{\nabla}, \vec{A} \) are polar vectors and \( \vec{B} \) is a pseudo vector.

Remember that pseudo vectors do not change sign under spatial inversion, whereas, real vectors do. One has to verify during every calculation, whether cross product is generating real vectors or pseudo vectors as well as whether a pseudo vector remains as pseudo vector throughout the calculation or not etc.

Some vector identities:

\[ A \times B = -B \times A \]
\[ \nabla \cdot (A \times B) = (\nabla \times A) \cdot B - A \cdot (\nabla \times B) \]
\[ A \times (B \times C) = (C \times B) \times A - B(A \cdot C) + C(A \cdot B) \]
\[ \nabla \times (A \times B) = A(\nabla \cdot B) - B(\nabla \cdot A) + (B \cdot \nabla)A - (A \cdot \nabla)B \]

References:


UNIT-1
CHAPTER-1
MECHANICS OF PARTICLES
పద మాధ్యమంలోనికి పదార్థాలు
శుభేష్ములు

మాధ్యమిక యువ కళాశాల (సాంస్కృతిక శాస్త్రము) అనే శాఖను ప్రారంభించారు.

1) నాటకం మాంగం విశ్లేషించాలి లేదా ప్రామాణిక యాత్రా పంపించాలి.

2) శిక్షణ నియమాలు (ప్రశంఖాల అంశం) ప్రథమ పదధాతి ప్రారంభించాలి.

3) కాల్చి అనుసారం నాటించండివి మాంగం శిక్షణ పంపించాలి.

4) వేర్వేరు పరిశీలన పంపించండి లేదా అంతా పంపించండి లేదా కృతి నిశ్చితం చేయాలి.

అవసరాలు

అవసరాలు ప్రామాణిక యాత్ర లో ఉండవచ్చుండి:

1) నాటకం మాంగం విశ్లేషించాలి లేదా ప్రామాణిక యాత్రా పంపించాలి.

2) శిక్షణ నియమాలు (ప్రశంఖాల అంశం) ప్రథమ పదధాతి ప్రారంభించాలి.

3) కాల్చి అనుసారం నాటించండివి మాంగం శిక్షణ పంపించాలి.

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5) కాల్చి అనుసారం నాటించండి లేదా అంతా పంపించండి లేదా కృతి నిశ్చితం చేయాలి.

6) నాటకం మాంగం నాటించండి లేదా అంతా పంపించండి లేదా కృతి నిశ్చితం చేయాలి.
Syllabus

Review of Newton’s Laws of Motion, Motion of variable mass system, Motion of a rocket, multistage rocket, Concept of impact parameter, scattering cross-section, Rutherford scattering-Derivation.

Learning Objectives

In this chapter students would learn in detail

1. Newton’s laws of motion
2. Variable mass system
3. Rocket equation and details of multistage rocket
4. Scattering cross section concept and Rutherford’s scattering derivation.

Learning Outcomes

By the end of the chapter, student would be able to

1. Identify specific properties of variable mass system and scattering phenomenon.
2. Describe the dynamics of variable mass system and scattering phenomenon.
3. Calculate the terminal velocity of rocket and scattering angle of charged particles. Apply various conservation laws to study the dynamics of system.
4. Identify various correction terms required to apply Newton’s laws to various systems where direct application fails. Identify the limits and limitations of Newton’s formalism.
5. Justify the initial conditions required to attain a required momentum for the given variable mass system and scattering system.
6. Develop prototype models for various applications of variable mass system and scattering problem.
సంపుర్ణ శాస్త్ర కర్మాంతర పాఠం మాత్రమే పంటండ్ల పేరు దొరికే సంస్థ యొక్క వర్గం తో పాఠం చేసింది.

1. రోమనం రాకెట్లపై పాఠిడిన దృశ్యానికి సుప్రాచితాం నుండి అప్రాచితాం వర్క సిస్టిస్ కల్స.
2. రోమనం రాకెట్లు ప్రతి విషయానికి దృశ్యానికి కాంపూటర్ జర్సిగే స్టిస్టిస్ క్స్ ఫిజిక్స్ ఫిరి బ్రిటింగ్ సిస్ట్మ్స్.
3. జియలజి ఫిజిక్స్ నియమాలమీద డైక్స్టుగా సిస్ట్మాలలో పూర్తి య్యా మెట్టయిల్స్.
4. జియలజి ఫిజిక్స్ నియమాలు స్ట్ముది పు అను ఫోర్్ విజ్యేషన్్ చేయటాం గుర్టాంచి ప్రేం మలలో వివిధ అను రాకెట్ చేస్టాం.
5. స్టేటాంగ్ ద్యేరా కార్బన్ ఏవిదాంగా నిక్షేపాలను ముఖ్యాంగా ఇస్కేషన్ లో సిస్ట్మ్స్.

పిన్నాటి ఎంపాంచం విధానాలు మీద

ఈ విద్యాంశంలో ఆంధ్రప్రదేశ్ సంస్థ యొక్క వర్గం తో పాఠం చేసిన మొదటి విషయం మరొక రాకెట్లు కాంపూటర్ జర్సిగే స్టిస్టిస్టిస్ క్స్ ఫిజిక్స్ ఫిరి బ్రిటింగ్ సిస్ట్మ్స్.

సంపూర్ణ శాస్త్ర కర్మాంతర పాఠం మాత్రమే పంటండ్ల పేరు దొరికే సంస్థ యొక్క వర్గం తో పాఠం చేసింది.

28
Course Outcomes specific to program and Future directions

By the end of this chapter students from specific programs would be see the:

1. Physics: Scattering is used in several material characterization techniques like SEM, DLS etc. Heavy celestial bodies like binary stars undergo mass transfer that requires variable mass system equations.

2. Chemistry: Law of mass action in chemistry deals with time variation of concentration of reactants.

3. Computer Science: Industrial automation requires programming of variable mass systems like powder dispensers and boiler taps. Especially vibrating variable mass systems are more prominent.

4. Geology: Seismic waves get scattered by hydrocarbon materials. Thus study of scattering of seismic waves gives information about petroleum and natural gas reserves.

5. Electronics: Several variable mass systems that we come across in industry like powder dispensers, boiler taps etc., need a knowledge of variable mass system.

6. Renewable Energy: Study of wind energy and ocean energy harvesting considers wind/water as a variable mass system during conversion process.

7. Statistics: Statistical analysis of scattering cross-section is one of the major activities in particle physics.

Familiar to Unfamiliar

In your 9th class, you might have learned about how unbalanced forces result in a change of motion of an object. You might have also learned about Newton’s laws, the concept of inertia, action-reaction principles and the concept of conservation of momentum. In your 11th class, you might have learned about Newton’s laws, law of conservation of momentum, various types of frictional forces and the method of solving for net force acting on objects using free-body diagrams. In this chapter you will be given a critical review on Newton’s three laws of motion, the variable mass
ఉన్నత గణితం| కాలికలేషన్ వసుతు వున్నద్ద ప్రటని యల్స దొర్కలేద్ద అనేది బీజంలు ఆ ప్రతిల్స చినిగా ఉపోద్ఘాతాంత్రికంగా ఉపుయోగ్వాంచబడింది. 1901 న్యాటన్ ఉప్యోగ్వాంచటాంత్రికంగా ప్రటటిల్స చేయబడింది. అనేక అధాయనాంతాలు ఆన్యాంతింగా బిాంద్ద వాడుతుని హేపరాా లన్ని చాప్ి ర్ి లో కాలుేలస్ అనువదించబడింది. అప్ుటికి డై నమిక్స్ కాలుేలస్ ఆతితాంతాలు ఇాంగ్లి ష్ట్మాధానాంతాలు అనేది వచేు తరువాత తరువాతనే ప్రటని యల్స గా ఆప్లేల్స్ గా ఆపాదించబడింది. దయచేసిన సిద్యానంతాలు మహాందాంపటి యల్స నుంచి శాస్త్రంలో కార్ణాంతాలు స్తధార్ణాంగా ఉనాంట్ర ఉంది. స్త్మాధానాంతాలు వసుతు వుగా మీద చదివేటప్పుడు స్త్ర్టప్రయ్య న్యాటన్ చేయబడింది. అటు సిద్యానంతాలు వకి ర్ి లో పాస్త్రి అనుమానం తము ఉపోద్ఘాతాంత్రికంగా ఉనాంట్ర కార్ణాంతాలు ఏవిడాంంగా ఈ భావనల వత్రకారులో తెలుసుకని లో ఫోర్్ ప్రటని యల్స ప్రటటిల్స చేసిన ద్యని ప్ద్యరాా లు చాప్ి ర్ి మార్టనది వదకటాం. మార్టనది వదకటాంగా వకి ర్ి లో వచేువారు సెయిన్ ఉప్రద్యాతము తరువాతనే తరువాతనే కాలుేలస్ అవగాహనక వాదించాలి. ఫోర్్ ప్రటని యల్స ప్రటటిల్స చేసిన ద్యని ప్ద్యరాా లు చాప్ి ర్ి అధాయనాంతాలు ఇాంగ్లి ష్ట్మాధానాంతాలు అనేది వచేు తరువాత తరువాతనే ప్రటని యల్స గా ఆప్లేల్స్ గా ఆపాదించబడింది. స్త్మాధానాంతాలు ఆపాదించబడింది అది బిాంద్ద వాడుతారు ఇాంగ్లి ష్ట్మాధానాంతాలు ఆపాదించబడింది. ఇాంగ్లి ష్ట్మాధానాంతాలు అనువదించబడింది.
1.1 Introduction

Theoretical developments in science came in three levels. The first and foremost methodology was natural philosophy where people argue and come to logical conclusions on any topic. Later the research turned out to be empirical in its nature. Here, people look for experimental evidence for every phenomenon and concept. Aristotle, Archimedes era comes into this category. The so called modern science has started with the contributions from Copernicus, Kepler and further by Ptolemy and Galileo. They have developed geometrical studies in physics. Attributing calculus to empirical theories of physics started with Isaac Newton and Leibnitz. Subsequently more fundamental theories were evolved.

Newton considered all the systems to be point-like particles at the primary level. This makes the calculations simpler and more effective. Here point-like object does not mean that the object is of point size. But for all mathematical and physical considerations, the object may be considered as a point object. For example, for the revolution of earth around the sun, earth may be considered as a point like object. But for apple falling on earth, earth cannot be considered as point like object. In the current exposition, in later chapters, the effect of finite size of objects on the statics, dynamics and kinematics of the system is described, which can be considered as an improvement over the earlier treatments. Subsequently the studies are extended to astronomical objects.

The causes and effects that we come across in Newtonian mechanics are majorly vectors. Therefore the development of vector analysis by Gibbs and Wilson in 1901 made Newtonian mechanics all the more exciting. This provided a geometrical approach to the Newtonian mechanics. But the major problem with Newtonian mechanics arises when multiple forces act on a body which sometimes may result in dynamic equilibrium. In such a case, solving
న్యూటన్ గమన నియమాలు పునః సమీక్ష

మొదటి గమన నియమము:

- బాహా బలాం ప్నిచేయనాంత వర్క
- వసుత వులు నిశుల సిు త్ర లో కాన్న
- స్త్మ వేగ సిు త్ర లో కాన్న ఉాండి ప్రతుాంది

\[
\frac{dP}{dt} \propto F \Rightarrow \frac{dP}{dt} = \frac{1}{k} F \Rightarrow F = \frac{dP}{dt} = \frac{d(mv)}{dt} = m \frac{dv}{dt} = ma \text{ Newtons}
\]

ఇకేద చర్ా ప్ి త్ర చర్ా వేరు వేరు వసుత వులమీద ప్ని చయాటాంతో అవి ఎనిటికీ కాాని్ల్స అవేవు. ఒక్వ వసుత వు మీద వాత్రరేక దిశలలో బలాలు ప్ని చేసినటి యిత్త అవి కాాని్ల్స అవుతాయి.

మూడవ గమన నియమము:

- ప్ి త్ర ఒకే చర్ాకి స్త్మానాంగా వాత్రరేక దిశలో ప్ి త్ర చర్ా ఉాంటుాంది

\[
F_{12} = -F_{21}
\]

ఇకేద చర్ా ప్ి త్ర చర్ా వేరు వేరు వసుత వులమీద ప్ని చయాటాంతో అవి ఎనిటికీ కాాని్ల్స అవేవు. ఒక్వ వసుత వు మీద వాత్రరేక దిశలలో అవాతం అవి కాాని్ల్స అవుతాయి.

పరిశీలన-1

మాధవ కాలుని సంమానాంగా గమన పునః సమీక్ష. మాధవ కాలుని సంమానాంగా గమన పునః సమీక్ష నుండి మనం మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము మనం గమన నియమము

E - Corner

https://en.wikipedia.org/wiki/Madhava_of_Sangamagrama
multiple analytical or geometrical equations may result in a zero
which may dampen the enthusiasm of the learner. Hundred years
after the formulation of Newtonian mechanics, Lagrange and
Hamilton had come up with a study of cause of the cause in
Newtonian mechanics. i.e.; they have started studying about the
cause of the force. That is, the potential which may be related to force
as \( \vec{F} = -\vec{V}V \). Here the potential is a scalar. Thus the difficulties in
handling direction information are circumvented. Discussion on the
details of Lagrangian and Hamiltonian dynamics may be found in
your higher studies.

The observations of Galileo, Copernicus and Kepler and a few
others have been consolidated and a profound calculus based
explanation was given by Sir Issac Newton in his book ‘Principia’
which was published in Latin in 1687 and later translated into
English in 1728. It is believed that the calculus presented in Newton
and Leibnitz approaches may have their origins in Indian
astronomical and mathematical studies of Bhaskara II and Madhava
of Sangamagrama, Kerala, India during 1400AD.

### 1.2 Review on Newton’s laws of motion

**First law:** Every body continues to be in its state of rest or uniform
motion as long as there is no external force acting on it.

**Second law:** The rate of change of momentum produced in a body is
directly proportional to the applied force.

\[
\frac{dP}{dt} \propto F \Rightarrow \frac{dP}{dt} = \frac{1}{k} F \Rightarrow F = \frac{dP}{dt} = \frac{d(mv)}{dt} = m \frac{dv}{dt} = ma \text{ Newtons}
\]

Here \( k \), the proportionality constant, becomes 1 when force is
measured in Newtons. i.e., when 1Kg massive body experiences
1m/Sec\(^2\) acceleration upon application of the force. Here mass \( m \) is
considered to be constant always.

**Third law:** For every action, there will be an equal and opposite
reaction.

\[ F_{12} = -F_{21} \]
1.2 Review on Newton’s laws of motion

1837-8. अखंड अस्लांकोड़. काव्य पर अनुभव तरीक़े गयाकारी. कामकाज़ पर तीन नियम लगाया गया है।

1. यदि चुम्बक तथा धर्माकार प्रति चुम्बकीय रूप से डाला जाता है, तो उसे हायदारी दे दी जानी चाहिए।

2. यदि एक दिशा में काम किया जाता है और दूसरी दिशा में किया जाता है, तो उसे दो दिशाओं में ही किया जाना चाहिए।

3. यदि एक दिशा में काम किया जाता है और दूसरी दिशा में किया जाता है, तो उसे दो दिशाओं में ही किया जाना चाहिए।

*Fig: Stanzas from Vaiseshika Sutras.*


[https://archive.org/details/thevaiasesikasut00kanauoft](https://archive.org/details/thevaiasesikasut00kanauoft)
Here action and reaction forces work on two different bodies. Hence they will never get cancelled.

Think ...

If mass remains constant always in Newton’s laws, do we need to modify or make any approximation to apply them to rocket motion?

Observation-1:

The Newton’s first law is also called law of inertia. According to Aristotle’s theory of impetus (impulse/force) in 3rd century BC, object comes into motion when there is an external impetus and stops when the impetus is stopped. Thus according to Aristotle force produces velocity. Without force velocity becomes zero. But a German philosopher Nicholas Cusanus, in his *De Ludo Globi* (The game of Bowling) (1463) explained that the impulse is carried by the body throughout its journey. He also described friction as a cause of Aristotle’s observation. This laid the stepping stone for Galileo’s experimentation on inertia. According to him, once force is applied and the body is set to free motion, the velocity remains forever. This is what drives planets around the sun. When there is a friction, the impetus reduces and body comes to rest. Thus it was concluded that the force causes a change in velocity. As long as force is not applied, there will not be any change in velocity. Nicolas Kuzans concepts of Inertia were similarly described in Kanada’s Viseshika Sutras in 600BC itself. But the concept of friction was not well described, except for the gravitational pull.

Mass is the control parameter for inertia and inertia is more for heavy objects. It is easier to bring a small four wheeler into motion and to stop it as compared to moving or stopping a heavily loaded truck.

Observation-2:

In second law, the proportionality constant becomes 1 when force is measured in Newtons. i.e., when a force produces \(1 \text{m/Sec}^2\) acceleration in a 1Kg object, then that force is equal to 1Newton.
1.2 Review on Newton’s laws of motion

పండద-2

ప్రావాయం 1.2

$\text{net force } F = ma$

36 kg ఎంచుకుందనే 1m/sec^2 సంఘటన లేదు ప్రతి 1వ్యాచ్యంలో

$ma = F$ తో సంపాదించారు. అయితే 12 ఎంచుకుందనే 2 ఎంచుకుందనే మిస్టర్ పూర్తి ప్రతి సంఖ్యలో

$F = 12 \times 4 = 3$ అవసరం సంపాదించారు. అయితే కాండియుండనే 3 ఎంచుకుందనే వింతలో తీసుకువారు. అయితే వేగం ఎంచుకుందనే 0.5 ఎంచుకుందనే వింతలో తీసుకువారు. (సింగ మండించిన సమాధానం మాండించిన మొదటి ఎంచుకుందనే వింతలో తీసుకువారు). ఎంచుకుంది ఎంచుకుందనే అది ఉంతుంది నియమాలో చేయబడిన లేదా LHS ఎంచుకుంది ఎంచుకుంది ఎంచుకుంది. అది అనేక పరిస్థితులు ఉంది ప్రత్యేకంగా ఇకేడ ఉంది. ఇకేడ అనుకాంద్యాం (P)

$ma = F/k$

యార్ కనిి $\text{LH}$ లాంటి ఇతర సిస్ట్మి ఉత్తాలి $\text{H}$ ఎంచుకుందనే వింతలో తీసుకువారు. ఇకేడ అనుకాంద్యాం (P2)

$F = 0$

$ma = 0$ ఎంచుకుందనే వింతలో తీసుకువారు. ఇకేడ అనుకాంద్యాం (P3)

ఎంచుకుంది ఎంచుకుందనే అది ఉంతుంది నియమాలో చేయబడిన లేదా LHS ఎంచుకుంది ఎంచుకుంది ఎంచుకుంది. అది అనేక పరిస్థితులు ఉంది ప్రత్యేకంగా ఇకేడ ఉంది. ఇకేడ అనుకాంద్యాం (P)

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$ma = 0$ ఎంచుకుందనే వింతలో తీసుకువారు. ఇకేడ అనుకాంద్యాం (P3)
Here the proportionality constant comes in the denominator so as to ensure that it normalizes the physical quantity. Suppose a kid measures the width of a door as 12 feet (12 ft_{kid}) with his small foot. But with the actual foot scale measures it as 4 feet (4 ft_{scale}). Then one has to compare the foot size of the kid with the actual scale and define the constant $k' = \frac{n \text{ft}_{kid}}{1 \text{ft}_{scale}} = 3$. Then the measurement made by the kid is standardized by dividing it with $k'$. Thus Width of door = $\frac{d_w \text{ft}_{kid}}{k'} = \frac{12}{3} = 4\text{ft}$. Suppose a giant measures the width of the door as 2 ft with his giant foot, then $k' = 0.5$. Thus the proportionality constant plays the role of standardizing units, apart from its actual purpose of proportionality. Hence it comes in the denominator of the independent parameter. Once the proportionality constant is fixed, the equation may be rewritten in any order as the rest will be purely mathematical.

Later, after the establishment of SI unit system, some proportionality constants were kept in numerator and some in denominator, especially in electromagnetism, in order to fit them into Maxwell’s equations.

**Observation-3:**

In Newton’s second law, it is strictly considered that the mass remains constant. Hence $F = ma$ is not just a special condition but a valid form of Newton’s second law. While considering the variable mass systems like rockets, for applicability of Newton’s laws, the total mass of the system (Rocket+Fuel) is assumed to be constant. Otherwise, in the strictest sense, one cannot apply Newton’s laws to variable mass systems. Thus in variable mass systems, constant mass required by Newton’s laws will be ensured by considering conservation of mass equation.

**Observation-4:**

From the first law, if $F = 0$, \( \frac{dp}{dt} = 0 \Rightarrow P = \text{const} \).

\( \Rightarrow v = \text{constant. (could even be zero)} \) as \( m \) is already a constant.
1.2 Review on Newton’s laws of motion

According to Newton’s first law of motion, if there is no net force acting on an object, it will remain at rest or move at a constant velocity in a straight line. The equation for this law is:

\[ \sum F = ma \]

Where \( \sum F \) is the net force acting on the object, \( m \) is the mass of the object, and \( a \) is the acceleration of the object.

According to Newton’s second law of motion, the acceleration of an object is directly proportional to the net force acting on the object and inversely proportional to the mass of the object. The equation for this law is:

\[ F = ma \]

Where \( F \) is the net force acting on the object, \( m \) is the mass of the object, and \( a \) is the acceleration of the object.

According to Newton’s third law of motion, for every action, there is an equal and opposite reaction. The equation for this law is:

\[ F_1 = -F_2 \]

Where \( F_1 \) is the action force and \( F_2 \) is the reaction force.

These laws help us understand the motion of objects under the influence of forces. Newton’s laws of motion are fundamental to the study of physics and are used in various applications, from designing vehicles to understanding the motion of planets.

The image also includes a note on the page number, indicating that this page is 38.
1.2 Review on Newton’s laws of motion

Thus, it looks as if, Newton’s first law is a special case of Newton’s second law. Mathematically it is true. Then why is it defined as a separate law? And why is it given higher priority than its original?

To understand that, one needs to look for the failure of first law i.e., are there any systems which exhibit a change in velocity even when the external force is zero?

Consider a person (Observer-1 (P₁)) travelling in a bus with a bag in front of him/her. The bus is moving at a speed of 60KMPH. Suppose a person (Observer-2 (P₂)) is chasing the bus on a bike at a speed of 40KMPH. There is a stationary observer (Observer-3 (P₃)) on the road who is observing the situation.

Now the speed of the bag as seen by P₁ is zero, as seen by P₂ is 20KMPH and as seen by P₃ is 60KMPH. Thus when there is no external force, every observer attributes a constant velocity to the object (which could also be zero).

When the bus stops suddenly, the bag moves without application of force, as seen by observer P₃.

If the bus is at rest, P₁ reports the speed of the bag as 0KMPH while P₂ reports it as -40KMPH and P₃ reports as 0KMPH. If the bus starts, suddenly from rest, then the bag moves backwards without application of any force.

One should not attach the acceleration or deceleration of the bus with the jerk produced in the bag. If that is the case, we should also feel some jerks while walking on earth without application of any external force as earth is in motion already.

Thus one may conclude that if there is any acceleration or deceleration between object frame and observer frame, Newton’s first law fails.

What could be the problem if Newton’s first law fails?

If Newton’s first law fails, newton’s second law also fails. Suppose by the time brakes are operated on the running bus, the observer P₁ touched the bag with a wish to push it forward a little bit. The bag moves forward due to both break operation and the push by P₁. But
1.2 Review on Newton’s laws of motion

\[ F_{\text{up}} = m_{\text{person}} \times a_{\text{up}}, \quad F_{\text{down}} = m_{\text{person}} \times (-a_{\text{down}}) \]

The force that come as correction term to Newtons laws when the observer is accelerating is called fictitious force or pseudo force. This is a real force and is a polar vector. But since it has no cause with-in observer frame of reference, it is called pseudo force. Here cause comes from outside. (Just like effect of motion of earth on rivers.) One may observe that the correction force arising as a result of cross product is also a polar vector as the cross product involves one polar vector and another pseudo vector. The effect of earth rotation on pendulum was firs demonstrated by Foucault by his pendulum experiment.

https://en.wikipedia.org/wiki/List_of_Foucault_pendulums
1.2 Review on Newton’s laws of motion

one cannot distinguish the real cause of the observed displacement in the bag. In other words, Newton’s second law fails, i.e., one cannot estimate the change in momentum produced just by substituting applied force into Newton’s second law equation. This suggests that Newton’s second law needs corrections.

Thus first law defines the arena where Newton’s second law is valid. Hence it is considered as an independent law and is given higher priority over second law. The frame of reference where Newton’s first law (Law of inertia) is valid is called inertial frame of reference. An inertial frame of reference of observer is the one which maintains a constant or zero speed difference with the object frame.

Corrections to Newton’s second law:

1. **Accelerating or decelerating bus:** When a bus is moving with constant or zero velocity and suddenly decelerated or accelerated, then the correction term for force is given by \( F_c = m_{\text{bag}} \times a_{\text{bus}} \). If the applied force is \( F_a \), then the total force is \( F = F_a + F_c \).

2. **Lift moving upwards or downwards:** When the lift moves upwards persons in the lift feel heavier and when it moves downwards, the persons feel lighter in the beginning. Thus the correction terms to force are \( F_{\text{up}} = m_{\text{person}} \times a_{\text{up}} \), \( F_{\text{down}} = m_{\text{person}} \times (-a_{\text{down}}) \).

3. **Charged particle in motion:** When a charged particle is in motion, it produces magnetic field given by \( B = \mu_0 I \times A \). When two such moving charges interact, there will be force due to electric field as well as force due to magnetic field. The force due to magnetic field is given by \( \vec{F}_m = q(\vec{v} \times \vec{B}) \). But this term is negligible in non-relativistic cases, where \( v \ll c \).

4. **Rotatory motion:** Suppose an object is travelling from the center towards the circumference along the radius of a rotating disc. For the observer outside, it looks as if it is travelling along a straight line along the radius. But for the observer on the disc, it looks like travelling in circular path. As we already know, to bring an object into circular motion,
1.2 Review on Newton’s laws of motion

**Fig: Wind patterns due to Coriolis effect.**

**Fig: Coriolis force and direction of Earth’s magnetic field.**

**Fig: Rivers turning right due to Coriolis effect.**
one needs to apply a force which is directed always perpendicular to the velocity, called the centripetal force. Thus there exists a centripetal acceleration between object frame and observer frame. Hence Newton’s laws fail. How do Newton’s laws fail in this case? They fail because, one observer reports that the object is moving in straight line with constant velocity (F=0), whereas another observer reports that the particle is moving in circle, which implies some centrifugal acceleration is observed in the particle. The corresponding force is called inertial centrifugal force. The correction term for this circular motion is \( \frac{dP'}{dt} = \frac{dP}{dt} + \omega \times P \).

Or \( F_c = \omega \times P \). This correction force is also called Coriolis force. This is responsible for the turning of rivers towards right in northern hemisphere.

**Fig: Pseudo force in rotatory motion.**

Is Newton’s second law considered as definition of force?

**No!** Because, a force may do so many other things as well other than producing acceleration in the object. For example, deformation, heat, vibrations etc. Hence Newton’s second law is only one possibility of application of force on objects. It is not to be considered as definition of force.

Does Newton’s second law act as definition of mass?

Newton’s law is defined for constant mass. i.e.; \( m = \frac{F}{a} \). Then can we use it as definition of mass? The answer is **No**. Because, with velocity, mass changes in relativistic cases. In such case, the correction to mass
Perform Foucault pendulum experiment in your institute.
Record the pattern for a long time and consolidate.
is given by \( m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \). Then the correction to Newton’s second law is given by \( F = \frac{d(mv)}{dt} \) where \( m \) is the relativistic mass.

The variation of mass \( m = \frac{F}{a} \) as a function of velocity of the object is given by

**Fig: Variation of inertial mass with velocity.**

Thus Newton’s second law cannot be used as a standard definition of mass. This type of definition of mass is called inertial mass. The other one is the gravitational mass given by \( m = F/g \). Here \( g \) is the acceleration produced by gravity.

**Is Newton’s second law universal?**

Newton’s second law is not a universal law. This is because the law works only when mass remains constant. But when the velocity of the object reaches relativistic limit, mass also changes with velocity. This law also fails when friction or any other form of energy losses occur in the system, or when fictitious forces exist in the system. Hence corrections are to be made even in non-relativistic regime to make it universal.

**Is Newton’s third law Universal?**

Newton’s third law of motion with a little rearrangement of terms can be written as \( F_{12} + F_{21} = 0 \) \( \Rightarrow \sum F_i = 0 \). Thus, it gives a
1.3 Incompressible Fluid Velocities

In incompressible fluids, the density remains constant throughout the flow. Therefore, the continuity equation reduces to:

\[ \sum F_i = 0 \]

where \( F_i \) is the force in the \( i \) direction. The pressure in such fluids is also constant across the flow.

\[ \sum P_i = \text{constant} \]

The force in the \( i \) direction is given by:

\[ F_i = m \frac{dv_i}{dt} \]

where \( m \) is the mass and \( v_i \) is the velocity in the \( i \) direction.

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where \( m \) is the mass and \( v_i \) is the velocity in the \( i \) direction.

\[ P_i = mv - - - (1) \]

The total force in the \( i \) direction is given by:

\[ p_f = dm(v - v_{rel}) + (m - dm)(v + dv) - - - (2) \]

where \( v_{rel} \) is the relative velocity and \( dm \) is the mass increment. Therefore:

\[ p_i = p_f \]

\[ mv = vdm - v_{rel}dm + mu + mdv - vdm - dmdv \]

where \( dv \) is the velocity increment in the \( i \) direction.
condition for dynamic equilibrium of systems. By using second law on this equation, one can write $\Sigma P_i = \text{constant}$. Thus Newton’s third law also ensures law of conservation of momentum. As long as homogeneity of space exists, momentum is conserved and Newton’s third law finds its validity. Thus Newton’s third law can be considered as a universal law, within non-relativistic regime. This law fails in relativistic electromagnetic field interactions, i.e., when a charge moves with relativistic speeds and produces magnetic force of considerable strength.

### 1.3 Motion of variable mass system

Consider an object of mass $m$ moving with velocity $v$ at time $t$. In a mean time $dt$, let a small mass $dm$ be ejected out of it with relative velocity $v_{rel}$. Still the ejected mass moves forward with reduced velocity $v - v_{rel}$. This activity results in a rise in the velocity of the remaining mass $m - dm$. Let the final velocity of remaining mass be $v + dv$. Then

$$P_i = mv$$

The momentum of final system at time $t + dt$ is

$$p_f = dm(v - v_{rel}) + (m - dm)(v + dv)$$

If there is no external force, momentum is conserved, i.e., ($P_i = p_f$) which gives

$$mv = vdm - v_{rel}dm + mv + mdv - vdm - dmdv$$

Since $dm$, $dv$ are small, the last term may be neglected. Thus

$$mdv = v_{rel}dm$$
1.3 Motion of variable mass system

Consider a variable mass system with mass capturing instead of mass ejection and derive equation of motion.

\[
\frac{dP}{dt} = F_{\text{ext}} \Rightarrow \frac{mdv - v_{\text{rel}} dm}{dt} = F_{\text{ext}}
\]

\[
\Rightarrow m \frac{dv}{dt} = F_{\text{ext}} + v_{\text{rel}} \frac{dm}{dt} = F_{\text{ext}} + F_{\text{reaction}} \tag{4}
\]

This equation shows that the rate of change of pressure is equal to the external force acting on the system plus the reaction force due to the change in mass.

**Activity**

Consider a variable mass system with mass capturing instead of mass ejection and derive equation of motion.
If there is any external force like gravity that acts on the system, then the equation of motion is given by

\[
\frac{dP}{dt} = F_{\text{ext}} \Rightarrow \frac{mdv - v_{\text{rel}} dm}{dt} = F_{\text{ext}}
\]

\[
\Rightarrow m \frac{dv}{dt} = F_{\text{ext}} + v_{\text{rel}} \frac{dm}{dt} = F_{\text{ext}} + F_{\text{reaction}} \tag{4}
\]

This is the equation of motion of a variable mass system. Here \( dm \) is a negative number, as it represents a reduction in mass. Hence it may be indicated with a negative sign, if necessary, to avoid confusion. The last term in the equation \( F_{\text{reaction}} \) is the reaction force generated in the main system due to the exhaust of a mass \( dm \) with velocity \( v_{\text{rel}} \).

1.4 Rocket Equation

Consider a rocket of mass \( m \) travelling with velocity \( v \) at some instant of time \( t \). After some time \( dt \), let a mass \( dm \) be ejected out of the system with relative velocity \( v_{\text{rel}} \). This results in a rise in the velocity of the remaining mass \( m - dm \) by a factor of \( dv \). The ejected mass actually travels forward with reduced velocity \( v - v_{\text{rel}} \) and the resultant mass \( m - dm \) travels with velocity \( v + dv \).

The initial momentum of the system at time \( t \) is,

\[
P_i = mv \tag{1}
\]

Final momentum of the system at time \( t + dt \) is

\[
P_f = dm(v - v_{\text{rel}}) + (m - dm)(v + dv) \tag{2}
\]

Let there be an external force, say gravity, \( F_{\text{ext}} = -mg \), acting on the system; then

\[
\frac{dP}{dt} = F_{\text{ext}} \Rightarrow \frac{P_f - P_i}{dt} = -mg
\]

\[
\Rightarrow \frac{[vdm - v_{\text{rel}} dm + mv + mdv - vdm - dmdv] - mv}{dt} = -mg
\]

Since \( dm \) and \( dv \) are small, their product may be neglected.
1.4 రాకెట్ ఈక్వేషన్

\[ P_i = mv \] — (1)

మిగతా 1+dt అంటే మాస్మాట్లో మారసిన వేగాన్ని 

\[ p_f = dm(v - v_{rel}) + (m - dm)(v + dv) \] — (2)

అనగా బయటకి వచున 

\[ dm \] మర్యాదలు చిని విలువలు ఐనాంద్దవలన వాతి లావలంటి ఉపేక్షించినప్పుడు,

\[ h = \int vdt = v_0 t + v_{rel}(m_0 \ln m_0 - m_0 - m \ln m + m) - \frac{1}{2} gt^2 \]
\[ m \frac{dv}{dt} = v_{rel} \frac{dm}{dt} - mg \]

This is the equation of motion of rocket moving under the influence of gravity. Here \( dm \) is a negative quantity. Hence a \(-ve\) sign may be introduced. From this, the change in velocity is given by

\[ \Rightarrow dv = -\frac{v_{rel}}{m} dm - gdt \]

Integrating over time from \( 0 \) to \( t \), velocity from \( v_0 \) to \( v \) and mass from \( m_0 \) to \( m \), gives

\[
\int_{v_0}^{v} dv = -v_{rel} \int_{m_0}^{m} \frac{1}{m} dm - g \int_{0}^{t} dt
\]

\[ \Rightarrow (v - v_0) = -v_{rel} (\ln m - \ln m_0) - gt \]

\[ \Rightarrow v = v_0 + v_{rel} \ln \frac{m_0}{m} - gt \]

This is the velocity equation of the rocket, with \( v_0 \) being initial velocity and \( m_0 \), the initial mass of the rocket. This is called **Tsiolkovsky rocket equation**.

The height reached by rocket is given by

\[ h = \int v dt = v_0 t + v_{rel} (m_0 \ln m_0 - m_0 - m \ln m + m) - \frac{1}{2} gt^2 \]

In the absence of gravity, (in outer space), one may write

\[ \Delta v = v - v_0 = v_{rel} \ln \frac{m_0}{m} \Rightarrow \frac{\Delta v}{v_{rel}} = \ln \frac{m_0}{m} \Rightarrow \Delta v = -\ln \frac{m}{m_0} \]

**Fig: Rocket mass ratio vs. change in velocity.**
\[ \Delta v = v - v_0 = v_{rel} \ln \frac{m_0}{m} = \Delta \frac{\nu}{v_{rel}} = \ln \frac{m_0}{m} = \Delta \ln \frac{m}{m_0} \]

This change in velocity can be expressed as follows:

\[ m = m_0 e^{-\frac{\Delta \nu}{v_{rel}}} \]

###火箭俄文的运动方程

火箭的总质量在射出速度的增加下也相应地变化，如下所示。

\[ m = m_e = m_s + m_{pl} \quad (1) \]

\[ m_0 = m_e + m_{pl} = m_{pl} + m_s + m_f \quad (2) \]

###惯性系的运动方程

在惯性系中，火箭的总质量在射出速度的增加下也相应地变化，如下所示。

\[ M_R = \frac{m_0}{m_e} = \frac{m_{pl} + m_s + m_f}{m_s + m_{pl}} \quad (3) \]

###质心速度

质心的速度为：

\[ \pi = \frac{m_{pl}}{m_0 - m_{pl}} = \frac{m_{pl}}{m_s + m_f} \quad (4) \]

###质心位移

质心的位移为：

\[ \varepsilon = \frac{m_s}{m_s + m_f} \quad (5) \]

###火箭的质量比

火箭的质量比为：

\[ M_R = \frac{1 + \pi}{\varepsilon + \pi} \quad (6) \]

###质心位移

质心的位移为：

\[ \lambda = \frac{m_{pl}}{m_0} \quad (7) \]

###质心速度

质心的速度为：

\[ M_R = \frac{1}{\varepsilon + (1 - \varepsilon)\lambda} \quad (6) \]

###质心速度

质心的速度为：

\[ \Delta v = v - v_0 = v_{rel} \ln M_R = -v_{rel} \ln (\varepsilon + (1 - \varepsilon)\lambda) \quad (7) \]
\[ m = m_0 e^{\frac{\Delta v}{v_{rel}}} \]

The variation of final mass of rocket as a function of change in velocity is shown in the figure above.

**Parameters of single stage rocket**

Consider a single stage rocket with empty structural mass (mass of a stage) \( m_s \), payload mass (mass of satellite or other higher stages if any) \( m_{pl} \) and propellant mass (mass of fuel) \( m_f \). Then the empty mass of the rocket is defined as

\[ m_e = m_s + m_{pl} \tag{1} \]

Full mass of the rocket is defined as

\[ m_0 = m_e + m_{pl} = m_{pl} + m_s + m_f \tag{2} \]

Other useful parameters are

**Propellant mass ratio**, which is defined as the ratio of full mass to empty mass of the rocket. i.e., the ratio of rocket mass with and without propellant. Its value is always greater than or equal to 1.

\[ M_R = \frac{m_0}{m_e} = \frac{m_{pl} + m_s + m_f}{m_s + m_{pl}} \tag{3} \]

**Pay load ratio** which is defined as the ratio of payload mass to rest of the mass of the rocket

\[ \pi = \frac{m_{pl}}{m_0 - m_{pl}} = \frac{m_{pl}}{m_s + m_f} \tag{4} \]

**Structure ratio** which is defined as the ratio of mass of empty structure to the mass of structure with propellant. i.e.; this ratio doesn’t take into account the payload mass \( (m_{pl}) \).

\[ \varepsilon = \frac{m_s}{m_s + m_f} \tag{5} \]

Substituting Eq. (4), Eq. (5) in Eq. (3), one obtains

\[ M_R = \frac{1 + \pi}{\varepsilon + \pi} \tag{6} \]

If we define overall payload fraction as \( \lambda = \frac{m_{pl}}{m_0} \) then

\[ \frac{1}{M_R} = \frac{m_e}{m_0} = 1 - \frac{m_s + m_f}{m_0} \frac{m_f}{m_s + m_f} \]

\[ \Rightarrow \frac{1}{M_R} = 1 - (1 - \lambda)(1 - \varepsilon) = \varepsilon + (1 - \varepsilon)\lambda \]
1.5 మాట్లాడు కాడను

మల్లిన రంగులు (l) ప్రాంతంలో మొత్తం ప్రాంతాలు ప్రాంతాలు లేదా ప్రాంతాలు మొత్తం ప్రాంతాలు ప్రాంతాలు లేదా ప్రాంతాలు ప్రాంతాలు లేదా ప్రాంతాలు ప్రాంతాలు లేదా ప్రాంతాలు ప్రాంతాలు లేదా ప్రాంతాలు ప్రాంతాలు లేదా ప్రాంతాలు ప్రాంతాలు లేదా ప్రాంతాలు ప్రాంతాలు 

మల్లిన రంగులు (l) ప్రాంతాలు ప్రాంతాలు ప్రాంతాలు లేదా ప్రాంతాలు ప్రాంతాలు లేదా ప్రాంతాలు ప్రాంతాలు లేదా ప్రాంతాలు ప్రాంతాలు 

మల్లిన రంగులు (l) ప్రాంతాలు ప్రాంతాలు ప్రాంతాలు లేదా ప్రాంతాలు ప్రాంతాలు 

మల్లిన రంగులు (l) ప్రాంతాలు ప్రాంతాలు ప్రాంతాలు 

మల్లిన రంగులు (l) ప్రాంతాలు 

మల్లిన రంగులు (l) ప్రాంతాలు 

మల్లిన రంగులు (l) 

మల్లిన రంగులు (l) 

మల్లిన రంగులు (l) 

1.5 మాట్లాడు కాడను
Or
\[ M_R = \frac{1}{\varepsilon + (1 - \varepsilon)\lambda} \]  \hspace{1cm} (6)

With these definitions, the change in velocity of the ideal rocket, which starts from rest, in the absence of gravity, is given by
\[ \Delta v = v - v_0 = v_{rel} \ln M_R = -v_{rel} \ln(\varepsilon + (1 - \varepsilon)\lambda) \]  \hspace{1cm} (7)

This equation tells us that the rocket efficiency will be more if the payload ratio (\(\lambda\)) is more. Here, efficiency deals with the amount of payload the rocket can carry, compared to the rest of its weight. That is possible only when the structure ratio (\(\varepsilon\)) of the rocket is low.

But one cannot reduce the structure ratio arbitrarily as a moderately high structure factor is necessary to bear the weight of the payload, propellant and at the same time to withstand the external pressure.

The performance of the rocket is defined by the ratio \(\Delta v / v_{rel}\), i.e., how fast it can attain the required maximum speed. Performance will be highest when \(\lambda = 0\). Performance will be zero when \(\lambda = 1\). i.e., when payload mass equals to rest of the mass of the rocket.

Thus, there is a trade-off between the efficiency and performance of a rocket in terms of payload ratio. A plot of payload ratio variations as a function of performance is shown in the figure below.

**Fig:** Plot of payload ratio vs performance of single stage rocket.
2. **Rockets**

   - When the rocket is fired, the exhaust gases are ejected from the nozzle with a high velocity. The exhaust velocity depends on the fuel and oxidizer used. The total impulse of the rocket depends on the mass flow rate of the exhaust gases and the area of the nozzle. The efficiency of the rocket engine is defined as the ratio of the actual impulse to the theoretical impulse.

   \[ \lambda = \prod_{i=1}^{N} \lambda_i \]

   \( \lambda_i = \frac{m_{0(i+1)}}{m_{0(i)}} \) for ith stage, where \( m_{0(i)} \) is the mass flow rate at the inlet of the ith stage, \( m_{0(i+1)} \) is the mass flow rate at the outlet of the ith stage, and \( N \) is the total number of stages.

   \[ \Delta v = \sum_{i=1}^{N} v_{exi} \ln[\epsilon_i + (1 - \epsilon_i)\lambda_i] \]

   Here, \( \epsilon_i \) is the exhaust velocity at the outlet of the ith stage.

   **Activit**

   Plot graphs of performance and efficiency for single-stage and multi-stage rockets using MATLAB.
1.5 Multi stage rocket

Multi stage rockets are used to enhance both efficiency and performance in an optimal manner. The majority of the weight of the rocket is contained in the propellant and the empty rocket. As the rocket rises higher and higher, its propellant gets consumed and the rocket becomes empty. However, it still has to carry the empty rocket, which is unnecessary. Multi stage rockets are designed to overcome these problems. Multi stage rockets are basically classified into two types.

1. Serial staging
2. Parallel staging.

In serial staging, each stage is fired one stage after the other in series. In parallel staging, several small stages are strapped onto a central sustainer. The strap on stages will be used to produce an initial thrust to overcome air drag. Sometimes, after a parallel stage, there may be a series stage implemented in order to reach the required height and velocity.

Usually in lower stages, solid or liquid propellants are used. The higher stages come into operation usually at high altitudes or at low air pressures or sometimes in vacuum. For this purpose, they can use low pressure combustion engines. Fuel also can be cryogenic liquid or gaseous type. This reduces the usage of complicated machinery parts which in turn gives rise to considerable weight reduction.

Multi stage rockets reduce the amount of space debris, as majority of the components fall back on to earth and can be collected for reuse. There are three types of reusable multistage rockets, depending on the type of landing system for the lower stages.

1. Breaking type: In this type, the detached lower stages fall back into sea or ocean by making a splash. To reduce the impact, they usually employ parachutes. In this method the capsule may encounter little to moderate damage due to splash impact while hitting water bodies during landing.
2. Space plane type: In this type, the vehicle can be used both as an aeroplane and as a rocket. Thus, it uses wings and
1.5 Multi stage rocket

Fig: RLV-TD of ISRO.

Fig: Discovery space shuttle.

Fig: Falcon-9 vertical landing space shuttle.
aeroplane principles in earth’s atmosphere and rocket principles in outer space. The thermal protection system while entering back to earth’s atmosphere is highly important in this type of systems. Indian Space Research Organization (ISRO) has successfully tested its RLV-TD (Reusable Launching Vehicle Technology Demonstrator).

3. Vertical landing type: In this type, the first stage empty part will carry an extra 10% fuel reserved for its return journey. This method is most successful due to its low cost and high safety to the returned rocket parts. They use Reaction Control System (RCS) technology to control the return journey. They have additional nozzles on the side ways of the capsule that can generate roll, yaw and pitch type of motion to control the movement of the capsule while re-entering into earth’s atmosphere and while landing.

Usually multi stage rocket systems are more prone to failure risk, as they involve additional machinery to separate the lower stage and fire the next stage on the fly. Multiple usage may not be possible due to quality deterioration in each landing, as a result of which they need to be refurbished for next usage.

The payload ratio for the multistage rocket is obtained by the product of individual stage payload ratios, as given below.

\[ \lambda = \prod_{i=1}^{N} \lambda_i \]

Here \( \lambda_i = \frac{m_{0(i+1)}}{m_{0(i)}} \) is the payload ratio of the \( i^{th} \) stage.

The change in velocity \( \Delta v \), produced by a multistage rocket, is given by

\[ \Delta v = \sum_{i=1}^{N} v_{ex_i} \ln[\epsilon_i + (1 - \epsilon_i)\lambda_i] \]

A plot of variation of payload ratio with performance for a multistage rocket is shown below. One can see that up to three stages, there is a significant improvement in performance and from
1.5 Multi stage rocket

Fig: RCS Mechanism for vertical landing.


E - Corner

Prepare a study report on various types of technologies and jet engines used in reusable launch vehicles.
the 4th stage onwards there is no significant effect of the number of stages on the performance of the rocket.

\[ N = 1, 2, 3, 10 \]

\[ \frac{\Delta u}{v_{rel}} \]

**Fig: Payload ratio vs Performance of multistage rocket.**

### 1.6 Rutherford scattering

Rutherford \( \alpha \) – particle scattering experiment is the ground breaking experiment that opened doors to a new era in mechanics namely old quantum mechanics. The phenomenon has been dealt at such a fundamental level that even simple conservation laws can be able to explain the observed experimental results. Thus stands as an evidence of simplistic fundamental characteristic of wonders of nature.

To begin with, consider an \( \alpha \) particle of charge \( 2e \) with initial velocity \( v_0 \) incident on a nucleus \( N \) of charge \( Ze \). The \( \alpha \) particle approaches the nucleus and repels back in a smooth hyperbolic path as shown in the figure. Let \( v_0 \) be the velocity at \( A \), let \( v \) be the velocity at point \( C \) and at point \( B \) velocity again attains a value of \( v_0 \). At point \( C \), the velocity of the \( \alpha \) particle is perpendicular to the radial vector connecting it with the nucleus. Let \( b \) be the distance of closest approach. i.e., the direct shortest distance between the nucleus and the \( \alpha \) particle at any instance. Let \( p \) be the \( \perp^{lr} \) distance between the
1.6 పార్స్కాన్న ప్యాటానిక్

పార్స్కాన్న క్రమానికే వెడడి ఖాతాల వలన 1.6 పార్స్కాన్న ప్యాటానిక్ ఉపయోగించడం కలిగింది. వాటిని పరిమితి కాలం తో కాదినంత మెటిగించడం సాధించాలి. ఈ పార్స్కాన్న ప్యాటానిక్ సంఖ్య పరిమితి కాలం తో కాదినంత మెటిగించడం సాధించాలి. అ మొత్తం విలువ పరిమితి కాలం తో కాదినంత మెటిగించడం సాధించాలి.
nucleus and the \( \alpha \) particle at any instance. This is called the impact parameter. Let \( \phi \) be the scattering angle. i.e., the angle between incident and emergent ray of \( \alpha \) particles.

Since there are no external forces acting on the system and since there is no energy loss in the process, one can apply laws of conservation of energy as well as momentum. Since the behaviour of \( \alpha \) particle is same in all directions around the nucleus, homogeneity of space exists and angular momentum also conserves. Applying angular momentum conservation at \( A \) and \( C \) yields

\[
mv_0 p = mv d \Rightarrow v = \frac{v_0 p}{d} \quad (1)
\]

and application of law of conservation of energy at \( A \) and \( C \) gives

\[
\frac{1}{2} mv_0^2 + 0 = \frac{1}{2} mv^2 + \frac{2Ze^2}{4\pi\epsilon_0 d} \quad (2)
\]

If the \( \alpha \) particle is incident along the radial vector, then it is called head-on collision and in that case impact parameter becomes zero \((p = 0)\) and only distance of closest approach \((d \neq 0)\) exists. Then, conservation of energy yields

\[
\frac{1}{2} mv_0^2 = \frac{2Ze^2}{4\pi\epsilon_0 b} \Rightarrow b = \frac{2Ze^2}{4\pi\epsilon_0 T_0} \quad (3)
\]

where \( T_0 \) is the initial kinetic energy of the \( \alpha \) particle.
1.6 మాత్రవణ కోషం

రూథర ఫో ర్స్సాకటంగి, C మధ్యలో M చెందిన 0 వీటిపై నిపుణ్ణమైన విశ్లేషణ. అంటే OC=OM=a (ప్రతిమన), ఆకు ΔMNO భాగం

\[ a = p \cot \theta \Rightarrow p = a \tan \theta \]  ---- (5)

\[ d = ON + OC = a / \cos \theta + a = a(\sec \theta + 1) \]  ---- (6)

Eq. (5) ప్రత్యేకించింది,

\[ d = a(\sec \theta + 1) = p \cot \theta (\sec \theta + 1) = p \frac{\cos \theta \left( \frac{1}{\cos \theta} + 1 \right)}{\sin \theta} \]

\[ \Rightarrow d = p \frac{2 \cos^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} = p \cot \frac{\theta}{2} \]

Eq. (5) ద్వారా Eq. (6) మర్పు ఎవుందుడు,

\[ b = \frac{(a + a \sec \theta)^2 - (a \tan \theta)^2}{a + a \sec \theta} \]

\[ = \frac{(a^2 + a^2 \sec^2 \theta + 2a^2 \sec \theta - a^2 \tan^2 \theta)}{a + a \sec \theta} \]

\[ = \frac{2a^2(\sec \theta + 1)}{a(\sec \theta + 1)} \]

\[ \Rightarrow b = 2a \]  ---- (7)

Eq. (5) మర్పు ద్వారా \( 2\theta + \phi = 180° \) ప్రత్యేకింది,

\[ \Rightarrow b = 2p \cot \theta = 2p \cot \left(90° - \frac{\phi}{2}\right) = 2p \tan \frac{\phi}{2} \]  ---- (8)

ప్రత్యేకింది, అంటే \( \phi \) సంఖ్యను అందించాలంటే, అంటే మూలాంలోని రెండవ పదం ఎలి ప్పుడూ లావితే, ఎకేడ సంఖ్య ఎలి ప్పుడూ 1 కాబటింది.

\[ \varepsilon = \frac{ON}{OC} = \frac{a / \cos \theta}{a} = \sec \theta = \sqrt{1 + \tan^2 \theta} = \sqrt{1 + \left(\frac{2p}{b}\right)^2} \]  ---- (9)

ఇచ్చిన విధానం సంఖ్య 1+ ఎలి ని రెండవ పదం ఎలి ప్పుడూ ప్పుడు లావితే ఉండాంది. కాబటి లాంటి మూలాంలోని రెండవ పదం ఎలి ప్పుడూ లావితే, కాబటి లాంటి మూలాంలోని రెండవ పదం ఎలి ప్పుడూ లావితే.
Substituting Eq. (1) and Eq. (3) in Eq. (2) gives a relation between the variables from the two studies as,

\[
\frac{2Ze^2}{4\pi\varepsilon_0 b} = \frac{1}{2}m\frac{p^2}{d^2} + \frac{2Ze^2}{4\pi\varepsilon_0 d} = \frac{p^2}{2} \frac{2Ze^2}{4\pi\varepsilon_0 b} + \frac{2Ze^2}{4\pi\varepsilon_0 d}
\]

\[
\Rightarrow \frac{1}{b} = \frac{p^2}{d^2} \frac{1}{b} + \frac{1}{d} \Rightarrow \frac{d^2 b}{b} = p^2 + \frac{d^2 b}{d} \Rightarrow d^2 = p^2 + db \Rightarrow b
\]

\[
= \frac{d^2 - p^2}{d} \quad -(4)
\]

Due to symmetry of the hyperbola, the points C and M fall on a circle with O as the center. Thus \(OC = OM = a\) (say), then from \(\Delta MNO\),

\[
a = p \cot \theta \Rightarrow p = a \tan \theta \quad -(5)
\]

From the figure

\[
d = ON + OC = a / \cos \theta + a = a(\sec \theta + 1) \quad -(6)
\]

Using Eq. (5) gives

\[
d = a(\sec \theta + 1) = p \cot \theta (\sec \theta + 1) = p \frac{\cos \theta}{\sin \theta} \left( \frac{1}{\cos \theta} + 1 \right)
\]

\[
= p \left( \frac{1 + \cos \theta}{\sin \theta} \right)
\]

\[
\Rightarrow d = p \frac{2 \cos^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} = p \cot \frac{\theta}{2}
\]

Substituting Eq. (5) and Eq. (6) in Eq. (4) results in

\[
b = \frac{(a + a \sec \theta)^2 - (a \tan \theta)^2}{a + a \sec \theta}
\]

\[
= \frac{(a^2 + a^2 \sec^2 \theta + 2a^2 \sec \theta - a^2 \tan^2 \theta)}{a + a \sec \theta}
\]

\[
= \frac{2a^2(\sec \theta + 1)}{a(\sec \theta + 1)}
\]

\[
\Rightarrow b = 2a \quad -(7)
\]

Using Eq. (5) and the observation that \(2\theta + \phi = 180^\circ\) gives
1.6 మిచ్చందక పద్ధతి

Eq. (6), Eq. (8) మీదాడం Eq. (7) వలన అంతరం కొండి, కనుక తరువాతి సమయం p సమీపాన్ని,

\[ d = a(1 + \sec \theta) = \frac{b}{2} (1 + \varepsilon) \]

\[ = \frac{b}{2} \left( 1 + \sqrt{1 + (2p/b)^2} \right) \] (10)

మొదలు లేదా, p=0 తో d=b, కనుక Eq.(4) వద్దను కనుగొనుకోండి. b మేఖల పరిమిత తో పరిమితం.

పోస్ట్రంగ్ కాంసెక్షన్

పోస్ట్రంగ్ కాంసెక్షన్ కాంసెక్షన్ అనేది, మొత్త కాంసెక్షన్ తప్పని కాంసెక్షన్ అంతే పంప్చుడు ఫండా బాయిర లో ఉన్నాంటే సెక్షన్ కణా లో లను ఉపయోగిసి, ప్ితాక్ష అత్రం తకే ఉద్యర ని కనుకేంట్రం,

\[ l = \pi p^2 l_0 \] (11)

p మేఖల ప+dp మారిని చేసి నిర్ణయించండి

\[ dl = 2\pi p dp l_0 \] (12)

ఈనిటి సమయం ఆధారంగా కాంసెక్షన్ ని మొత్తం సంతరం ని ప్రతిఫీల్‌కు కనుగొనుకోండి. అనంతం కాంసెక్షన్+dp మారి పంపే ఫండా బాయిరం ఆధారంగా కాంసెక్షన్ ని ప్రతిఫీల్‌కు కనుగొనుకోండి.

\[ \sigma I_0 d\omega = \sigma I_0 2\pi \sin \phi d\phi \] (13)

Eq.(12) మేఖల పరిమిత పరిమిత పరిమిత పరిమిత పరిమిత పరిమిత

\[ \sigma I_0 2\pi \sin \phi d\phi = -2\pi p dp l_0 \] (14)

ఆధారం తో పరిమిత p సమయం పంపే ఫండా బాయిరం మొత్తం సంతరం ని ప్రతిఫీల్‌కు కనుగొనుకోండి. పంపే ఫండా బాయిరం తో తెలుగు ఆధారం ని ప్రతిఫీల్‌కు కనుగొనుకోండి,

\[ \sigma = \frac{-2\pi p dp l_0}{I_0 2\pi \sin \phi d\phi} = -\frac{p dp}{\sin \phi d\phi} \] (15)

Eq.(8) పరిమిత

\[ p = \frac{b}{2} \cot \frac{\phi}{2} \Rightarrow dp = -\frac{b}{4} \cosec^2 \phi/2 d\phi = -\frac{b}{4} \frac{1}{\sin^2 \phi/2} d\phi \]

అనంతరం

\[ \sigma = \frac{4Z^2e^4}{(4\pi\varepsilon_0)^2T_0^2 16 \sin^4 \phi/2} = \frac{Z^2e^4}{16\pi^2\varepsilon_0^2m^2v_0^2 \sin^4 \phi/2} \] (16)
\[ b = 2p \cot \theta = 2p \cot \left(90^\circ - \frac{\phi}{2}\right) = 2p \tan \frac{\phi}{2} \quad - - - (8) \]

Thus, the scattering angle \( \phi \) will be more for smaller values of impact parameter \( p \).

From figure, one can define eccentricity as

\[ \varepsilon = \frac{ON}{OC} = \frac{a}{\cos \theta} = \sec \theta = \sqrt{1 + \tan^2 \theta} = \sqrt{1 + \left(\frac{2p}{b}\right)^2} \quad - - - (9) \]

Here eccentricity \( \varepsilon > 1 \) always, since the second term in the square root is square of a number which is always positive. Thus square root of \( 1+ \) a positive number will always be greater than 1, so that the trajectory of the \( \alpha \) particle is a hyperbola.

Using Eq. (6), Eq. (8) and Eq. (9), the distance of closest approach is given by

\[ d = a(1 + \sec \theta) = \frac{b}{2}(1 + \varepsilon) \]

\[ = \frac{b}{2} \left(1 + \sqrt{1 + \left(\frac{2p}{b}\right)^2}\right) \quad - - - (10) \]

For head on collision, \( p = 0 \Rightarrow d = b \), thus verifying the relation in Eq.(4). The value of \( b \) is given by Eq. (3).

**Scattering cross section**

The scattering cross section gives the number of particles scattered into the given solid angle, out of all the scattered particles or the incident particles. Let \( I_0 \) be the intensity of incident beam. Then the intensity of particles that are incident, within impact parameter range \( p \), are

\[ I = \pi p^2 I_0 \quad - - - (11) \]

The number of particles that fall within the range of \( p \) and \( p + dp \) is

\[ dl = 2\pi p \, dp \, I_0 \quad - - - (12) \]
1.6 Rutherford scattering

Rutherford scattering is a collision process in which a charged particle is scattered by another charged particle. The scattering is highly elastic, with a small cross-section.

1. **Scattering off a nucleus**: The scattering is governed by the interaction between the incident particle and the nucleus.
2. **Angular momentum**: The scattering is determined by the conservation of angular momentum.
3. **Energy conservation**: The scattered particle exchanges energy with the target.
4. **Relativistic effects**: In the relativistic case, the scattering cross-section is modified.

**E - Corner**

https://personal.math.ubc.ca/~cass/rutherford/r.pdf
http://www.olabs.edu/in/?sub=75&brch=12&sim=88&cnt=1
https://phet.colorado.edu/en/simulations/rutherford-scattering
https://demonstrations.wolfram.com/RutherfordScattering/
http://galileoandeinstein.physics.virginia.edu/more_stuff/Applets1/rutherford/rutherford.html

**Activit**

Derive scattering cross-section for relativistic case of Rutherford scattering problem.
Consider a screen at a distance of $r$ from the target. Then the number of particles that scatter within the solid angle between $\phi$ and $\phi + d\phi$ is

$$\sigma I_0 d\omega = \sigma I_0 \ 2\pi \sin \phi \ d\phi$$

From Eq. (12) and Eq. (13),

$$\sigma I_0 \ 2\pi \sin \phi \ d\phi = -2\pi p \ dp \ I_0$$

Here $-ve$ sign indicates an increment in $\phi$ with a decrement in $p$. Thus the magnitude of scattering cross-section is given by

$$\sigma = \frac{-2\pi p \ dp \ I_0}{I_0 \ 2\pi \sin \phi \ d\phi} = \frac{-p \ dp}{\sin \phi \ d\phi}$$

From Eq.(8),

$$p = \frac{b}{2} \cot \frac{\phi}{2} \Rightarrow dp = -\frac{b}{4} \cosec^2 \frac{\phi}{2} \ d\phi = -\frac{b}{4} \frac{1}{\sin^2 \frac{\phi}{2}} \ d\phi$$

Thus

$$\sigma = \frac{p \ dp}{\sin \phi \ d\phi} = \frac{b \cos \frac{\phi}{2}}{2 \sin \frac{\phi}{2}} \frac{b}{4 \sin^2 \frac{\phi}{2}} \ d\phi = \frac{b^2}{16 \sin^4 \frac{\phi}{2}}$$

Or

$$\sigma = \frac{4Z^2 e^4}{(4\pi \epsilon_0)^2 T_0^2 \ 16 \ \sin^4 \frac{\phi}{2}} = \frac{Z^2 e^4}{16 \pi^2 \epsilon_0^2 m^2 v_0^4 \sin^4 \frac{\phi}{2}}$$
1.7 Future insights

Hugh D. Young, Roger A. Freedman - Sears and Zemansky’s University Physics with Modern Physics-Pearson Education (2015)


Here the scattering cross-section is inversely proportional to square of kinetic energy and to the 4\textsuperscript{th} power of sine of scattering angle. From the above equation, one may observe that

1. For larger nuclei, larger is the value of \(Z\) and hence larger is the scattering angle.
2. For high kinetic energy particles, scattering cross section will be low.
3. Larger the angle of scattering \(\phi\), smaller is the scattering cross section.
4. For head on collision, scattering cross section is the lowest.

**Solved Problems & Exercises**

**Problem 1. Astronaut’s tug of war**: Two astronauts A and B on a spacewalk decide to play tug of war by pulling on either end of a rope. Suppose A is stronger than B and mass of the rope is neglected. Find the motion of the rope if each astronaut pulls on the rope as hard as possible.

Sol: The force diagram is as follows.

\[
\begin{align*}
M_A &\quad F_A \\
F_A &\quad a_A \\
M_r &\quad F_B \\
F_B &\quad a_B \\
M_B &
\end{align*}
\]

Let \(F_A\) and \(F_B\) are forces exerted on the rope by the Astronauts A and B respectively. And \(F_A'\) and \(F_B'\) are the forces exerted by the rope on the astronauts A and B respectively.

Hence from Newton’s third law, \(F_A = F_A'\) and \(F_B = F_B'\). Now the equation of motion of the rope is \(F_A - F_B = M_r a_r = 0 \Rightarrow F_A = F_B\).

\(\Rightarrow\) \(F_A' = F_B'\) i.e the astronauts must pull with the same force. No matter how much stronger the astronaut A than B, A is unable to pull harder than B!!
Problem 2. String of N cars: Consider a string of N cars, each of mass M and pulled by a force F. Find the force that pulling last n cars.

\[ \text{Sol: Given mass of each car is M hence total mass of the system is NM} \]

\[ \text{and the cars are joined and thus constrained to have same acceleration ‘a’. } \]

\[ F \text{ is the force of pull of the cars} \]

Hence from Newton’s second law, \[ F = (NM)a \Rightarrow a = \frac{F}{NM} \]

The net force experienced by each car to the right is \[ F_{Net} = Ma = M \times \frac{F}{NM} = \frac{F}{N} \]

\[ \therefore \text{ The force that pulling last n cars = sum of all net forces} \]

\[ \text{experienced by n cars} = n \times \frac{F}{N} \]

From above force diagrams, we can observe that, the normal force exerted by the floor and weight of the car cancel each other in each case of the masses because there is no vertical acceleration.

Hence equation of motion for the 3\textsuperscript{rd} car or the force on 3\textsuperscript{rd} car is

\[ F_3 = Ma = M \times \frac{F}{3M} = \frac{F}{3} \]

Equation of motion for middle car is

Force by 1\textsuperscript{st} car – Force by 3\textsuperscript{rd} car = \[ F_2 - F_3' = Ma \]
Solved Problems & Exercises

But from Newton’s third law, $F'_{3} = F_3 \Rightarrow F_2 - \frac{F}{3} = M \times \frac{F}{3M} = \frac{F}{3}$

$$F_2 = \frac{2F}{3}$$

Equation of motion for the 1st car is

Force by 1st car – Force by 3rd car $= F_2 - F'_{3} = Ma$

But from Newton’s third law, $F'_{3} = F_3 \Rightarrow F_2 - \frac{F}{3} = M \times \frac{F}{3M} = \frac{F_2}{3}$

**Problem 3.** A block of mass 2 kg is at rest on a horizontal table. The coefficient of friction between the block and the table is 0.1. A horizontal force of 3 N is applied to the block. Find the speed of the block after it has moved a distance 10m.

IIT JAM 2015

Solution: The speed of block at 10m distance $v = \sqrt{2as} = \sqrt{2 \times a \times 10}$

The net force acting on block

$ma = \text{applied force} - \text{frictional force} = 3 - F_{\text{frictional}}$

$F_{\text{frictional}} = \mu mg = 0.1 \times 2 \times 9.8 = 1.96$

$ma = 3 - 1.96 = 1.04 \Rightarrow a = \frac{1.04}{2} = 0.52 \text{ m/s}^2$

$\therefore v = \sqrt{2 \times 0.52 \times 10} = 3.22 \text{ m/s}$

**Problem 4:** A raindrop falling vertically under gravity gather moisture from the atmosphere at a rate given by $\frac{dm}{dt} = kt^2$, Where $m$ is the instantaneous mass, $t$ is the time $k$ is a constant. The equation of motion of the raindrop is

$$m \frac{dv}{dt} + v \frac{dm}{dt} = mg$$

If the drop starts falling at $t$ is equal to zero, with zero initial velocity and initial mass $m_0$ (given $m_0 = 2gm, k = \frac{12gm}{s^3}$ and $g = 1000 cm/s^2$), Find the velocity $v$ of the drop after 1 sec.

IIT JAM 2011
Sol: Given \( \frac{dm}{dt} = kt^2 \Rightarrow m(t) = \frac{kt^3}{3} + C = \frac{kt^3}{3} + m_0 \) [Since here at \( t=0 \), \( m=m_0 \)]

At \( t = 1 \) sec, \( m(1) = \frac{12 \times 1^3}{3} + 2 = 6 \) gm

\[ m \frac{dv}{dt} + v \frac{dm}{dt} = mg \Rightarrow \frac{d}{dt}(mv) = mg \Rightarrow m_1 v_1 - m_0 v_0 = m_1 g \ dt \]

\( \Rightarrow v_1 = \frac{g \ dt}{1} = 1000 \text{ cm/sec} \)

(Here at \( t=1 \) sec, instantaneous mass \( = m_1 \) and \( dt = 1 \) sec)

**Problem 5.** Show that greater the value of \( \frac{M_{\text{fuel}}}{M_{\text{vehicle}}} \), the greater be speed attained by rocket.

**Sol:**

We know that

\[ v = v_o + u \ log_e \left( \frac{M_o}{M} \right) \]

When the fuel is burnt out completely, the remaining mass corresponds to the mass of the vehicle \( (M_{\text{vehicle}} = M_v) \).

Above equation can be written as

\[ v = v_o + u \ log_e \left( \frac{M_o}{M_v} + 1 - 1 \right) \]

\[ v = v_o + u \ log_e \left( 1 + \frac{M_o}{M_v} - 1 \right) \]

\[ v = v_o + u \ log_e \left( 1 + \frac{M_o - M_v}{M_v} \right) \]

We know that

\[ M_{\text{fuel}} = M_o - M_v \]

Therefore

\[ v = v_o + u \ log_e \left( 1 + \frac{M_{\text{fuel}}}{M_{\text{vehicle}}} \right) \]

It clearly shows that, if \( \frac{M_{\text{fuel}}}{M_{\text{vehicle}}} > 1 \) then only final velocity \( v \) of the rocket increases.
i.e. greater the $\frac{M_{fuel}}{M_{vehicle}}$ value, then the greater is the speed attained by the rocket

**Problem 6:** The first and second stages of a two stages rocket have weight 100kg and 10kg and carry 800kg and 90kg of fuel supply. The velocity of the ejected gases relative to the rocket is 1.5km/sec. Find the final velocity attained by the rocket. Given ($\log_e 10 = 2.3$ and $\log_{10} 2 = 0.3$)

Sol:

The velocity of the rocket at an instant of time is given by

$$v = v_o + u \log_e \frac{M_o}{M}$$

First stage

$M_o = 100 + 10 + 800 + 90 = 1000kg$

$M = 100 + 10 + 90 = 200kg$

Rocket initial velocity $v_o = 0$.

And gasses ejecting velocity $u = 1.5km/sec$

$$v_1 = v_o + u \log_e \frac{M_o}{M}$$

$$v_1 = 0 + 1.5 \log_e \frac{1000}{200}$$

$$v_1 = 2.41 \text{ km/sec}$$

For second stage

$M_o = 10 + 90 = 100kg$

$M = 10kg$

Rocket initial velocity $v_o = v_o = 2.41 \text{ km/sec}$.

And gasses ejecting velocity $u = 1.5km/sec$

$$v_2 = v_1 + u \log_e \frac{M_o}{M}$$
\[ v_2 = 2.41 + 1.5 \log_e \frac{100}{10} \]
\[ v_2 = 2.41 + 3.45 \]
\[ v_2 = 5.48 \text{ km/sec} \]

**Problem 7:** A rocket having an initial mass \( M_0 \) starts from rest. When it attains a velocity \( v \), its mass becomes \( M \). What is the ration of \((M_0/M)\) when the velocity of exhaust gases is equal to \( v \) (numerically).

**Sol:**

The velocity \( v \) of the rocket at any instant of time \( t \) is given by

\[ v = v_o + u \log_e \frac{M_0}{M} \]

Given that \( v_o = 0 \) and \( u = v \)

Therefore, \( v = 0 + v \log_e \frac{M_0}{M} \)

\[ v = v \log_e \frac{M_0}{M} \]

\[ \frac{M_0}{M} = e = 2.717 \]

**Problem 8.** 5 Mev \( \alpha \) particles are scattered back from a metal foil with \( Z = 79 \). Calculate the maximum volume in which the positive charge of the atom is likely to be concentrated.

**Sol:** Given the alpha particles are scattered back. Ie at the point of closest approach ‘b’, the particle kinetic energy is converted in to potential energy

\[ \therefore K.E = P.E \Rightarrow \frac{1}{2}mv_0^2 = \frac{2Ze^2}{4\pi\varepsilon_0 b} \]

\[ => 5 \times 10^9 \times 1.69 \times 10^{-19} = \frac{2 \times 79 \times (1.69 \times 10^{-19})^2}{\left(\frac{1}{9 \times 10^9}\right)b} \]

\[ \Rightarrow b = \frac{2 \times 79 \times (1.69 \times 10^{-19})^2 \times 9 \times 10^9}{5 \times 10^9 \times 1.69 \times 10^{-19}} = 480 \times 10^{-19} = 0.048 \text{ fm} \]
\[ \Rightarrow \text{Maximum volume} = \frac{4}{3} \pi b^3 = \frac{4}{3} \times 3.14 \times 0.048^3 \text{ m}^3 = 0.000463 \text{ m}^3 \]

**MCQs**

1. The scattering cross section has dimensions of
   a) Volume b) Area c) Mass d) Density

   AUCET 2020
   Ans: b

2. In a Rutherford scattering experiment, the number of scattered particles per unit area is proportional to
   a) \(1/ \sin^4 \frac{\phi}{2}\) b) \(Z\) c) \(Z^2\) d)\(V^2\)

   AKNU 2020
   Ans: a and c

3. In a Rutherford’s scattering experiment of \(\alpha\) – particles by metallic foils, with the increase of atomic number of nucleus, the scattering angle is
   a) Remains unchanged b) Decreases c) Increases d) First decreases and then increases

   AKNU 2020
   Ans: c

4. A rocket burns 0.05kg of fuel per second and ejects the burnt gases with a velocity of 5000 m/s, the reaction is
   a) 250 N b) 225 N c) 210 N d) 230 N

   AKNU 2020
   Ans: a
5. A rocket of mass $M_0$, takes off with a constant velocity $v_0$, in a
uniform gravitational field. The rocket loses fuel mass, as it is
propelled by the gas, which is ejected with a velocity $u$ relative
to the rocket. At a later time $t_f$, is given that the mass of the
rocket is $M_f$ the velocity of the rocket $v_f$ is equal to

\[ a) u \ln \left( \frac{M_0}{M_f} \right) - g t_f \quad b) -u \ln \left( \frac{M_0}{M_f} \right) \quad c) u \ln \left( \frac{M_0}{M_f} \right) \quad d) u \ln \left( \frac{M_0}{M_f} \right) - g t_f \]

HCU 2019

Ans: a

6. A particle whose mass varies with the time, moves under the
influence of a force $\vec{F}$. If $T$ is the kinetic energy and $\vec{p}$ is the
momentum of this particle, which of the following equation is
valid

\[ a) \frac{dT}{dt} = \frac{\vec{F} \cdot \vec{p}}{m} \quad b) \frac{d(mT)}{dt} = \frac{\vec{F} \cdot \vec{p}}{m} \quad c) \frac{d(mT)}{dt} = m \vec{F} \cdot \vec{p} \quad d) \frac{d(mT)}{dt} = \vec{F} \cdot \vec{p} \]

HCU 2016

Ans: a

Hint : As per the dimensions option a is correct

7. Consider a Rutherford’s scattering experiment in which 2 MeV
alpha particles were incident on a thin gold Au$^{197}$ foil. The
distance of closest approach is

a) 228 fm   b) 567 fm   c)114 fm   d)197 fm

HCU 2016

Ans:c

\[ b = \frac{2Z e^2}{4 \pi \varepsilon_0 \rho_0} = \frac{2 \times 79 \times (1.6 \times 10^{-19})^2}{2 \times 10^9 \times 1.6 \times 10^{-19} \times 9 \times 10^9} = 114 \text{ fm} \]

Here

\[ \frac{1}{4 \pi \varepsilon_0} = 9 \times 10^9 \]

and $T_0 = 2 \text{ MeV} = 2 \times 10^9 \times 1.6 \times 10^{-19} J$
8. If the distance between two bodies of masses $m_1$ and $m_2$ is doubled, the gravitational force between them becomes
   \[ a) \text{ double} \quad \quad \quad b) \text{ half} \quad \quad \quad c) \text{ one forth} \]
d) Unchanged

HCU 2014

Ans : c

9. Consider two particles moving along the $x$-axis. In terms of their coordinates $x_1$ and $x_2$, their velocities are given as $\frac{dx_1}{dt} = x_2 - x_1$, $\frac{dx_2}{dt} = x_1 - x_2$ respectively. When they start moving from their initial locations of $x_1(0) = 1$ and $x_2 = -1$, the time dependence of both $x_1$ and $x_2$ contains a term of the form $e^{at}$, where $a$ is constant. The value of $a$ (an integer) is

IIT JAM 2017

Ans: $a = -2$

Hint: Let $x_1 = Ae^{at}$ and $x_2 = Be^{at}$. Given $\frac{dx_1}{dt} = aAe^{at} \Rightarrow$ 

\[ ax_1 = x_2 - x_1 \quad ----(1) \]

\[ \frac{dx_2}{dt} = aBe^{at} \Rightarrow ax_2 = x_1 - x_2 \quad ----(2) \]

(1) − (2) gives $a(x_1 - x_2) = 2(x_2 - x_1) \Rightarrow a = -2$

10. A particle moves in a circular path in the $xy$-plane centered at the origin. If the speed of the particle is constant, then its angular momentum

   a) About the origin is constant both in magnitude and direction

   b) About (001) is constant in magnitude but not in direction

   c) About (001) varies both in magnitude and direction
d) About (001) is constant in direction but not in magnitude

IIT JAM 2016

Ans: a and b

11. A rod is hanging vertically from a pivot. A particle, travelling in horizontal direction, collides with the rod as shown in the figure. For the rod-particle system, consider the linear momentum and the angular momentum about the pivot. Which of the following statement are not true

a) Both the linear momentum and angular momentum are conserved
b) Linear momentum is conserved but angular momentum is not
c) Linear momentum is not conserved but angular momentum is conserved
d) Neither and linear momentum nor angular momentum are conserved

IIT JAM 2015

Ans: a

12. Two points N and S are located in the Northern and Southern hemisphere, respectively, on the same longitude. Projectiles P and Q are fired from N and S respectively, towards each other. Which of the following options is correct for the projectiles as the approach the equator? a) Both P and Q will move towards the East b) Both P and Q will move towards the west c) P will move towards the east and Q towards the west d) P will move towards the west and Q towards the East

IIT JAM 2014

Ans: b
Problems

Grade your understanding

1. In a variable mass system, the ejected mass ($\Delta m$) moves opposite to the remaining mass ($m - \Delta m$) [ ]
2. Rocket motion needs modified Newton’s laws of motion [ ]
3. Newton’s Laws of motion are universal [ ]
4. The rate of change of momentum of uniformly moving variable mass is zero [ ]
5. Cryogenic liquid or gaseous type fuels reduces considerably the weight of the rocket [ ]
6. The trajectory of $\alpha$ particle in Rutherford scattering is either hyperbola or a straight line [ ]
7. Mass of satellite or other higher stages excluding empty structural mass is called payload mass [ ]
8. Performance of the rocket will be zero when, when payload mass equals to rest of the mass of the rocket [ ]
9. The angle between incident and emergent ray of $\alpha$ particles is called impact parameter [ ]
10. For high kinetic energy particles, scattering cross section will be low [ ]


13. A 400kg rocket is set for vertical firing. Exhaust speed of the gases is 490m/s. The rate at which gas is to be ejected to give an upward acceleration of g to the rocket
   a) 160kg/s     b) 16kg/s     c) 1.9g/s     d) 80kg/s
   Ans: b

14. If M is mass of rocket, r is rate of ejection of gases and u is velocity of gasses with respect to rocket then acceleration of the rocket is
   a) $ru/(M-rt)$   b)$(M-rt)/ru$   c)$ru/(M+rt)$   d)$ru/M$
   Ans: a

15. A rocket takes off from Earth and reaches a speed of 100 m/s in 10.0 s. If the exhaust speed is 1500 m/s and the mass of fuel burned is 100 kg, what was the initial mass of the rocket? (neglect the gravity on rocket)
   a) 1501 kg     b) 1601 kg     c)1551 kg     d)1651 kg
   Ans: c
<table>
<thead>
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<tr>
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<td><strong>Centrifugal force</strong></td>
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<td><strong>Isotropy</strong></td>
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<td><strong>Impulse</strong></td>
</tr>
<tr>
<td><strong>Intensity of particles</strong></td>
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<tr>
<td><strong>Laws of motion</strong></td>
</tr>
<tr>
<td><strong>Payload</strong></td>
</tr>
<tr>
<td><strong>Point like Particle</strong></td>
</tr>
<tr>
<td><strong>Propellant</strong></td>
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<tr>
<td><strong>Radial vector</strong></td>
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<tr>
<td><strong>Review</strong></td>
</tr>
<tr>
<td><strong>Rocket staging</strong></td>
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<td>Solid angle</td>
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<td>Thrust</td>
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UNIT-I

Chapter-2

Rigid body Dynamics

డృఢ వస్తువుల గతశాస్త్రం
CHAPTER 2

ప్రవేశము

1. దృఢవల గాణం, మండలం నండి మీద సందర్భాలు.

2. సంక్షేపాని మేలుకపోతుంది మిగిలిన పరిమితులు సంతరనం.

3. ప్రతి వేల మిగిల సాధన పద్ధతి.

4. ఏమిటి ప్రభుత్వాని మేలుకపోతుంది రెండు పరిమితులు పరిమితులు సంతరనం.

5. ప్రవేశము, ప్రవేశం ఆసక్తి ఇది సందర్భాలు మాత్రమే, మాత్రమే రెండు పరిమితులు సంతరనం.

అభసన ఫ్రూండ్

1. మాడి నైను ప్రమాణాని ప్రశ్నలు సాధన అంశాల ప్రాంతాలు సంతరనం.

2. సంబంధాల సాధన ఆలోచన పరిమితులు మాత్రమే సంతరనం.

3. ప్రవేశం ఆసక్తి ఇది ప్రశ్నలు సాధన అంశాల ప్రాంతాలు సంతరనం.

4. ఆసక్తి ఎక్కడ ప్రశ్నలు సాధన అంశాల ప్రాంతాలు సంతరనం.

5. ఆసక్తి ఎక్కడ ప్రశ్నలు సాధన అంశాల ప్రాంతాలు సంతరనం.

6. ప్రవేశం ఆసక్తి ఇది ప్రశ్నలు సాధన అంశాల ప్రాంతాలు సంతరనం ప్రాంతాలు ప్రాంతాలు సంతరనం అంశాల ప్రాంతాలు.
RIGID BODY DYNAMICS

Syllabus

Rigid body, rotational kinematic relations, Equation of motion for a rotating body, Angular momentum and Moment of inertia tensor, Euler equations, Precession of a spinning top, Gyroscope, Precession of atom and nucleus in magnetic field, Precession of the equinoxes

Learning Objectives

In this chapter students would learn about,

1. Rigid body definition, kinematic relations of rotatory motion.
2. Angular momentum and moment of inertia tensor.
3. Equation of motion of rotating body.
4. Euler equations and precession of top
5. Gyroscope, Precession of atoms in magnetic field, Precession of Equinoxes.

Learning Outcomes

By the end of the chapter, student would be able to

1. Define various physical quantities related to rotatory motion of rigid bodies.
2. Explain angular momentum in various dimensional spaces.
3. Calculate moment of inertia tensor for various systems.
4. Analyze Euler angles in various systems of interest.
5. Justify the effects of precession in various systems of interest.
6. Develop prototype models and equations for momentum of inertia and precession effects in rigid bodies.
Syllabus

ధ్యంలంలనంషఫమభషశాయంసమషలంరంలదృఢవంంకఅవసంచగల.

1. కసం: వలరసరణ, పరలరసరణ; ఆలసకరుంంంఅంతవరుందృఢవలదకరం.

2. లం: లంఇనమషపనత.

3. కంటు: లదవలవరచలంయంఆఇనమంనంఅవసరం.

4. గరసం: ధకావపంవడంమయంపనత.

5. ఎల్యు: ధమంయంయంచబనలూంచంయంఆఇనమంనంఅవసరం.

6. నదకశ: శలవవసంఆఇనమంపండాడుడ.

7. గం: ధకయంగంకషణంఆఇనమంనంరసరణంనంఅవసరం.

పాతంంఅపతంఈవ తరగరవవసగ, గ, సహజ మక్రమ ఉపగలంపచయం.
Course Outcomes specific to program and Future directions

By the end of this chapter students from specific programs would be able to identify the need of rigidbody dynamics in the following fields.

1. Physics: besides Precession of equinoxes, precession of atoms; Euler equations guide the motion of rigid bodies ranging from bicycle to spaceship.
2. Chemistry: Moment of inertia and precession plays a major role in molecular spectroscopy.
3. Computer Science: simulation of video games to mars rovers require knowledge about moment of inertia and precession.
4. Geology: Study of moment of inertia plays a major role in understanding the mass distribution in various geological structures.
5. Electronics: Designing sensors that are attached to various rotating machine parts requires knowledge about moment of inertia and precession.
6. Renewable energy: Flywheel energy storage system works on the principle of moment of inertia. Knowledge about Precession of equinoxes is required in estimating the tidal energy.
7. Statistics: Statistical analysis of various industrial machinery requires knowledge of moment of inertia and precession.

Familiar to Unfamiliar

In your 11th class you might have learned about the concept of rigid body, center of mass, moment of inertia; calculation of moment of inertia for symmetric and asymmetric objects, kinematic and dynamic equations of rotatory motion. In this chapter, we shall be building on the above knowledge and some applications of the concepts.
చాలా సమయం ఇమ్మ్లు నుండి ప్రస్తుతికరణ తరువాతం ఉన్నది. ఈ ప్రపంచంలో ప్రతి మల్ల అధిక బాలుల గ్రహణమాయం సంచి సట్లు (అంధ/బ్లై) అధికంగా ఉంటాయి. బ్రహ్మాండ ఉపషండి ఉదయం లూ అధికం ప్రమాణం. వర్ష పూణ, మంచిందు అయితే, మొదట నీటి మాంసాన్ని సూపార్చిన చివరి 7వ సంవత్సరం అది మాంసాన్ని కలిగి ఉంటుంది. దూర్మాంస మాంసాన్ని కలిగి ఉంటుంది. మాంసాన్ని కలిగి ఉంటుంది. 8వ సంవత్సరం మాంసాన్ని కలిగి ఉంటుంది. మాంసాన్ని కలిగి ఉంటుంది. 9వ సంవత్సరం మాంసాన్ని కలిగి ఉంటుంది. మాంసాన్ని కలిగి ఉంటుంది. 10వ సంవత్సరం మాంసాన్ని కలిగి ఉంటుంది. మాంసాన్ని కలిగి ఉంటుంది. 11వ సంవత్సరం మాంసాన్ని కలిగి ఉంటుంది. మాంసాన్ని కలిగి ఉంటుంది. 12వ సంవత్సరం మాంసాన్ని కలిగి ఉంటుంది. మాంసాన్ని కలిగి ఉంటుంది.

యమండి సాంప్రదాయం ప్రధానంగా వి రంగాలకు లమ్బాదంగా ఉంటుంది. ఈ ప్రపంచంలో ప్రతి మల్ల అధిక బాలుల గ్రహణమాయం సంచి సట్లు (అంధ/బ్లై) అధికంగా ఉంటాయి. బ్రహ్మాండ ఉపషండి ఉదయం లూ అధికం ప్రమాణం. వర్ష పూణ, మంచిందు అయితే, మొదట నీటి మాంసాన్ని సూపార్చిన చివరి 7వ సంవత్సరం అది మాంసాన్ని కలిగి ఉంటుంది. దూర్మాంస మాంసాన్ని కలిగి ఉంటుంది. మాంసాన్ని కలిగి ఉంటుంది. 8వ సంవత్సరం మాంసాన్ని కలిగి ఉంటుంది. మాంసాన్ని కలిగి ఉంటుంది. 9వ సంవత్సరం మాంసాన్ని కలిగి ఉంటుంది. మాంసాన్ని కలిగి ఉంటుంది. 10వ సంవత్సరం మాంసాన్ని కలిగి ఉంటుంది. మాంసాన్ని కలిగి ఉంటుంది. 11వ సంవత్సరం మాంసాన్ని కలిగి ఉంటుంది. మాంసాన్ని కలిగి ఉంటుంది. 12వ సంవత్సరం మాంసాన్ని కలిగి ఉంటుంది. మాంసాన్ని కలిగి ఉంటుంది.

https://www.sutori.com/story/atomic-theory-timeline--GtGsBHe8bqzrzFeP6tEng4Vo
https://www.mcgoodwin.net/pages/otherbooks/tlc_rerumnatura.html
https://en.wikipedia.org/wiki/Atomism
https://wiki.seg.org/wiki/Seismic_attenuation
https://www.youtube.com/watch?v=shdLjlkRaS8
2.1 Introduction

The earliest theories of atomism have their origins in India and Greece. Uddalaka Aruni in Brihadaranyaka Upanishad describes about indivisibility of atoms (anu/kana) in 8th century BC. According to him, matter is composed of three elements namely Fire/light, water and food/plants. His ideas were further developed by Ajivaka and Charvaka schools in 7th century BC. They considered four elements, which are the earth, water, air, fire that constitute matter. Subsequently, Kanada or Kasyapa of 5th century BC considered matter constituted of earth, water, air, fire and sky/ether. Kanada proposed paramanu (the ultimate atom) as indestructible. He also proposed that atoms will be of four types that constitute matter. In 4th century BC Greek philosophers Lucretius and Democritus proposed atoms are indistinguishable and there are infinite types of atoms with different shapes and sizes that constitute matter. They have proposed that there must be open space (void) between atoms which is responsible for fluidity of liquids and gases and for solids to have very less void. These theories consider the nature to be composed of 4 elements namely air, water, fire and earth. Later, in 3rd century BC, Aristotle proposed atoms do not exist but the entire nature is composed of some combination of four fundamental elements namely earth, fire, water and air. Democritus’s ideas were set aside by Aristotle’s ideas for about 2000 years. In 18th century Lavoisier proved experimentally that water is composed of hydrogen and oxygen and can be decomposed. This resulted in Democritus ideas coming into lime light again. This was further developed by John Dalton leading to classification of elements based on their atomic weights. According to the current understanding, there are four fundamental states of matter, namely, solids, liquids, gases and plasma. There are several other states which are observable at extremely low temperatures or at extremely high energies.
2.1 ఇంటర్నెషన్

మధ్యమ పశ్చిమ సాహిత్యంలో, ముందు - చంద్ర నృషాతా కలెక్టివేషన్ నిర్మాణం గురించి పలు ప్రశ్నలు ఉన్నాయి. ఇక్కడ చంద్ర నృషాతా జ్యోతిస్తు చాలా విస్తరించిన వివరణలు ఉండాలంటాయి.

ఇక్కడ నృషాతా చంద్రం చేసిన ముందు ఎంత మాటం ఉంది ఇక్కడ అధికంగా వివరించబడింది. ప్రత్యేకించి చంద్రం మాటం ఉంది ఇది సమాధానం చేసేవిస్తా ప్రశ్నలు ఉన్నాయి.

ఇక్కడ చంద్రం మాటం ఉంది ఇది ఉంది ఇది మాటం ఉంది ఇది మాటం ఉంది ఇది మాటం ఉంది ఇది మాటం ఉంది ఇది మాటం ఉంది ఇది మాటం ఉంది ఇది మాటం ఉంది.
Mechanics of solids is studied by using Newtonian and further by Euler–Lagrange and Hamilton mechanisms. Mechanics of liquids are studied by fluid dynamics. Mechanics of gases are studied by Maxwell’s kinetic gas equations. Plasma dynamics are also modelled similar to fluid dynamics or kinetic gas equations. The fundamental assumption in fluid dynamics is that the shear modulus of fluids is zero. For solid materials, stress is a function of strain; but for fluids, shear stress is a function of time rate of change of strain. The most ideal and simplest model for fluids is the Newtonian fluid. In these fluids, viscous strain is linearly proportional to rate of change of stress. In general, liquids are considered incompressible i.e., their density does not change with stress. Eg: Water. But gases are considered compressible i.e., their density changes with change in stress. Eg: air.

A rigid body is considered as incompressible under the application of external stress, force etc. i.e.; the distance between any two points remains constant in rigid bodies. In practice, this is an approximation that would work out in specific limits only. Any material body can be deformed if suitably high amounts of pressure is applied. Mechanics of rigid body can be simplified by the introduction of center of mass. Center of mass is obtained by calculating the first moment of mass. In physics, moment is calculated by multiplying the given physical quantity with radial distance.

\[ MR = \sum_i m_i r_i = (m_1 r_1 + m_2 r_2 + \cdots) \Rightarrow R = \frac{1}{M} \sum_i m_i r_i \]

Here \( m_1, m_2 \ldots \) are individual masses. \( r_1, r_2 \ldots \) are the radial distances from origin to the respective masses. \( M = m_1 + m_2 \ldots \) indicate the total mass of the system and \( R \) represents the radial distance of origin to the center of mass.

Here first moment of mass gives center of mass and second moment of mass gives moment of inertia. i.e., the moment of center of mass. To understand the flexibility while dealing with center of mass, consider the following case.
2.1 Introduction

Consider two masses \( m_1, m_2 \) in a certain gravitational field. \( r_1 \) and \( r_2 \) are the distances between the centers of mass of \( m_1 \) and \( m_2 \) and the centers of mass of the entire system. \( R \) is the distance between the centers of mass of the entire system. The masses are in a gravitational field, where

\[
r = r_2 - r_1 \quad ---(1)
\]

In this case, the potential energy \( u \) of the system is given by

\[
u = \frac{m_1 m_2}{r_2} - \frac{m_1}{r_1} - \frac{m_2}{r_2} = \frac{m_1 m_2}{R^2} - \frac{m_1}{r_1} - \frac{m_2}{r_2}
\]

Then

\[
M = m_1 + m_2 \quad ---(3)
\]

The force \( F \) between the masses is given by

\[
F = \frac{m_1 m_2}{r^2} \quad ---(4)
\]

or

\[
\frac{\ddot{r}_1}{m_1} = -\frac{F(r)}{m_1} \quad ---(5)
\]

\[
\frac{\ddot{r}_2}{m_2} = \frac{F(r)}{m_2} \quad ---(5)
\]
Consider two masses $m_1, m_2$ separated by a distance $r$. Let $r_1$ and $r_2$ be the coordinates of the masses from some observer frame of reference. Let $R$ be the center of mass of the system. Then from the vector diagram,

$$r = r_2 - r_1 \quad \text{(1)}$$

Center of mass is the point at which the entire mass of the system is considered to be concentrated for all point particle assumptions in Newtonian mechanics. Thus

$$m_1R + m_2R = m_1r_1 + m_2r_2$$

This equation means, keeping mass $m_1$ at $r_1$ and mass $m_2$ at $r_2$ is equivalent to keeping both the masses at $R$. Thus one can convert a rigid body into a point particle. This equation can be rearranged to obtain

$$R = \frac{m_1r_1 + m_2r_2}{m_1 + m_2} = \frac{m_1r_1 + m_2r_2}{M} \quad \text{(2)}$$

where

$$M = m_1 + m_2 \quad \text{(3)}$$

From the vector diagram, one can obtain the vectors in relative coordinate system, where distances measured by keeping the center of mass as origin are as given below

$$r_{1c} = r_1 - R = r_1 - \frac{m_1r_1 + m_2r_2}{m_1 + m_2} = \frac{m_2(r_1 - r_2)}{m_1 + m_2} = \frac{m_2r}{m_1 + m_2}$$
2.1 Introduction

Eq.(1) to Eq.(4) are followed by Eq.(5) to Eq.(10):

\[ \ddot{r} = F(r) \left( \frac{1}{m_1} + \frac{1}{m_2} \right) = \frac{F(r)}{\mu} \quad (5) \]

and

\[ \mu = \frac{m_1 m_2}{m_1 + m_2} \quad (6) \]

Next, let us consider a simple example

\[ I = m_1 r_1^2 + m_2 r_2^2 = MR^2 + \mu r^2 \quad (7) \]

wherein \( r \) represents the radius of the orbit, and \( R \) represents the radius of the orbit. Solving this equation for \( r \) results in the equations of motion. The equations of motion are then used to find the energy of the system.

\[ E = \frac{1}{2} m_1 \dot{r}_1^2 + \frac{1}{2} m_2 \dot{r}_2^2 + V(|r_2 - r_1|) \]

\[ = \frac{1}{2} M \dot{R}^2 + \frac{1}{2} \mu \dot{r}^2 + V(r) \quad (7) \]

wherein \( \dot{r}_1, \dot{r}_2 \) represent the velocities of the two masses, and \( \dot{R} \) represents the velocity of the center of mass.

\[ p_1 = m_1 \dot{r}_1, \quad p_2 = m_2 \dot{r}_2 \quad P = M \dot{R} \quad p = \mu \dot{r} \quad (8) \]

The above equations lead to the expressions

\[ E_k = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} = \frac{p_1^2}{2M} + \frac{p_2^2}{2\mu} \quad (9) \]

where \( E_k \) represents the kinetic energy of the system. Solving this equation for \( p_1 \) and \( p_2 \) results in the momentum of the system.

\[ p_1 = m_1 \dot{r}_1 = m_1 \dot{R} + \mu \dot{r} \quad p_2 = m_2 \dot{r}_2 = m_2 \dot{R} - \mu \dot{r} \quad (10) \]
Consider a gravitational force of attraction between the two masses and except for that assume that no external forces act on the system. Then, the force exerted by each mass on the other will be equal and opposite.

\[ F(r) = -m_1\ddot{r}_1 = m_2\ddot{r}_2 \]

or

\[ \ddot{r}_1 = -\frac{F(r)}{m_1} \quad (4) \]

\[ \ddot{r}_2 = \frac{F(r)}{m_2} \quad (5) \]

Substituting Eq. (4) and Eq. (5) in Eq. (1) gives

\[ \ddot{r} = F(r)\left(\frac{1}{m_1} + \frac{1}{m_2}\right) = \frac{F(r)}{\mu} \quad (5) \]

where

\[ \mu = \frac{m_1m_2}{m_1 + m_2} \quad (6) \]

is the reduced mass of the system.

With these definitions, one can easily verify that

\[ l = m_1r_1^2 + m_2r_2^2 = MR^2 + \mu r^2 \quad (7) \]

This is the famous parallel axis theorem, which states that the moment of inertia of an object in any coordinate system is the sum of the moment of inertia of the center of mass of the object and moment of inertia of the rest of the particles about the center of mass of the body.

If the center of mass of the object is stationary, then the moment of inertia is given by \( l = \mu r^2 \).

The total energy \( E \) of the system is given by
2.1 Introduction

The introduction of the document contains information about the mathematical equations and references to external sources. The text is in both English and Telugu.

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**E - Corner**


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\[ \theta = \frac{l}{r} \]  (1)
2.2 Rotational Kinematic relations

Here the terminology of rotational kinematics will be introduced in order to derive the kinematic relations for rotational motion. They are as follows

**Angular position:** Angular position of an object is defined as the angle made by the line joining the object to a fixed point, with reference to a fixed line passing through the same fixed point.
2.2 Rotational Kinematic relations

\[ \Delta \theta = \theta_2 - \theta_1 \quad (2) \]

\[ \omega_{avg} = \frac{\Delta \theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1} \quad (3) \]

\[ \omega = \frac{d\theta}{dt} \quad (4) \]

\[ \alpha_{avg} = \frac{\Delta \omega}{\Delta t} = \frac{\omega_2 - \omega_1}{t_2 - t_1} \quad (5) \]

\[ \alpha = \frac{d\omega}{dt} \quad (6) \]

Eq. (6) \Rightarrow

\[ d\omega = \alpha dt \Rightarrow \int d\omega = \int \alpha dt \Rightarrow \omega - \omega_0 = \alpha t \]

\[ \Rightarrow \omega = \omega_0 + \alpha t \quad (7) \]

Eq. (4) \Rightarrow Eq. (7) \Rightarrow

\[ d\theta = \omega dt = (\omega_0 + \alpha t)dt \Rightarrow \int d\theta = \int (\omega_0 + \alpha t)dt \]
Angular position usually describes the position of rotating objects about an axis perpendicular to the plane containing the reference line and the line connecting the object to the reference point.

Angular position is a vector quantity whose direction is given by the right hand thumb rule, where curl of fingers gives the direction of rotation starting from fixed axis and thumb gives the direction of angular position vector. This vector is always perpendicular to the plane of rotation.

To describe angular position, angle is measured in radians. It is the ratio of length of the arc to the length of the radius of the circle.

\[
\theta = \frac{l}{r}
\]  

Angle is positive if measured in anticlockwise direction and is negative if measured in clockwise direction from the reference axis.
2.2 Rotational Kinematic relations

\[ \theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2 \]

\[ \Rightarrow \theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \quad -(8) \]

\[ \Rightarrow \theta - \theta_0 = \omega_0 \frac{\omega - \omega_0}{\alpha} + \frac{1}{2} \alpha \frac{(\omega - \omega_0)^2}{\alpha^2} \]

\[ = \frac{\omega_0 \omega - \omega_0^2}{\alpha} + \frac{1}{2} \frac{\omega_0^2 + \omega^2 - 2\omega_0 \omega}{\alpha} = \frac{\omega^2 - \omega_0^2}{2\alpha} \]

\[ \Rightarrow \omega^2 - \omega_0^2 = 2\alpha (\Delta \theta) = 2\alpha (\theta - \theta_0) \quad -(9) \]
2.2 Rotational Kinematic relations

**Angular displacement:** The change in angular position at two different time stamps is called angular displacement. This is defined as

\[ \Delta \theta = \theta_2 - \theta_1 \quad - - - (2) \]

Angular displacement is positive for anti-clockwise rotation and negative for clockwise rotation. The direction of angular displacement vector is also given by the right hand thumb rule.

**Angular velocity and Angular speed:** It is the rate of change of angular displacement. The average value of angular velocity is given by

\[ \omega_{avg} = \frac{\Delta \theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1} \quad - - - (3) \]

Instantaneous angular velocity is given by

\[ \omega = \frac{d\theta}{dt} \quad - - - (4) \]

The magnitude of angular velocity vector is called angular speed.

**Angular acceleration:** The rate of change of angular velocity is called angular acceleration. The average angular acceleration is given by

\[ \alpha_{avg} = \frac{\Delta \omega}{\Delta t} = \frac{\omega_2 - \omega_1}{t_2 - t_1} \quad - - - (5) \]

and the instantaneous angular acceleration is given by

\[ \alpha = \frac{d\omega}{dt} \quad - - - (6) \]

**Kinematic Equations:**

From Eq. (6),

\[ d\omega = \alpha dt \Rightarrow \int d\omega = \int \alpha dt \Rightarrow \omega - \omega_0 = \alpha t \]

\[ \Rightarrow \omega = \omega_0 + \alpha t \quad - - - (7) \]

From Eq. (4) and Eq. (7)

\[ d\theta = \omega dt = (\omega_0 + \alpha t)dt \Rightarrow \int d\theta = \int (\omega_0 + \alpha t)dt \]
Think ...

In 2 dimensions, the motion exists in 2d, but the result of cross product of two vectors lies in a direction perpendicular to the plane. Then does angular momentum exist in 2d?

Did You Know?

In 2 dimensions, angular momentum is a scalar. Its value is given by \[ L = x p_y - y p_x \]

2.3 Angular momentum and Moment of Inertia Tensor

\[ \Rightarrow \theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2 \]

\[ \Rightarrow \theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 - \cdots (8) \]

Substituting the value of \( t \) from Eq. (7) into Eq. (8) gives,

\[ \theta - \theta_0 = \omega_0 \frac{\omega - \omega_0}{\alpha} + \frac{1}{2} \alpha \frac{(\omega - \omega_0)^2}{\alpha^2} \]

\[ = \frac{\omega_0 \omega - \omega_0^2}{\alpha} + \frac{1}{2} \frac{\omega_0^2 + \omega^2 - 2 \omega_0 \omega}{\alpha} = \frac{\omega^2 - \omega_0^2}{2\alpha} \]

\[ \Rightarrow \omega^2 - \omega_0^2 = 2\alpha (\Delta \theta) = 2\alpha (\theta - \theta_0) - \cdots - (9) \]

<table>
<thead>
<tr>
<th>Translational</th>
<th>Rotational</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equations of motion: in kinematics</td>
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</tr>
<tr>
<td>a) ( v = u + at )</td>
<td>1. ( \omega = \omega_0 + at )</td>
</tr>
<tr>
<td>b) ( s = s_0 + ut + \frac{1}{2}at^2 )</td>
<td>2. ( \theta = \theta_0 + \omega t + \frac{1}{2} \alpha t^2 )</td>
</tr>
<tr>
<td>c) ( v^2 - u^2 = 2a(s - s_0) )</td>
<td>3. ( \omega_2^2 = \omega_1^2 + 2\alpha(\theta - \theta_0) )</td>
</tr>
</tbody>
</table>

2.3 Angular momentum and Moment of Inertia Tensor

Consider an object of mass \( m \) moving in a straight line with velocity \( v \) and that there is a stationary observer perpendicular to the direction of motion. Then the angular momentum is given by

\[ L = r \times p = m(r \times v) = mvr \sin \theta = mr_\perp v \quad -- (1) \]

![Fig: Angular momentum in rectilinear motion.](image)
2.3 Angular momentum and Moment of Inertia Tensor

Angular momentum and Moment of Inertia Tensor

\[ L = \sum m_i r_i \times (r_i - R) = 0 \quad \text{(5)} \]

\[ \sum m_i \dot{r}_i = \sum m_i \dot{r}_{ic} = \sum m_i (\dot{r}_i - \dot{R}) = 0 \quad \text{(6)} \]

\[ L = R \times \sum m_i V + r_{ic} \times \sum m_i v_{ic} = R \times MV + r_{ic} \times \sum m_i v_{ic} \]

\[ L = R \times P + L_{CM} \quad \text{(7)} \]
Thus angular momentum is purely observer dependent. If there is a radial vector perpendicular to the direction of motion, then angular momentum is observed. If the object executes rectilinear motion, angular momentum varies from point to point in magnitude. Direction always will be perpendicular to the plane containing object and observer. If the observer is in the same direction as the moving mass (which amounts to a one dimensional case), then angular momentum does not exist.

Circular motion is a special case, where parallel component of radial vector is always zero. Or it is the case where radial vector is always perpendicular to the velocity vector. Eg: Ceiling fan, wind turbine etc.

\[
L = \sum m_i (r_i \times v_i)
\]

Consider the radial vector of center of mass from origin to be \( R \) and let \( r_{ic} \) be the radial distance between the center of mass and the \( i^{th} \) particle. Then one can write,

\[
r_i = R + r_{ic} \quad \text{and} \quad v_i = V + v_{ic}
\]

Substituting Eq. (3) in Eq. (2) gives

\[
L = \sum m_i (R + r_{ic}) \times (V + v_{ic})
\]
### 2.3 Angular momentum and Moment of Inertia Tensor

Angular momentum and Moment of Inertia Tensor are defined as follows:

\[ L = \sum r_i \times m_i \mathbf{v}_i = \sum m_i (r_i \times (\mathbf{\omega} \times r_i)) \]  

where \( \mathbf{\omega} \times r_i \) is the cross product of angular velocity \( \mathbf{\omega} \) and position \( r_i \). This equation represents the angular momentum of a system.

The moment of inertia tensor is given by:

\[ \Lambda = (A.C)B - (A.B)C \]

where \( A, B, C \) are matrices. This equation describes how the moments of inertia are related.

The components of the moment of inertia tensor are:

\[ L_x = \sum m_i (x_i^2 + y_i^2 + z_i^2) \mathbf{\omega}_x - \sum m_i x_i (x_i \mathbf{\omega}_x + y_i \mathbf{\omega}_y + z_i \mathbf{\omega}_z) \]

\[ L_y = \sum m_i (z_i^2 + x_i^2) \mathbf{\omega}_y - \sum m_i y_i x_i \mathbf{\omega}_x - \sum m_i y_i z_i \mathbf{\omega}_z \]

\[ L_z = \sum m_i (x_i^2 + y_i^2) \mathbf{\omega}_z - \sum m_i z_i x_i \mathbf{\omega}_x - \sum m_i z_i y_i \mathbf{\omega}_y \]

These equations provide the Moment of Inertia Tensor for a system of particles.
2.3 Angular momentum and Moment of Inertia Tensor

\[ L = \sum m_i R \times V + \sum m_i R \times v_{ic} + \sum m_i r_{ic} \times V + \sum m_i r_{ic} \times v_{ic} \]

By the definition of center of mass

\[ \sum m_i R = \sum m_i r_i \Rightarrow \sum m_i r_{ic} = \sum m_i (r_i - R) = 0 \]

Differentiating the above equation WRT time, gives

\[ \sum m_i \dot{R} = \sum m_i \dot{r}_i \Rightarrow \sum m_i v_{ic} = \sum m_i (\dot{r}_i - \dot{R}) = 0 \]

Substituting Eq. (6) and Eq. (5) in Eq. (4) gives

\[ L = R \times \sum m_i V + r_{ic} \times \sum m_i v_{ic} = R \times MV + r_{ic} \times \sum m_i v_{ic} \]

Thus the total angular momentum of system of particles is the sum of angular momentum of the center of mass and the angular momentum of the rest of the particles W.R.T. center of mass. Here the first term represents the angular momentum due to the movement of center of mass. The second term represents the rotation of the rest of the particles keeping center of mass stationary. This is because for a rigid body, if center of mass is stationary, the only other motion possible for rest of the particles is rotation about center of mass; as other movements are restricted by the rigidness of the body.
2.3 Angular momentum and Moment of Inertia Tensor

\[
\begin{pmatrix}
L_x \\
L_y \\
L_z
\end{pmatrix} = \begin{pmatrix}
I_{xx} & I_{xy} & I_{xz} \\
I_{yx} & I_{yy} & I_{yz} \\
I_{zx} & I_{zy} & I_{zz}
\end{pmatrix} \begin{pmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{pmatrix}
\]

\[
l_{xx} = \sum m_i (y_i^2 + z_i^2), \quad l_{xy} = -\sum m_i x_i y_i, \quad l_{xz} = -\sum m_i x_i z_i
\]
\[
l_{yy} = \sum m_i (z_i^2 + x_i^2), \quad l_{yx} = -\sum m_i y_i x_i, \quad l_{yz} = -\sum m_i y_i z_i
\]
\[
l_{zz} = \sum m_i (x_i^2 + y_i^2), \quad l_{zx} = -\sum m_i z_i x_i, \quad l_{zy} = -\sum m_i z_i y_i
\]

Do You Know?

A rank one tensor is called a Vector and a rank zero tensor is called a scalar (But not vice versa). There are some other physical quantities that spread, are usually tensors in Physics. Eg: Thermal & electrical conductivity, Polarizability, stress, strain.

ఇకడ I ఒక -2 న. ఎంకంవలస 3 లంబ శలంలంటం పతంయం. ఇతర పరస్థంత లంబ శలంలంటం పతంయం.ఉత్ప లల ఉన్ని సంఖ్యా రంచబం. ఇకడు I ఒక -2 న. ఎంకంవలస 3 లంబ శలంలంటం పతంయం. ఇతర పరస్థంత లంబ శలంలంటం పతంయం.ఉత్ప లల ఉన్ని సంఖ్యా రంచబం.
2.3 Angular momentum and Moment of Inertia Tensor

If the system has more than one axis of rotation, then the intersection of those axes may lead to a stationary point. Thus the least possible symmetry element in the 3D rotation of an object is the rotation about a fixed point. Eg: Table fan.

A more general expression for angular momentum of rigid body is given by

$$L = \sum r_i \times m_i v_i = \sum m_i (r_i \times (\omega \times r_i))$$  \hspace{1cm} (8)

We know that $A \times (B \times C) = (A.C)B - (A.B)C$; with this the above equation becomes,

$$L = \sum m_i [(r_i \cdot r_i) \omega - (r_i \cdot \omega) r_i]$$

$$= \sum m_i r_i^2 \omega - \sum m_i (r_i \cdot \omega) r_i$$  \hspace{1cm} (9)

Here $r_i^2 = r_i \cdot r_i = (x_i \hat{i} + y_i \hat{j} + z_i \hat{k}) \cdot (x_i \hat{i} + y_i \hat{j} + z_i \hat{k}) = x_i^2 + y_i^2 + z_i^2$ using this, the $x$ \textit{component} of angular momentum is given by

$$L_x = \sum m_i (x_i^2 + y_i^2 + z_i^2) \omega_x - \sum m_i x_i (x_i \omega_x + y_i \omega_y + z_i \omega_z)$$
A Gentle Introduction to Tensors for Machine Learning with NumPy - Machine Learning Mastery

In deep learning it is common to see a lot of discussion ...

https://machinelearningmastery.com/introduction-to-tensors-for-machine-learning/

https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.552.4933&rep=rep1&type=pdf


2.3 Angular momentum and Moment of Inertia Tensor

\[ L_x = \sum m_i(y_i^2 + z_i^2) \omega_x - \sum m_i x_i y_i \omega_y - \sum m_i x_i z_i \omega_z. \]

Similarly

\[ L_y = \sum m_i(z_i^2 + x_i^2) \omega_y - \sum m_i y_i x_i \omega_x - \sum m_i y_i z_i \omega_z \]

\[ L_z = \sum m_i(x_i^2 + y_i^2) \omega_z - \sum m_i z_i x_i \omega_x - \sum m_i z_i y_i \omega_y \]

These can be written in matrix form as

\[
\begin{pmatrix}
L_x \\
L_y \\
L_z
\end{pmatrix} =
\begin{pmatrix}
\sum m_i(y_i^2 + z_i^2) & -\sum m_i x_i y_i & -\sum m_i x_i z_i \\
-\sum m_i y_i x_i & \sum m_i(z_i^2 + x_i^2) & -\sum m_i y_i z_i \\
-\sum m_i z_i x_i & -\sum m_i z_i y_i & \sum m_i(x_i^2 + y_i^2)
\end{pmatrix}
\begin{pmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{pmatrix}
\]

or

\[
\begin{pmatrix}
L_x \\
L_y \\
L_z
\end{pmatrix} =
\begin{pmatrix}
l_{xx} & l_{xy} & l_{xz} \\
l_{yx} & l_{yy} & l_{yz} \\
l_{zx} & l_{zy} & l_{zz}
\end{pmatrix}
\begin{pmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{pmatrix}
\]

where

\[ l_{xx} = \sum m_i(y_i^2 + z_i^2), \quad l_{xy} = -\sum m_i x_i y_i, \quad l_{xz} = -\sum m_i x_i z_i \]

\[ l_{yy} = \sum m_i(z_i^2 + x_i^2), \quad l_{yx} = -\sum m_i y_i x_i, \quad l_{yz} = -\sum m_i y_i z_i \]

\[ l_{zz} = \sum m_i(x_i^2 + y_i^2), \quad l_{zx} = -\sum m_i z_i x_i, \quad l_{zy} = -\sum m_i z_i y_i \]

This matrix of \( l \) is called moment of inertia matrix. This is a symmetric matrix. This matrix indicates that the angular momentum variations along a particular axis can affect the angular momentum along the other two mutually perpendicular directions as well. This matrix with off diagonal elements is also called a Tensor. In colloquial language, a tensor is a multi-dimensional vector. The rank of the tensor is decided by the number of degree of freedom minus the number of constraints. Here \( l \) is a rank-2 tensor. This is because the system is 3-dimensional (3 degrees of freedom) and there is one fixed point (One constraint.)
Two ladybugs rest without slipping on a rotating platter that is increasing its angular velocity. Ladybug A is closer to the rotation axis than bug B. Which statement correctly describes the relationship between the bugs’ angular accelerations (\(a\)) and centripetal accelerations (\(a_{\text{rad}}\))?

1) \(a_A > a_B\) and \(a_{\text{rad},A} > a_{\text{rad},B}\)
2) \(a_A < a_B\) and \(a_{\text{rad},A} < a_{\text{rad},B}\)
3) \(a_A = a_B\) and \(a_{\text{rad},A} < a_{\text{rad},B}\)
4) \(a_A = a_B\) and \(a_{\text{rad},A} = a_{\text{rad},B}\)
5) \(a_A = a_B\) and \(a_{\text{rad},A} > a_{\text{rad},B}\)

Two ladybugs rest without slipping on a rotating platter that is increasing its angular velocity. Ladybug A is closer to the rotation axis than bug B. Which statement correctly describes the relationship between the bugs’ angular accelerations (\(a\)) and centripetal accelerations (\(a_{\text{rad}}\))?
2.3 Angular momentum and Moment of Inertia Tensor

For systems with fixed axis of rotation, \( \omega \) and \( r_i \) will be perpendicular to each other, then \( x_i, \omega = x_i, \omega = y_i, \omega = y_i, \omega = z_i, \omega = z_i, \omega = 0 \). Then the angular momentum matrix is given by

\[
\begin{pmatrix}
L_x \\
L_y \\
L_z
\end{pmatrix} = \begin{pmatrix}
\sum m_i (y_i^2 + z_i^2) & 0 & 0 \\
0 & \sum m_i (z_i^2 + x_i^2) & 0 \\
0 & 0 & \sum m_i (x_i^2 + y_i^2)
\end{pmatrix} \begin{pmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{pmatrix}
\]

![Fig: Rotation about a fixed axis.](image)

Thus in this case, the moment of inertia tensor becomes a vector. Since the description of a line requires at least two points to be fixed, the constraints will be two and thus it becomes a rank-1 tensor or a vector. One can write angular momentum as,

\[ L = I \omega = I_x \omega_x + I_y \omega_y + I_z \omega_z \]

The axes \( x, y, z \) are called the principle axes of the rotating system. When the rotational axis coincides with the principal axis of the system, then the off-diagonal elements of the inertia tensor becomes zero. i.e., if the body rotates along one of the principal axes, then the angular momentum along other principal axes can be altered independently. If the principal axes pass through the center of mass of the body, then they are called “central principal axes”.

In a symmetric matrix of moment of inertia, if any two of the diagonal elements are equal then it is called symmetric top and if all the three diagonal elements are equal, then it is called spherical top. If two of the diagonal elements are equal and if third one is zero, then it is called linear rotor. If any of the three diagonal elements are not equal to each other, then it is called asymmetric top.
భావనల పంచం రొంద విషయాలు

సూత్రల పొలిస్తే వేరే తరుగితగా ఈయనికాన్ని ఇతరవన్నాసాకని. అందువలస సూత్రాలను అవడించాం అందువలసాన్ని ఇది లభయిస్తుంది:

\[ L = r \times p \]

ఈ సూత్ర పోటు మాత్రమే మానం పరిమాణాన్ని అంచనాసాన్ని సూచిస్తుంది. ఈ పోటు మాత్రమే సూత్ర అనేకం అనంతం అంచనాసాన్ని సూచిస్తుంది:

\[ \frac{dL}{dt} = \frac{d}{dt} (r \times p) = \frac{dr}{dt} \times p + r \times \frac{dp}{dt} = v \times mv + r \times F \]

\[ = 0 + r \times F = \tau \]

ఈ సూత్ర పోటు మాత్రమే సూత్ర అనేకం అంచనాసాన్ని సూచిస్తుంది. ఈ పోటు మాత్రమే సూత్ర అనేకం అంచనాసాన్ని సూచిస్తుంది.

\[ L = I\omega \]

ఈ సూత్ర పోటు మాత్రమే సూత్ర అనేకం అంచనాసాన్ని సూచిస్తుంది. ఈ పోటు మాత్రమే సూత్ర అనేకం అంచనాసాన్ని సూచిస్తుంది:

\[ \frac{dL}{dt} = I \frac{d\omega}{dt} + I \frac{d\Theta}{dt} \]

ఈ సూత్ర పోటు మాత్రమే సూత్ర అనేకం అంచనాసాన్ని సూచిస్తుంది. ఈ పోటు మాత్రమే సూత్ర అనేకం అంచనాసాన్ని సూచిస్తుంది:

\[ \frac{dL}{dt} = I \frac{d\omega}{dt} + I \frac{d\Theta}{dt} \]

ఈ సూత్ర పోటు మాత్రమే సూత్ర అనేకం అంచనాసాన్ని సూచిస్తుంది. ఈ పోటు మాత్రమే సూత్ర అనేకం అంచనాసాన్ని సూచిస్తుంది:

\[ \tau = \frac{dL}{dt} = I\alpha \]

ఈ సూత్ర పోటు మాత్రమే సూత్ర అనేకం అంచనాసాన్ని సూచిస్తుంది. ఈ పోటు మాత్రమే సూత్ర అనేకం అంచనాసాన్ని సూచిస్తుంది:

\[ \tau = \frac{dL}{dt} = I\alpha \]

ఈ సూత్ర పోటు మాత్రమే సూత్ర అనేకం అంచనాసాన్ని సూచిస్తుంది.

https://www.lehman.edu/faculty/anchordoqui/chapter19.pdf
http://teacher.pas.rochester.edu/PHY235/LectureNotes/Chapter11/Chapter11.htm
https://www.brown.edu/Departments/Engineering/Courses/E4/notes_old/RigidKinematics/rigkin.htm
2.4 Equation of motion for a rotating body

Consider a point particle executing rotatory motion. Then angular momentum is given by

\[ L = r \times p \]

The time variation of angular momentum is given by

\[
\frac{dL}{dt} = \frac{d}{dt}(r \times p) = \frac{dr}{dt} \times p + r \times \frac{dp}{dt} = \vec{v} \times m\vec{v} + r \times \vec{F} = 0 + r \times \vec{F} = \tau
\]

Thus for a point particle, the rate of change of angular momentum gives the torque \( \tau \) produced in the system. This is analogous to Newton’s second law for rotatory motion.

For system of particles, angular momentum is given by

\[ L = I\omega \]

and the time variation of angular momentum is given by

\[
\frac{dL}{dt} = \frac{d}{dt}(I\omega) = \frac{dI}{dt} \omega + I \frac{d\omega}{dt}
\]

Here moment of inertia remains independent of time as long as the object retains its shape or mass distribution. Thus the first term vanishes in the above equation, giving rise to

\[
\frac{dL}{dt} = I \frac{d\omega}{dt} = I\alpha = \tau
\]

Thus for a rigid body, the equation of motion is given by

\[ \tau = \frac{dL}{dt} = I\alpha \]

Here \( I \) is a rank-2 tensor for 3 dimensional objects.
ఆల సకర

ఆల సకర

పలం

ఉనై పం ఫోట/తరణం

ఉనై,

వలం

చలన

సకర

ఆల

పలం

అయడం,

అయవర

రణం

వంచ

బల.

ఆ సఖా ఏకపగంచం మరియు సమయం కపయం రడం దపగంచం.అక   హర   ం

అంం,

అక

A

కసమయ

వంధ

భమణ

చలనం

సంటుకుండా

ఇయవ.

యవ

= dA/dt

ఇకడ

ω

అక

ఫక

కసమయ

ఇయవ.

ఈ

సంబం

ఉపం

భమణ

చలనం

సంటుకుండా

ఇయవ.

అంచబం

= dA/dt

ఇకడ

×

= (dA)

space

= (dA)

body

+ ω × A

ఆ లా/ష వర్షాత్రింంచం

ఉన్నాడంపాలితే అలాంటి చూపురాలు జాగ్రత్తవంపాలితే

అవి

సమయం

మాట్లాడంపాలితే

ఇయవ.

ఎందుకందా, కూడా ఎంతం వెండం పేరికింది మాట్లాడంపాలితే

అందులే

τ = \frac{dL}{dt} + \omega \times L = \frac{d(l\omega)}{dt} + (\omega \times L)

ఆను

\omega \times L = i(\omega_2 L_3 - L_2 \omega_3) + j(L_1 \omega_3 - \omega_1 L_3) + k(\omega_1 L_2 - L_1 \omega_2)

అందులే

\tau_1 = l_1 \frac{d\omega_1}{dt} + \omega_2 l_3 \omega_3 - l_2 \omega_2 \omega_3 = l_1 \frac{d\omega_1}{dt} + (l_3 - l_2)\omega_2 \omega_3

\tau_2 = l_2 \frac{d\omega_2}{dt} + \omega_3 l_1 \omega_1 - l_3 \omega_3 \omega_1 = l_2 \frac{d\omega_2}{dt} + (l_1 - l_3)\omega_3 \omega_1
2.5 Euler Equations

Euler proposed equation of motion of objects when the observer is in moving frame or especially in rotating/accelerating frames of reference. In doing so, he has introduced pseudo forces which can’t be explained by cause and effect relations of Newtonian framework.

Consider an arbitrary vector $A$ on a rotating body coordinate system and consider that the vector is not changing as a function of time. Suppose the same vector is observed from an external fixed frame, then the time rate of change of vector $A$ is purely due to the rotation of the body frame. That is given by

$$\frac{dA}{dt} = \omega \times A$$

here $\omega$ is the angular velocity of the rotating frame.

Then a general expression for the time rate of change of vector in the space frames of reference is given by

$$\left(\frac{dA}{dt}\right)_{\text{space}} = \left(\frac{dA}{dt}\right)_{\text{body}} + \omega \times A$$

here $dA/dt$ is the time rate of change of vector $A$ in the body/rotating frame of reference.

Using this relation, the Newton’s second law for rotatory motion is given by

$$\tau = \frac{dL}{dt} + \omega \times L = \frac{d(I\omega)}{dt} + (\omega \times L)$$

here

$$\omega \times L = i(\omega_2 L_3 - L_2 \omega_3) + j(L_1 \omega_3 - \omega_1 L_3) + k(\omega_1 L_2 - L_1 \omega_2)$$

then

$$\tau_1 = I_1 \frac{d\omega_1}{dt} + \omega_2 L_3 \omega_3 - I_2 \omega_2 \omega_3 = l_1 \frac{d\omega_1}{dt} + (L_3 - L_2)\omega_2 \omega_3$$

$$\tau_2 = I_2 \frac{d\omega_2}{dt} + \omega_3 L_1 \omega_1 - I_3 \omega_3 \omega_1 = l_2 \frac{d\omega_2}{dt} + (L_1 - L_3)\omega_3 \omega_1$$
2.5 Euler Equations

\[ \tau_3 = l_3 \frac{d\omega_3}{dt} + \omega_1 l_2 \omega_2 - l_1 \omega_1 \omega_2 = l_3 \frac{d\omega_3}{dt} + (l_2 - l_1) \omega_1 \omega_2 \]

Assuming \( \omega = \text{constant} \) and \( \tau = \text{constant} \)...

\[ \frac{l_1}{dt} \frac{d\omega_1}{dt} + (l_3 - l_2) \omega_2 \omega_3 = 0 \]
\[ \frac{l_2}{dt} \frac{d\omega_2}{dt} + (l_1 - l_3) \omega_3 \omega_1 = 0 \]
\[ \frac{l_3}{dt} \frac{d\omega_3}{dt} + (l_2 - l_1) \omega_1 \omega_2 = 0 \]

Integrating with respect to \( t \), we get...

\[ \frac{d}{dt} \left( \frac{1}{2} l_1 \omega_1^2 + \frac{1}{2} l_2 \omega_2^2 + \frac{1}{2} l_3 \omega_3^2 \right) = 0 \Rightarrow \frac{d}{dt} (E_k) = 0 \]

\[ E_k = \text{Kinetic Energy} = \text{Constant} \]

Integrating again, we get...

\[ \frac{1}{2} \frac{d}{dt} \left( l_1 \omega_1^2 + l_2 \omega_2^2 + l_3 \omega_3^2 \right) = 0 \Rightarrow \frac{1}{2} \frac{d}{dt} (L^2) = 0 \Rightarrow \frac{2L \frac{dL}{dt}}{2} = 0 \]

Thus, \( L = \text{constant} \).
These are called Euler’s equations of motion for rigid body.

Conservation of Kinetic Energy and Angular momentum:
When external torque is zero, \( \tau = 0 \) Angular momentum and hence rotational kinetic energy conserves,
Then Euler equations become
\[
I_1 \frac{d\omega_1}{dt} + (I_3 - I_2) \omega_2 \omega_3 = 0 \\
I_2 \frac{d\omega_2}{dt} + (I_1 - I_3) \omega_3 \omega_1 = 0 \\
I_3 \frac{d\omega_3}{dt} + (I_2 - I_1) \omega_1 \omega_2 = 0
\]
Multiplying above Euler equations with \( \omega_1, \omega_2, \omega_3 \) respectively and adding gives,
\[
\frac{d}{dt}\left(\frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \omega_2^2 + \frac{1}{2} I_3 \omega_3^2\right) = 0 \Rightarrow \frac{d}{dt}(E_k) = 0
\]
\( \Rightarrow E_k = \text{Kinetic Energy} = \text{Constant} \).

Multiplying the above Euler equations with \( I_1 \omega_1, I_2 \omega_2, I_3 \omega_3 \) and adding gives,
\[
\frac{1}{2} I_1^2 \frac{d\omega_1^2}{dt} + \frac{1}{2} I_2^2 \frac{d\omega_2^2}{dt} + \frac{1}{2} I_3^2 \frac{d\omega_3^2}{dt} = 0
\]
\( \Rightarrow \frac{1}{2} \frac{d}{dt}(I_1^2 \omega_1^2 + I_2^2 \omega_2^2 + I_3^2 \omega_3^2) = 0 \Rightarrow \frac{1}{2} \frac{d}{dt}(L^2) = 0 \Rightarrow \frac{2L}{2} \frac{dL}{dt} = 0 
\]
If \( L = 0 \), rotation does not take place, so \( dL/dt = 0 \Rightarrow L = \text{Constant} \).
While momentum conserves, rotational kinetic energy also conserves only when moment of inertia is independent of time.
మరియు ఖనియత ప్రపంచం

మనిషి చెందిన ప్రపంచం మనిషి విశ్వస్తత ప్రాంతం ప్రపంచ ప్రాంతం నీటి మనిషి విశ్వస్తత ప్రాంతం.

ప్రాంతం మనిషి విశ్వస్తత ప్రాంతం సమ్మేధ ప్రాంటం.

ప్రాంతం మనిషి విశ్వస్తత ప్రాంతం సమ్మేధ ప్రాంటం.

ప్రాంతం మనిషి విశ్వస్తత ప్రాంతం సమ్మేధ ప్రాంటం.

ప్రాంతం మనిషి విశ్వస్తత ప్రాంతం సమ్మేధ ప్రాంటం.

Did You Know?

precision: The quality of being accurate

Precession: Rotation of axis of rotation around another axis.
2.6 Precession of spinning top

The slow rotation of axis of rotation of a body around another fixed axis is called precession.

Consider a top spinning about a vertical axis. Then the angular momentum vector $L$ will be along the vertical axis. Suppose the axis of rotation of the top now makes an angle $\theta$ with the vertical axis. In such a case, if the center of gravity of the top is above the ground level, then the gravity that acts downwards produces a torque on the top. Let $r$ be the distance of center of gravity from the ground.

![Fig: Precession of top](image)

The torque produced is given by

$$\tau = r \times mg = rmg \sin \alpha = rmg \sin(180^\circ - \theta) = rmg \sin \theta.$$ 

Here to calculate the cross product, one needs to extend $r$ upwards or $mg$ backwards to make them either both converging or both diverging vectors. Only then the angle between them can be defined.

The resultant torque will be pointing into the plane and thus pulls the axis of rotation inwards. At every point the torque will be
మ్యూస్ (ఫ్యాక్స్ మానవు)

T - Corner
https://www.youtube.com/watch?v=GeyDf4ooPdo
https://www.youtube.com/watch?v=tLMpdBjA2SU

హుదుంచి (ప్రయోగ సంపాదన)

మూలాకారి ఎడితరి ఎడితరి ఎడితరి ఎడితరి ఎడితరి ఎడితరి ఎడితరి ఎడితరి ఎడితరి ఎడితరి ఎడితరి ఎడితరి

తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో

తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో

తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో

తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో

తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో తెలుగు లో

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తెలుగు లో తెలుగు లో తెలుగు లో తెల�ం(ఫ్యాక్స్ మానవు)
perpendicular to the plane containing \( r \) and \( mg \). Thus results in a rotation of the vector \( L \) around the fixed axis.

From the diagram, the radius of the precession circle is given by \( L \sin \theta \).

The torque produced by gravity results in a change in the direction of the angular momentum vector \( L \). Let \( \Delta L \) be the change in angular momentum produced. Then the corresponding precession angular velocity is given by

\[
\omega_p = \frac{\Delta \phi}{\Delta t} = \frac{\text{arc length/radius}}{\Delta t} = \frac{\Delta L}{\Delta t \ L \ \sin \theta} = \frac{\tau}{L \ \sin \theta}
\]

\[
= \frac{mgr \ \sin \theta}{L \ \sin \theta} = \frac{mgr}{L} = \frac{mgr}{I \Omega} = \frac{mgr}{mR^2 \Omega} = \frac{gr}{R^2 \Omega}.
\]

Here \( R \) is the radius of the top, that gives a measure of the bulginess of the top. \( \Omega \) is the angular velocity of rotation of top.

Thus if \( r \) is larger, precession is larger. i.e.; if the center of gravity is far away from ground level, precession effects will be more.

If \( \Omega \) is higher, precession effects will be low. i.e., if the top has high rotational velocity it will not precess. Only when the rotational velocity is considerably low, precession takes dominance.

The precession effect is independent of the mass of the object but is more affected by the mass distribution. i.e.; by the ratio \( r/R^2 \). Further, it is majorly controlled by the value of \( R \), as it appears as a squared term. Thus if the top is bulgy, precession will be low. In other words, if the top is bulgy, center of gravity should be shifted up enormously to make it precess.

2.7 Gyroscope

Gyroscope was named so by the French physicist, Leon Foucault. In Greek the word gyrus means rotation and scope means the one that displays. The gyroscope was used to demonstrate the rotation of earth, along with Foucault’s pendulum. Hence the name.
2.7 Gyroscope

A gyroscope is a device that uses the principles of angular momentum to maintain its orientation. The device's motion is governed by the laws of physics, specifically the conservation of angular momentum. When a gyroscope is set in motion, it resists any change in its angular momentum, which is why it is able to maintain its orientation.

In the figure, the shaded region represents the Moon's orbit around the Earth, and the unshaded region represents the Earth's orbit around the Sun. The angle between these two orbits is known as the obliquity of the ecliptic. The gyroscope's behavior is demonstrated by its tendency to maintain a constant orientation in space, as shown in the figure.

For more information, visit the following links:

https://www.youtube.com/watch?v=eTjGTxSevHE
https://www.youtube.com/watch?v=cquvA_1pEsA
https://courses.lumenlearning.com/suny-osuniversityphysics/chapter/11-3-precession-of-a-gyroscope/

Fig: Lunar nodes
A gyroscope is basically a rotating heavy wheel whose axis of rotation can be rotated in mutually perpendicular directions. In other words, it is a flywheel fitted with two mutually perpendicular gimbals. Since the central wheel is heavy, it won’t allow any change in its axis of rotation.

**Fig: Gyroscope**

Let $M$ be the mass of the flywheel in the gyroscope. Let $R$ be the radius of it and $\Omega$ be its rotational frequency. Then its angular momentum is given by $\mathbf{L} = I \mathbf{\Omega} = MR^2 \mathbf{\Omega}$.

Let the gyroscope axle be pivoted on a point object kept at a distance of $r$ from its center of gravity. Here the mass of the wheel $Mg$ acts downwards. Then it results in a rotational force (Torque) perpendicular to both $L$ and $Mg$.

$$\mathbf{\tau} = r \times Mg = Mgr \sin \theta = Mgr \quad (\because \theta = 90^\circ)$$

**Fig: Gyroscope working principle**

Due to this torque, the flywheel disc rotates around the pivot. The precession frequency of the gyroscope wheel around the pivot is given by
Did You Know?

The line joining earth moon usually makes an angle of 5°. Only twice in a year, all the three orbs coincide. The crossing points are called lunar nodes. During that time only eclipses are observed. That is the only time when sun, moon and earth fall on the same line. In Indian astronomy as well as in indian astrology, they are named as Rahu and Kethu. They are 180° apart. The orbit of the moon precesses around earth with a period of 18.6 yaers. i.e.; the date os solar/lunar eclipse coincides for every 18.6 years. Every year eclipse date shifts back nearly by 20 days. That is why in Indian astronomy and astrology it is said that all planets revolve round the sun in anticlockwise direction and Rahu, Ketu revolve round the sun in clockwise direction. Also during eclipse time, people on earth experience higher gravitational pull and thus highest/lowest tides. The component of those highest gravity vectors on to everyday vectors connecting earth, sun and moon system is called the Rahu Kaala and Kethu Kaala. That is the time on earth when higher gravitational effects will be observed in a day. That includes high tide and low tide time as well at a given place. Solar eclipse occurs during Rahu kaala and lunar eclipse occurs during Kethu Kaala.


https://en.wikipedia.org/wiki/Lunar_node
2.7 Gyroscope

\[
\omega_p = \frac{d\phi}{dt} = \frac{\text{arc length}/\text{radius}}{dt} = \frac{dL/L}{dt} = \frac{\tau}{L} = \frac{M\dot{r}}{I\Omega} = \frac{gr}{R^2\Omega}
\]

From the above figure, since for small values of \(d\phi\), \(\Delta L\) becomes small and remains perpendicular to \(L\). Then

\[
|L \pm \Delta L/2|^2 = |L^2 + L \cdot \Delta L + \Delta L^2/4| \approx L^2 \quad (\because \Delta L = 0)
\]

Thus the angular momentum of the flywheel in the gyroscope doesn’t change as it precesses around the pivot. i.e.; the flywheel doesn’t fall from the pivot as long as it rotates.

In addition, if \(\Omega\) is set to a very large value, then \(\omega_p\) tends to zero. In other words, the flywheel doesn’t even precess. Thus any object tied to the gyroscope wheel keeps its orientation irrespective of fluctuations in the orientations. If the body of any vehicle is connected to the gimbal wheels and if the gyroscope wheel is distracted from its original direction manually, then the gimbal wheels will move in opposite direction to restore the original orientation of the flywheel. Then the total vehicle moves along with the gimbal wheels. This mechanism is used in steering space vehicles where there won’t be any support or material medium to push and take a turn.

In space ships, manual force is needed to be applied, instead of gravity in the above equations. Then the steering force generated will be in a direction perpendicular to the plane of both the angular momentum and the applied force direction.

**Fig: Gyroscopic navigation**
బలం క కటన్వరు ఉంయబన కణం కనాకతం అకపతాయం ఎభం
ఆతవరు ఈబ్యమంలంబం సమయం రగ హణం
లంబంచందూ గ హణం ఉనాత ఇవబం ఉతం ఉతన అడ్డం.

అమాత్ర తుమకు రాగారం మట్టం తింటే విషయం

సంవతరం సమయం కణం కర ణం
అయంత గ హణం
20 రాగముల గణం
అయంత గ హణం

తనంలో సంసమన మాత్రమే కణం ఇవబం ఉతం
తనంలో సంమయం రగ హణం

తనంలో సంవతరం సమయం కణం కర ణం
అయాంత గ హణం

江西省ప్రపంచ యుద్ధంలో సంసమన మాత్రమే, అయాంత గ హణం
20 రాగముల గణం

అయాంత గ హణం

ముందు జీత్వలో కొంత కాలం, అయాంత గ హణం
20 రాగముల గణం

అయాంత గ హణం

F = q(v × B) = \frac{q}{t} (l × B) = l (l × B)
2.8 Precession of atom and nucleus in magnetic field

When a charged particle moves, it produces magnetic field. If the path of the particle is circular, then the magnetic dipole moment is given by

\[ \mu = IA = \frac{q}{t} \pi r^2 = \frac{q}{(\text{dist.})} \pi r^2 = \frac{q}{\left(\frac{2\pi r}{v}\right)} \pi r^2 = \frac{qvr}{2} = \frac{q}{2m} mvr \]

\[ = \frac{qL}{2m} \]

**Fig: magnetic moment of charge in circular loop**

This magnetic moment is a vector whose direction is along the direction of the area vector. i.e.; perpendicular to the plane of rotation given by right hand thumb rule.

Suppose a charged particle is kept in external magnetic field, then the force experienced by charged particle is given by the Fleming’s left hand rule. If the field is parallel to the velocity of the particle, then there will not be any force on the particle. If the field is perpendicular, the charge moves in circular path in a plane perpendicular to the field.

The magnetic force on the charged particle is given by

\[ F = q(v \times B) = \frac{q}{t} (l \times B) = I (l \times B) \]

The torque produced on the charged particle due to magnetic field is given by

\[ \tau = I (A \times B) = \mu \times B \]
2.8 Precession of atom and nucleus in magnetic field

\[ \tau = I (A \times B) = \mu \times B \]

\[ \tau = \mu \times B = \frac{q}{2m} L \times B = \gamma L \times B = \gamma BL \sin \theta \]

\[ \omega_{\text{Larmor}} = \frac{\Delta \phi}{\Delta t} = \frac{\Delta L}{\Delta t \sin \theta} = \frac{\tau}{L \sin \theta} = \frac{\gamma B L \sin \theta}{L \sin \theta} = \gamma B \]

\[ \omega_{\text{Larmor}} = -\gamma B \]

\[ g_J = g_L \frac{J(J+1) - S(S+1) + L(L+1)}{2J(J+1)} + g_S \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \]

\[ g_F = g_J \frac{F(F+1) - I(I+1) + J(J+1)}{2F(F+1)} + g_I \frac{\mu_N}{\mu_B} \frac{F(F+1) + I(I+1) - J(J+1)}{2J(J+1)} \]

\[ g_F \approx g_J \frac{F(F+1) - I(I+1) + J(J+1)}{2F(F+1)} \]

\[ g_L = 1 \quad g_S = 2 \]

\[ \mu_B = -\frac{e}{2m_e} \quad \mu_N = +\frac{e}{2m_p} \]

\[ \mu_N \ll \mu_B. \]
When a charged particle in atoms, like electron, which is already in circular motion is kept in external magnetic field, it undergoes precession. Then

\[ \tau = \mu \times B = \frac{q}{2m} L \times B = \gamma L \times B = \gamma BL \sin \theta \]

Fig: Precession of electron in magnetic field.

The precession frequency, known as Larmor frequency, is given by

\[ \omega_{\text{Larmor}} = \frac{\Delta \phi}{\Delta t} = \frac{\Delta L}{\Delta t \sin \theta} = \frac{\tau}{L \sin \theta} = \frac{\gamma B L \sin \theta}{L \sin \theta} = \gamma B \]

For electron, \( \gamma \) is negative \((-e/2m)\), hence

\[ \omega_{\text{Larmor}} = -\gamma B \]

If there are multiple charged particles in the given atom, then the resultant precessional frequency is given by

\[ \omega_{\text{Larmor}} = \frac{eg}{2m} B \]

here \( g \) is the Lande \( g \)–factor of the system.

If the system involves orbital, spin angular momenta as well as nuclear magnetic momenta then
2.8 Precession of atom and nucleus in magnetic field

Larmor Precession. 16 Aug. 2020,
http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/larmor.html
https://en.wikipedia.org/wiki/Larmor_precession
https://en.wikipedia.org/wiki/Lande%20g-factor
2.9 Precession of Equinoxes

\[ g_J = g_L \frac{J(J + 1) - S(S + 1) + L(L + 1)}{2J(J + 1)} + g_S \frac{J(J + 1) + S(S + 1) - L(L + 1)}{2J(J + 1)} \]

or by using \( g_L = 1 \) and \( g_S = 2 \) one can get

\[ g_J = 1 + \frac{J(J + 1) + S(S + 1) - L(L + 1)}{2J(J + 1)} \]

Here \( L \) is the orbital angular momentum and \( S \) is the spin angular momentum and \( J \) is the total angular momentum of the electron.

If the nuclear charge also precesses in the external field, then

\[ g_F = g_J \frac{F(F + 1) - I(I + 1) + J(J + 1)}{2F(F + 1)} + g_l \frac{\mu_N}{\mu_B} \frac{F(F + 1) + I(I + 1) - J(J + 1)}{2J(J + 1)} \]

Here \( I \) is the spin angular momentum of nucleons and \( F = I + J \) the total angular momentum of the atom. Also, \( \mu_B = -e/2m_e \) the magnetic moment of electron (also called Bohr magneton) and \( \mu_N = +e/2m_p \) is the magnetic moment of proton. Since mass of proton is very large compared to that of electron, \( \mu_N \ll \mu_B \). Thus the above equation becomes,

\[ g_F \approx g_J \frac{F(F + 1) - I(I + 1) + J(J + 1)}{2F(F + 1)} \]

2.9 Precession of Equinoxes

Earth revolves around the sun in an elliptical orbit with a tilt of \( 23^\circ \) of the axis of rotation. Due to this tilt, around June 21st in northern hemisphere it will be longest day. This is called summer solstice. For Indians, the Sun rays seem to be coming from north corner. Or sun rises in the northeast corner and sets in the northwest corner. Around December 21st, in northern hemisphere longest night will be recorded. This is called winter solstice. For Indians, Sun rays
మామలు ప్రకారం కొలువైన 23 1/2° సంవత్సరాలు ప్రస్తుతి ఎంపిక లాంటి రాశిస్తూంది. ఈ రాశి సంవత్సరం, అసలు అంశం 21వ మంది నాడు నడిచాడుతూ 12 మంది నాడును రాసించాడు. ఈ రాశి 23 1/2 సంవత్సరం తొలగించడం జరిగింది. సత్యాలగా, సాధారణంగా మతం మారి ఫలితాలు వచ్చాం. ఈ రాశి ప్రత్యేకంగా మతం నడిచడం తెలుగు రాశిశేకం అయ్యింది. మామలు ఈ ప్రకారం 21వ సంవత్సరం యొక్క ఆసనం లభించడానికి ప్రత్యేకంగా వేసారు. మామలు ఈ ప్రకారం 21వ సంవత్సరం యొక్క ఆసనం లభించడానికి ప్రత్యేకంగా వేసారు.

ఈ ఉత్తరాలు ప్రత్యేకంగా మతం నడిచడం తెలుగు రాశిశేకం అయ్యింది. మామలు ఈ ప్రకారం 21వ సంవత్సరం యొక్క ఆసనం లభించడానికి ప్రత్యేకంగా వేసారు. మామలు ఈ ప్రకారం 21వ సంవత్సరం యొక్క ఆసనం లభించడానికి ప్రత్యేకంగా వేసారు.

ఈ పాత్ర పరిశీలన ప్రకారం మతం నడిచడం తెలుగు రాశిశేకం అయ్యింది. మామలు ఈ ప్రకారం 21వ సంవత్సరం యొక్క ఆసనం లభించడానికి ప్రత్యేకంగా వేసారు. మామలు ఈ ప్రకారం 21వ సంవత్సరం యొక్క ఆసనం లభించడానికి ప్రత్యేకంగా వేసారు.

ఈ పాత్ర పరిశీలన ప్రకారం మతం నడిచడం తెలుగు రాశిశేకం అయ్యింది. మామలు ఈ ప్రకారం 21వ సంవత్సరం యొక్క ఆసనం లభించడానికి ప్రత్యేకంగా వేసారు. మామలు ఈ ప్రకారం 21వ సంవత్సరం యొక్క ఆసనం లభించడానికి ప్రత్యేకంగా వేసారు.
seem to be coming from south corner. Or sun rises in the south east corner and sets in the south west corner. From June to December sun rising point shifts from north-east corner to south-east corner. From December to June the opposite happens. This journey of sun in the sky from north to south is called “Dakshinayana” (A journey towards the South) and the other one is called “Uttaraayana” (A journey towards the North).

Fig: Solstices and Equinoxes.

In between December and June, around March 21st and around September 23rd, the sun rises exactly in the east and sets exactly in the west. They are called Equinoxes. Then there will be equal day and night throughout the earth.

But this tilt angle of earth is actually a part of precession of earth’s axis of rotation around another fixed axis. This precession occurs because of the mutual interaction of gravitational forces of the sun and the moon on the earth’s orbit.
Due to this precession, by 14,000 year, north pole of earth would point towards Vegas star. At present it is pointing towards Polaris. This precession time period is 25,772 years. In other words, there will be a shift of $1^\circ$ for every 72 years.

At present Equinoxes occur at around September 23rd and 21st March.

Thus after 3221 years, the equinoxes shift to around 12th November and 10th May.
After 6443 years, the equinoxes shift to December 21st and June 21st.

After 12886 years, the equinoxes precess by 180°. Thus in northern hemisphere during December, there will be summer and during June, there will be winter.
Fig: Equinoxes after 12,886 years.

After 25,772 years, again the configuration of equinoxes will be similar to the present configuration.

Further reading

https://www.britannica.com/science/precession-of-the-equinoxes
https://personal.math.ubc.ca/~cass/courses/m309-01a/tsang/precession.html
1. In the early 20th century, the standard format for music recording was a plastic disc that held a single song and rotated at 78 RPM. What was the angular velocity of such a disc?
Sol. 
If we measure angles in units of revolutions and the time in units of minutes then 78 RPM is the angular velocity using standard physics units of radians per second, however, we have
Angular velocity $\omega = 78 \text{ RPM} = \frac{78 \times 2\pi}{60} = 8.2 \text{ radian/sec}$

2. What is your radial acceleration due to the rotation of the Earth if you are at the equator?
Sol:
We Know that, The distance from the axis of rotation of the Earth to a point on the equator where you stand is the same as the radius of the Earth.
Solved Problems and Exercises

\[ R = 6.4 \times 10^6 \text{m} \]

Your angular velocity

\[ \omega = \frac{2\pi \text{radians}}{1 \text{day}} = \]

\[ \frac{2\pi}{24 \times 60 \times 60} \text{radian/sec} = \frac{2 \times 3.14}{86,400} \text{radian/sec} = 7.3 \times 10^{-5} \text{radian/sec} \]

Hence your radial acceleration due to the Earth’s rotation is

\[ a_r = \omega^2 r \]

=6.4 \times 10^6

=0.034 \text{m/sec}^2

3. A neutron star is initially observed to be rotating with an angular velocity of 2.0 \text{ sec}^{-1}, determined via the radio pulses it emits. If its angular acceleration is \(-1 \times 10^{-8} \text{ sec}^{-2}\) which is assumed to be constant. How many rotations will it complete before it stops? (in reality the angular acceleration is not always constant, sudden changes occur and are referred to as starquakes)

Sol:

We know that, If \(\omega_0\) and \(\omega\) be the angular velocities at time \(t = 0\) and \(t\) respectively and \(\theta = \theta\) at initial position then

\[ \omega^2 = \omega_0^2 + 2\alpha(\Delta\theta) \]

Given attimet, its stops rotations ie \(\omega = 0\) and \(\omega_0 = 2.0 \text{ sec}^{-1}, \alpha = -1 \times 10^{-8} \text{ sec}^{-2}\)

\[ \Rightarrow \Delta\theta = -\frac{\omega_0^2}{2\alpha} = -\frac{4}{2 \times (-1 \times 10^{-8})} \text{radians} \]

\[ = 2 \times 10^8 \text{radians} = \frac{2 \times 10^8 \text{radians}}{2\pi \text{radians}} \]

\[ = 3.2 \times 10^7 \text{rotations} \]

[In general neutron stars are formed from the supernova explosion of a massive star. But after formation they continue collapsing due to neutron degeneracy pressure(described by Pauli exclusion principle) to form a balck hole]
4. Your arm has a mass of 3.0 kg, and its center of mass is 30 cm from your shoulder. What is the gravitational torque on your arm when it is stretched out horizontally to one side, taking the shoulder to be the axis?

Sol: The total gravitational force acting on your arm is 

\[ |F| = (3.0 \text{ kg})(9.8 \text{ m/s}^2) = 29 \text{ N}. \]

For the purpose of calculating the gravitational torque, we can treat the force as if it acted at the arm’s center of mass. The force is straight down, which is perpendicular to the line connecting the shoulder to the center of mass, so

\[ |F| = F \perp = 29 \text{ N}. \]

This force acts at the center of the arm,

\[ r = 30 \text{ cm} = 0.30 \text{ m}, \]

so the torque is

\[ \tau = r F \perp = 8.7 \text{ Nm}. \]

5. The barbell shown in the figure consists of two small, dense, massive balls at the end of a very light rod. The balls have masses of 2.0 kg and 1.0 kg and the length of the rod is 3 m. Find the moment of inertia of the rod (1) rotation about its centre of mass and (2) for rotation about the centre of the more massive ball.

( This example shows that the moment of inertia depends on the choice of axis. For example, it is easier to wiggle a pen about its Centre than about one and)
Solved Problems and Exercises

Sol: 1) Moment of inertia of the rod about its centre of mass:
From figure, 2 kg mass is at a distance 1m and 1 kg mass is at a
distance 2m from centre of mass.

\[ I = m_1 r_1^2 + m_2 r_2^2 = 2 \times 1 + 1 \times 4 = 6 kg m^2 \]

Here the less massive ball contributes more to moment of inertia.

2) Moment of inertia of the rod about its centre of mass of more
massive ball.
Here the big ball itself also theoretically contributes to moment of
inertia.Since the balls are said to be small and dense, we can neglect
their small contribution.

Hence moment of inertia about more massive ball

\[ I = m_1 r_1^2 = 1 \times 3^2 = 9 kg m^2 \]

6. In the men's Olympic hammer throw a steel ball of radius
6.1 cm is hung on the end of a wire of length 1.22 m .What
fraction of the ball's angular momentum comes from its
rotation as opposed to its motion through space?
Sol:Given radius of the ball \( r = 6.1 cm \) and length of the wire \( l = 1.22 m \)

Let \( T \) be the time taken for the ball to complete rotation once around
the circle, \( \Rightarrow T = \frac{2\pi l}{v} \Rightarrow v = \frac{2\pi l}{T} \)

\( \Rightarrow \) Angular momentum due to its motion through space \( L_{space} = mvl = \frac{2\pi ml^2}{T} \) \--(1)

Also the time it takes to revolve once around its own axis is same as
\( T \),

Then Angular momentum due to its rotation about its own axis

\[ L_{axis} = I\omega = \left( \frac{2}{5} mr^2 \right) \times \left( \frac{2\pi}{T} \right) \] \---(2)
The fraction of the ball's angular momentum comes from its rotation as opposed to its motion through space is

\[ \frac{L_{\text{axis}}}{L_{\text{space}}} = \frac{\left(\frac{2}{5}m r^2\right) \times \left(\frac{2\pi}{7}\right)}{(\frac{2}{7} r^3)} = \frac{2}{5} \times \left(\frac{2}{7} r^3\right) \]

7. Suppose two objects have the same mass and the same shape, but one is less dense, and larger by a factor \( k \). (1) How do their moments of inertia compare? (2) What if the densities are equal rather than the masses?

Ans: (1) This is like increasing all the distances between atoms by a factor \( k \). All the \( r \)'s become greater by this factor, so the moment of inertia is increased by a factor of \( k^2 \).

(2) This introduces an increase in mass by a factor of \( k^3 \), so the moment of inertia of the bigger object is greater by a factor of \( k^5 \).

8. Consider a simple rigid body consisting of two particles of mass \( m \) separated by a massless rod of length \( 2l \). The midpoint of the rod is attached to a vertical axis that rotates at angular speed \( \omega \) around the \( z \) axis. The rod is skewed at angle \( \alpha \), as shown in the sketch. Find the angular momentum of the system.

(source-Kleppner D., Kolenkow R. - An Introduction to Mechanics (2013, CUP) - libgen.lc)
Solved Problems and Exercises

Sol: Given length of the rod is 2l. The rod is massless and the given masses with mass m are particles of negligible size.

To find the value of $L$, first let us resolve angular momentum along the rod $\omega_{II}$ and perpendicular to the rod $\omega_{\perp}$ as shown.

Because the masses are particles with negligible size, $\omega_{II}$ produces no angular momentum. Consequently, the angular momentum is due entirely to $\omega_{\perp}$. Because $L$ is parallel to $\omega_{\perp}$, we can use the result from fixed axis rotation $L = I\omega_{\perp}$, where the moment of inertia about the direction of $\omega_{\perp}$ is $ml^2 + ml^2 = 2ml^2$.

The magnitude of the angular momentum is

$$L = I\omega_{\perp} = 2ml^2 \cos \alpha$$

$L$ points along the direction of $\omega_{\perp}$. Hence $L$ swings around with the rod; the tip of $L$ traces out a circle about the $z$ axis called precession.

(This reveals the fundamental property that angular momentum is not necessarily parallel to angular velocity, in contrast to the case of fixed axis rotation where $L$ and $\omega$ are parallel and related by $L = I\omega$.

9. Show that, a freely falling coin wobbles twice as fast as it spins as shown in fig.
We know that for a thin coin, moments of inertia are as follows.

\[
I_x = m\tau^2 \\
I_z = I_y = \frac{1}{2} m\tau^2
\]

So for a coin spinning along the z-axis \(I_z = 2I_x = 2I_y = 2I_\perp\)

The rotational speed \(\Omega\) relative to coin axes is

\[
\Omega = \frac{(I_z - I_\perp)}{I_\perp} \omega_z
\]

The rotational speed \(\Omega_{space}\) relative to an observer fixed in space is

\[
\Omega_{space} = (\Omega + \omega_z) = \frac{I_z}{I_\perp} \omega_z = 2\omega_z
\]

Thus, a freely falling coin wobbles twice as fast as it spins.

10. A particle of mass \(m\) is located at \(x = 2, y = 0, z = 3\). Find its moments and products of inertia relative to the origin.

(Source: Kleppner D., Kolenkow R. - An Introduction to Mechanics (2013, CUP) - libgen.lc)
11. As a bicycle changes direction, the rider leans inward creating a horizontal torque on the bike. Part of the torque is responsible for the change in direction of the spin angular momentum of the wheels. Consider a bicycle and rider system of total mass $M$ with wheels of mass $m$ and radius $b$, rounding a curve of radius $R$ at speed $V$. The center of mass of the system is $1.5b$ from the ground.

(a) Find an expression for the tilt angle $\alpha$.

(b) Find the value of $\alpha$, in degrees, if $M = 70$ kg, $m = 2.5$ kg, $V = 30$ km/hour and $R = 30$ m.

(c) What would be the percentage change in $\alpha$ if spin angular momentum were neglected? (Source: Kleppner D., Kolenkow R. - An Introduction to Mechanics (2013, CUP) - libgen.lc)
Solved Problems and Exercises

Sol:

The torque \( \tau_b \) about the center of mass is into the paper.

\[
\tau_b = N(1.5b) \tan \alpha - f(1.5b) \\
= Mg(1.5b) \tan \alpha - (1.5b) \frac{MV^2}{R}
\]

The total spin angular momentum (two wheels) is

\[
L_s = 2I_0\omega_s = 2mb^3 \frac{V}{b} = 2mbV \\
\tau_b = L_s\Omega = L_s \cos \alpha \frac{V}{R}
\]

\[
Mg(1.5b) \tan \alpha - (1.5b) \frac{MV^2}{R} = 2mb \frac{V^2}{R} \cos \alpha
\]

\[
\tan \alpha = \frac{V^2}{Rg} \left( 1 + \frac{4m}{3M} \cos \alpha \right)
\]

Because \( m/M \ll 1 \), the second term in parentheses is a small correction and it is adequate to take \( \cos \alpha = 1 \).

\[
\tan \alpha \approx \frac{V^2}{Rg} \left( 1 + \frac{4m}{3M} \right)
\]

Converting units, using \( g = 32 \text{ ft/s}^2 \),

\[
V = \left( \frac{20 \text{ miles}}{\text{hour}} \right) \times \left( \frac{5280 \text{ ft}}{\text{mile}} \right) \times \left( \frac{1 \text{ hour}}{3600 \text{ s}} \right) = 29.3 \text{ ft/s}
\]

\[
\frac{4m}{3M} = 0.048 \quad \frac{V^2}{Rg} = 0.268
\]

\[
\tan \alpha = (0.268)(1.048) = 0.28
\]

\[
\alpha \approx 16^\circ
\]

If spin is neglected, the term in \( m/M \) should be omitted. Then \( \alpha \approx 15^\circ \). The spinning wheels increase the tilt angle by only about a degree, not a substantial effect.

**MCQs**

**Section A**

MCQs

1. If angular momentum \( (h) \), velocity \( (c) \) and mass \( (M) \) are taken as fundamental units, the dimension of length in this system is
Solved Problems and Exercises

1. The position coordinate of an object of mass m are given by 
\[ x = \cos(\omega t), y = \sin(\omega t), z = \text{constant} \]. The z component of angular momentum is

a) \( m\hat{z} \)  

b) \( m\hat{\omega} \)  

c) \( m\omega z \)  

d) \( m\omega y \)

Ans: b (Hint: \( \vec{L} = \vec{r} \times m\vec{\omega} \))

2. The mass per unit length of a rod of 2m length as \( \rho = \frac{3}{2} \). The moment of inertia in \( kgm^2 \) of the rod about a perpendicular axis passing through the tip of the rod (at x=0) is

\[ a) 18 \quad b) 14 \quad c) 12 \quad d) 8 \]

Ans: d (Hint: \( I = \frac{1}{3}ML^2 \) and \( M = \rho L \))

3. The angular velocity of the Earth about its axis increases then the value of ‘g’ at equator

a) Does not change  

b) Increases  

c) Decreases  

d) Become zero

Ans: c (Hint: \( g' = g - \omega^2 R \cos^2 \lambda \) and at equator \( \lambda = 0^0 \) => \( g' = g - \omega^2 \))

5. The moment of inertia of a circular disc R about its centre is

\[ a) MR^2 \quad b) \frac{MR^2}{2} \quad c) \frac{2MR^2}{5} \quad d) \frac{2MR^2}{3} \]

Ans: a

6. The rate of precession of spinning top is inversely proportional to the (AUCET 2020)
Solved Problems and Exercises

a) Angular momentum  
   b) Angle  
   c) Mass  
   d) Torque  

AUCET 2020

Ans: a \( \text{Hint:} \Omega = \frac{mgr}{I} \omega \)

7. Which of the following is an example for axial vector
   a) Displacement  
   b) Torque  
   c) Linear velocity  
   d) Force

AKNU 2020

Ans: b

8. When inertia tensor operates on angular velocity \( \omega \), it produces --- vector
   a) Force  
   b) Torque  
   c) Angular momentum  
   d) Angular velocity

AKNU 2020

Ans: c

9. An acrobat obeys
   a) Law of conservation of energy  
   b) Law of conservation of mass  
   c) Law of conservation of momentum  
   d) Law of conservation of angular momentum

AKNU 2020

Ans: d

10. The precessional period of Earth’s axis of rotation is about
    a) 440 years  
    b) 26000 years  
    c) 365 days  
    d) 24 hours

    Ans: b

Section B
Solved Problems and Exercises

11. An automobile develops 100 HP power when rotating at a speed of 1000 rpm. The torque acting is

a) 712.7 N-m  
   b) 74.6 N-m  
   c) 643.4 N-m  
   d) 314 N-m

Ans: a

Hint: \( \tau = \frac{p}{\omega} \) and 1 HP = 746 watt

12. A wheel is rotating at a frequency of \( f_0 \) Hz about a fixed vertical axis. The wheel stops in \( t_0 \) seconds, with constant angular deceleration. The number of turns covered by the wheel before it comes to rest is given by

a) \( f_0 t_0 \)  
   b) \( 2f_0 t_0 \)  
   c) \( \frac{f_0 t_0}{2} \)  
   d) \( \frac{f_0 t_0}{4} \)

IIT JAM 2020

Ans: c

Hint: \( \omega_0 = 2\pi f_0 \) and \( \omega = \omega_0 - at \) \( \rightarrow \alpha = \frac{2\pi f_0}{t_0} \) Then substitute \( \alpha \) value in the equation

\[
\theta = \omega_0 t - \frac{1}{2} at^2
\]

Hence no. Of turns = \( \frac{\theta}{2\pi} \)

13. If the diameter of the Earth is increased by 4\% without changing the mass, then the length of the day is _______ hours. (Take the length of the day before the increment as 24 hours. Assume the Earth to be a sphere with uniform density). (Round off to 2 decimal places)

IIT JAM 2019

Ans: 25.95 hours

Hint: Use law of conservation of angular momentum \( I_1 \omega_1 = I_2 \omega_2 \)

14. The rotational kinetic energy of a thin disc, of mass 20 kg and radius 70 cm, rotating like a merry go round at an angular speed of 120 rad/min, is

a) 5.9 J  
   b) 9.8 J  
   c) 19.6 J  
   d) 21.6 J

HCU 2019
Solved Problems and Exercises

Ans: B

Hint: \( K \cdot E_{\text{rot}} = \frac{1}{2} I \omega^2 \) and for disc \( I_{\text{cm}} = \frac{1}{2} MR^2 \)

15. If a rigid body with \( N \) particles is constrained to be fixed at a single point, then the number of degrees of freedom is
a) 3  b) 6  c) \( N \)  d) 0

HCU 2012

Ans: a

Hint: Ringid body without any constraint has 6 degrees of freedom, 3 translational 3 rotational. If translation is blocked, it will have only 3 degrees of freedom.

16.

The figure shows a thin square sheet of metal of uniform density along with possible choices for a set of principal axes (indicated by dashed lines) of the moment of inertia, lying in the plane of the sheet. The correct choice(s) for the principal axes would be

(A) \( p, q, \) and \( r \)  (B) \( p \) and \( r \)  (C) \( p \) and \( q \)  (D) \( p \) only

IIT JAM 2012

Ans:

Hint: \( I_z \) about center of mass due to \( p \) and \( q \) are same

17.

A particle is moving in a plane with a constant radial velocity of 12 m/s and constant angular velocity of 2 rad/s. When the particle is at a distance \( r = 8 \) m from the origin, the magnitude of the instantaneous velocity of the particle in m/s is

(A) \( 8\sqrt{15} \)  (B) 20  (C) \( 2\sqrt{37} \)  (D) 10.

IIT JAM 2016
Solved Problems and Exercises

Ans: b

Hint: \( v_r = 12 \frac{m}{s}, v_\theta = \omega r \) and \( v = \sqrt{v_r^2 + v_\theta^2} \)

18.

A uniform disk of mass \( m \) and radius \( R \) rolls, without slipping, down a fixed plan inclined at an angle 30° to the horizontal. The linear acceleration of disc(in m/s²) is

a) 5.25  b) 3.26  c) 2.5  d) 5.5

IIT JAM 2015

Ans: b

Hint: Equation of motion is \( mg \sin \theta - F = ma \) and torque= \( FR = I \alpha \) and \( I = \frac{mR^2}{2} \)

19. A particle moves in a circular path in the xy-plane centred at the origin. If the speed of the particle is constant, then the angular momentum

a) about the origin is constant both in magnitude and direction
b) about (0,0,1) is constant in magnitude and but not in direction
c) about (0,0,1) varies both in magnitude and direction
d) about (0,0,1) is constant in direction and but not in magnitude

IITJAM 2014

Ans: a and b

Hint: Angular momentum will be constant in xy-plane:

20. Seven uniform disks, each of mass and radius \( r \) are inscribed inside regular hexons, as shown. The moment of inertia of this system of seven disks, about an axis passing through central disc and perpendicular to the plane of disks, is
Solved Problems and Exercises

IIT JAM 2015

Ans: d

Hint: \( \frac{mr^2}{2} + 6 \times \left( \frac{mr^2}{2} + 4mr^2 \right) = \frac{55mr^2}{2} \)

Section C:

23. The moment of inertia of a disc about one of its diameters is \( I_M \). The mass per unit area of the disc is proportional to the distance from its centre. If the radius of the disc is \( R \) and its mass is \( M \), the value of \( I_M \) is

(A) \( \frac{1}{2}MR^2 \)  (B) \( \frac{2}{3}MR^2 \)  (C) \( \frac{3}{10}MR^2 \)  (D) \( \frac{3}{5}MR^2 \)

IIT JAM 2014

Ans:C

Sol: Mass density \( \sigma = \rho r \), \( M = \int_0^R \rho r \cdot r dr d\theta \Rightarrow \rho = \frac{3M}{2\pi R^3} \)
24.

A disc of radius $R_1$ having uniform surface density has a concentric hole of radius $R_2 < R_1$. If its mass is $M$, the principal moments of inertia are

(A) $\frac{M(R_1^2 - R_2^2)}{2} \frac{M(R_1^2 - R_2^2)}{4} \frac{M(R_1^2 + R_2^2)}{4}$

(B) $\frac{M(R_1^2 + R_2^2)}{2} \frac{M(R_1^2 + R_2^2)}{4} \frac{M(R_1^2 + R_2^2)}{8}$

(C) $\frac{M(R_1^2 + R_2^2)}{2} \frac{M(R_1^2 + R_2^2)}{4} \frac{M(R_1^2 + R_2^2)}{8}$

(D) $\frac{M(R_1^2 - R_2^2)}{2} \frac{M(R_1^2 - R_2^2)}{4} \frac{M(R_1^2 - R_2^2)}{8}$

Ans: b

Sol: $I_z = \int_{R_1}^{R_2} r^2 \, dm = \frac{M}{\pi (R_1^2 - R_2^2)} \int_{R_1}^{R_2} 2\pi r \, r^2 \, dr = \frac{M(R_1^2 + R_2^2)}{2}$

$I_x + I_y = I_z \text{ and by symmetry, } I_x = I_y \Rightarrow I_x = I_y = \frac{M(R_1^2 + R_2^2)}{4}$

Consider a uniform thin circular disk of radius $R$ and mass $M$. A concentric square of side $R/2$ is cut out from the disk (see figure). What is the moment of inertia of the resultant disk about an axis passing through the centre of the disk and perpendicular to it? (See figure)

(A) $l = \frac{M R^2}{4} \left[ 1 - \frac{1}{40 \pi} \right]$

(B) $l = \frac{M R^2}{2} \left[ 1 - \frac{1}{40 \pi} \right]$

(C) $l = \frac{M R^2}{4} \left[ 1 - \frac{1}{24 \pi} \right]$

(D) $l = \frac{M R^2}{2} \left[ 1 - \frac{1}{24 \pi} \right]$

25. IIT JAM 2017
Ans: B

Sol: \[ I = I_{\text{disc}} - I_{\text{square}} = \frac{MR^2}{2} - \frac{Mr(2a^2)}{2} \]
\[ M' = \frac{M}{\pi R^2} X \frac{R}{2} X \frac{R}{2} = \frac{M}{4\pi} \text{ and } a = \frac{R}{2} \]
\[ I = \frac{MR^2}{2} \left[ 1 - \frac{1}{48\pi} \right] \]

26. A spherical ball of ice has radius \( R_0 \) and is rotating with an angular speed \( \omega \) about an axis passing through its centre. At time \( t=0 \), it starts acquiring mass because the moisture (at rest) around it starts to freeze on its uniformity. As a result its radius increases as \( R(t) = R_0 + at \). Here \( a \) is constant. The curve which best describes its angular speed with time is

Ans: B

Sol: \( I(t) \omega(t) = \text{constant} \) and \( M(t)R(t) = \text{constant} \)

\[ \omega(t) = \frac{\text{constant}}{M(t)[R_0+at]^2} \Rightarrow \omega(t) \propto \frac{1}{t^2} \]
Grade Your Understanding

1. Angular momentum vector and angular velocity vectors are always in all cases, points along the direction axis of rotation

2. In general, rotational quantities depend on more factors than linear quantities

3. Precession becomes slower as the spin of top slows down

4. In general, the change in an axial vector is perpendicular to that vector itself

5. A spinning top has zero total momentum, but have finite kinetic energy

6. Finite angular displacement is a vector quantity

7. The tensor of inertia with respect to principal axes has a diagonal form

8. The angular momentum vector $\vec{L}$ depends on the choice of origin of the coordinate systems

9. The mass and the radius of a flywheel that aids a car should both be smaller to get the car up to speed by giving almost all its kinetic energy

10. Reducing the mass of the tires and wheel rims of a racing bike rather than an identical reduction in the mass of bike’s frame would allow the racer to achieve greater accelerations

11. Flying spacecraft cannot be in the shape of a cylinder because the cylinder is inherently unstable about its axial axis

12. Work and torque have same dimensions N-m, hence torque is a form of energy

## Glossary

**Glossary: Rigid body dynamics**

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angular moment</td>
<td>Property of any rotating body that gives Moment of momentum about an axis of rotation</td>
</tr>
<tr>
<td>Bulgingness</td>
<td>The property possessed by a round shape that is flattened at the poles</td>
</tr>
<tr>
<td>Coordinate system</td>
<td>A system that specifies each point in space uniquely by a set of coordinates</td>
</tr>
<tr>
<td>Constraint</td>
<td>A parameter that the system must obey throughout the problem</td>
</tr>
<tr>
<td>Centre of gravity</td>
<td>The point where the gravity appears to act. It focuses on weight of the body</td>
</tr>
<tr>
<td>Centre of mass</td>
<td>The mean position of the mass in an object. It focuses on mass of the body</td>
</tr>
<tr>
<td>Degrees of freedom</td>
<td>The number of independent parameters that define the state of a mechanical system</td>
</tr>
<tr>
<td>Dynamics</td>
<td>The branch of mechanics concerned with the motion of bodies under the action of force</td>
</tr>
<tr>
<td>Equation of motion</td>
<td>The equations that describe the behavior of physical system in terms of its motion as a function of time</td>
</tr>
<tr>
<td>Equinoxes</td>
<td>An event in which a planet’s subpolar point passes through its equator</td>
</tr>
<tr>
<td>Fly Wheel</td>
<td>A mechanical heavy wheel having high rotational inertia ie most of its weight is well out from the axis. The high inertia opposes the speed fluctuations of an engine.</td>
</tr>
<tr>
<td>Gyro compass</td>
<td>An automatic non magnetic compass to find geographical direction. Based on fast spinning top/disc and the rotation of the Earth</td>
</tr>
<tr>
<td>Kinematics</td>
<td>The branch of mechanics concerned with the motion of bodies without considering the force</td>
</tr>
<tr>
<td>Magnetic dipole</td>
<td>A magnet with two poles S &amp;N or magnet equivalent to a current loop</td>
</tr>
<tr>
<td>Glossary</td>
<td>Definition</td>
</tr>
<tr>
<td>---------------</td>
<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Nutation</td>
<td>An uneven wobbling/Nodding motion in the axis of rotation of a largely axially symmetric object</td>
</tr>
<tr>
<td>Principal axes</td>
<td>Any of three mutually perpendicular axes about which the moment of inertia of a body is maximum</td>
</tr>
<tr>
<td>Point mass</td>
<td>Any material body with negligible dimension and internal structure</td>
</tr>
<tr>
<td>Pseudo</td>
<td>A physically apparent but nonexistent force felt by an observer in a non-inertial frame (A frame undergoing acceleration)</td>
</tr>
<tr>
<td>Precession</td>
<td>Rotation of rotational axis of a rotating body another fixed axis</td>
</tr>
<tr>
<td>Radian</td>
<td>An SI unit of measuring angle whose corresponding arc in a circle is equal to the radius of the circle</td>
</tr>
<tr>
<td>Rectilinear</td>
<td>An object has a rectilinear motion when it moves along a straight line</td>
</tr>
<tr>
<td>Rigid Body</td>
<td>A solid body which does not undergo any change in shape or size (Stretch, compress or bend) by the application of external force</td>
</tr>
<tr>
<td>Solstice</td>
<td>Latin words Sol for Sun and sister for To stand still</td>
</tr>
<tr>
<td>Symmetry element</td>
<td>A line, a plane or a point in or through an object about which a rotation leaves the object in an orientation same as original</td>
</tr>
<tr>
<td>Translational motion</td>
<td>An object has a Translational motion when it shifts from one point to another along any possible path</td>
</tr>
<tr>
<td>Tensor</td>
<td>Generalization of scalar and vectors which are to be used to describe any physical quantity mathematically</td>
</tr>
</tbody>
</table>
UNIT-II
Chapter-3
MOTION IN A CENTRAL FORCE FIELD
పరిమిత ఫాంటి ప్రతి చలనం
చాలా వింపలం
వాతావరణం దానిలో వాతావరణం నేటి ప్రస్తుతం లేదు.
1. చాలా వింపలం దానిలో వాతావరణం నేటి ప్రస్తుతం లేదు.
2. చాలా వింపలం దానిలో వాతావరణం నేటి ప్రస్తుతం లేదు.
3. చాలా వింపలం దానిలో వాతావరణం నేటి ప్రస్తుతం లేదు.
4. GPS వింపలం దానిలో వాతావరణం నేటి ప్రస్తుతం లేదు.
5. చాలా వింపలం దానిలో వాతావరణం నేటి ప్రస్తుతం లేదు.

చాలా వింపలం
1. చాలా వింపలం దానిలో వాతావరణం నేటి ప్రస్తుతం లేదు.
2. చాలా వింపలం దానిలో వాతావరణం నేటి ప్రస్తుతం లేదు.
3. చాలా వింపలం దానిలో వాతావరణం నేటి ప్రస్తుతం లేదు.
4. చాలా వింపలం దానిలో వాతావరణం నేటి ప్రస్తుతం లేదు.
5. చాలా వింపలం దానిలో వాతావరణం నేటి ప్రస్తుతం లేదు.
6. చాలా వింపలం దానిలో వాతావరణం నేటి ప్రస్తుతం లేదు.
Motion in a Central Force Field

Syllabus

Central forces, definition and examples, characteristics of central forces, conservative nature of central forces, Equation of motion under a central force, Kepler’s laws of planetary motion- Proofs, Motion of satellites, Basic idea of Global Positioning System (GPS), weightlessness, Physiological effects of astronauts

Learning Objectives

In this chapter students would learn about,

1. Central forces properties, and their conservative nature.
2. General equation of motion under central forces.
3. Proofs of Kepler laws
4. Basic ideas of GPS
5. Weightlessness and physiological effects of astronauts.

Learning Outcomes

By the end of the chapter, student would be able to

1. Describe the properties of central forces and also the Kepler laws.
2. Interpret Kepler laws based on the other conservative laws.
3. Apply central force properties to specific systems of interest.
4. Analyse Kepler laws to arrive at satellite motion and weightlessness conditions.
5. Justify the physiological effects on astronauts in weightless state.
6. Develop prototype models for proper space flight maneuver suitable for various types of satellites and space crafts.
ధనాల చేత అనే రౌధర ప్రత్యేక పరిశీలన సమాచారాలను కలిగిన కార్యాలో కూడా అల్లపుల అంచంలో ఉండగల

1. భూచారం: ఆరోగ్య రూపాలు ఉపయోగాలయ్యాయ్యా హలూడు పాఠానిక పరిశీలన సమాచారాలు కలిగి
నిర్ణయం చేయడానికి ఉపయోగపడతాయి.

2. జాబీ: రేపు సాధారణంగా కనిపించే ప్రత్యేకత మేరిక మారుగలు క్రింద మారుగలు మరియు పరిశీలనలను ప్రారంభించారు.

3. కార్యాలీకరణ ఉపసంహరణ అనేక కార్యాల సమాచారాలు సమాచారం ప్రారంభం కృతి
నించడానికి ఉపయోగపడతాయి.

4. పాఠకు: ఇంటిని సంభవించి, పాఠానికం, పాఠానికం చెందిన సంఖ్యలు పరిశీలించడానికి ప్రారంభం కృతి
నించడానికి ఉపయోగపడతాయి.

5. అధీనస్థ: మరింత ఉపయోగం ఉపయోగం ఉపయోగం ఉపయోగం ఉపయోగం ఉపయోగం ఉపయోగం
నించడానికి ఉపయోగపడతాయి.

6. కార్యాలీకరణ ఉపసంహరణ ఉపసంహరణ ఉపసంహరణ ఉపసంహరణ ఉపసంహరణ
నించడానికి ఉపయోగపడతాయి.

7. సంస్థలు: కార్యాల మారింది మారింది మారింది మారింది మారింది మారింది
నించడానికి ఉపయోగపడతాయి.

పరిశీలన సమాచారాల ఊర

మూడు రోజులలో పడి చేత పనిచేసే పాఠానికం ఉపయోగపడతాయి. మొత్తం 115 కార్యాలు ఉపయోగపడతాయి. మొదటి 50 కార్యాలు మారివేయబడిన మరియు
అయినంత పరిశీలన ఉపయోగపడతాయి. మొదటి 15 కార్యాలు ఉపయోగపడతాయి. మరియు ప్రతి 20 కార్యాలు ఉపయోగపడతాయి.
ప్రతి 20 కార్యాలు ఉపయోగపడతాయి. మారివేయబడిన 30 కార్యాలు ఉపయోగపడతాయి. మారివేయబడిన 40 కార్యాలు ఉపయోగపడతాయి.
మారివేయబడిన 50 కార్యాలు ఉపయోగపడతాయి. మారివేయబడిన 60 కార్యాలు ఉపయోగపడతాయి. మారివేయబడిన 70 కార్యాలు ఉపయోగపడతాయి. మారివేయబడిన 80 కార్యాలు ఉపయోగపడతాయి. మారివేయబడిన 90 కార్యాలు ఉపయోగపడతాయి. మారివేయబడిన 100 కార్యాలు ఉపయోగపడతాయి. మారివేయబడిన 110 కార్యాలు ఉపయోగపడతాయి. మారివేయబడిన 115 కార్యాలు ఉపయోగపడతాయి.
Course Outcomes specific to program and Future directions

By the end of this chapter students from specific programs would be able to identify the need of central forces in the following fields.

1. Physics: The branch of physics called astronomy has its origins in Kepler problem and other central force problems.
2. Chemistry: Planetary ball milling method, which is used for nanomaterial preparation, requires knowledge of central force equation of motion.
4. Geology: Astrogeology, a branch of geology that deals with asteroids, craters on moon etc..
5. Electronics: Selection of suitable orbit for various types of satellites requires knowledge about central force problems.
6. Renewable energy: Kepler problem is required for the proper prediction of solar energy received on earth through out an year.
7. Statistics: Statistical analysis of models of planet formation requires knowledge about central forces.

Familiar to Unfamiliar

In your 8th class, you are introduced about the solar system, planets, natural and artificial satellites. In your 9th class you are introduced with Kepler laws for planetary motion. You are also introduced about free fall. In your 11th class, you are given detailed explanation of Kepler laws, satellites like geo stationary and polar satellites and also about weightlessness. In this chapter, you would learn about central force problem which is responsible for planetary motion in fixed orbits. You would derive Kepler laws and introduced with various types of artificial satellites and their orbits. You would also be introduced with weightlessness and various physiological changes that astronauts undergo during their space travel.
3.1 పచయం

ధర్మానంత్రం రేం పదార్థాలు సాధారణంగా సిద్ధం నిశ్చలించబడింది. అడుగు ప్రతి తప్పని మతానుసరించడానికి, ప్రతిభ నిర్ధారణలు విషయం పిండి సహకరించబడినా మనం నిలిస్తుంది. తిరంగ కప్ప విడివడం కారణంగా, అది మనం నిర్ధారణలను విషయం పిండి సమర్పించడానికి తప్పనీ ప్రతిభ నిర్ధారణలు వుంది. 3.1 పచయంలో నిర్ధారణలను వేసిన తరువాత మనం ఆభం తప్పవచ్చు. అయితే దానికి మూడు సంవత్సరాలు ఎక్కడ అభం తప్పవచ్చు. అధికారుల తప్పని నిపుణుతుంది. మనం ఆభం తప్పించడానికి మూడు సంవత్సరాలు ఎక్కడ మనం ఆభం తప్పవచ్చు. 

మూడు సంవత్సరాలు (మార్చి 31 & జాన్యూస్ 31) ఎందుకంటే ఎందుకంటే వెలుగు లక్షణాలు లేని మనం నిర్ధారణలు వేయకుండా తప్పనించాలని చెబుతుందని అభం తప్పవచ్చు. ప్రతి వందబడితే, ప్రతిలో మూడు సంవత్సరాలు (మార్చి 31) ఎందుకంటే ఎందుకంటే వెలుగు లక్షణాలు లేని మనం నిర్ధారణలు వేయకుండా తప్పనించాలని చెబుతుందని అభం తప్పవచ్చు. ప్రతి వందబడితే, ప్రతి వందబడితే మనం నిర్ధారణలు వేయకుండా తప్పనించాలని చెబుతుందని అభం తప్పవచ్చు. 

మూడు సంవత్సరాలు (మార్చి 31 మార్చి 31) ఎందుకంటే ఎందుకంటే వెలుగు లక్షణాలు లేయంది మనం నిర్ధారణలు వేయకుండా తప్పనించాలని చెబుతుందని అభం తప్పవచ్చు. మనం అయితే ఒకరేటి మార్చి 31 నిర్ధారణాలు వేయకుండా తప్పనించాలని చెబుతుందని అభం తప్పవచ్చు. మనం అయితే ఒకరేటి మార్చి 31 నిర్ధారణాలు వేయకుండా తప్పనించాలని చెబుతుందని అభం తప్పవచ్చు. మనం అయితే ఒకరేటి మార్చి 31 నిర్ధారణాలు వేయకుండా తప్పనించాలని చెబుతుందని అభం తప్పవచ్చు. మనం అయితే ఒకరేటి మార్చి 31 నిర్ధారణాలు వేయకుండా తప్పనించాలని చెబుతుందని అభం తప్పవచ్చు.
3.1 Introduction

Study of astronomy is ever fascinating since the caveman’s times. Sun, moon and stars are the natural time keepers for human beings since then. The phase of moon gives the information about which date we are in, while the location of sun rise on eastern sky gives the information about month. The location of stars in clear night sky tells us about the year we are in. As we have already come across in the previous chapter, the pole star location shifts to Vegas by 14,000 year. Similarly observation of various star movements in sky can be used to obtain information about the year.

Observation of these environmental changes slowly converted human beings from a life style of hunter-gatherer to that of an agricultural colony man. This had happened around 12,000 years ago. This lead to production of surplus amount of food thus enabling people to find time to discover other secrets of nature.

At first people have thought that earth is flat and floating on water. Some of the Indian Puranas also described the same. But the earliest full fledged text available on astronomy, the vedanga Jyothsha by Lagadha from 1000 BC, contains descriptions on solar months, lunar months and leap month (Adhimasa) etc., in detail. Major developments in astronomy were reported in Babylonia also during 7th century BC. They have described the transit of Venus across the sun on earthen slates/tablets. Further breakthrough in the subject came from Aaryabhatiya and Aryabhata Siddhanta in 5th century BC.

Chinese astronomy dates back to 6th century BC. In 4th century BC Greeks have developed 3D working models of planetary motion as

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**Did You Know?**

Sun rises in the east and sets in the west only twice in a year during Equinoxes. In India, on Summer solstice day, sun rises in the southeast corner and on Winter solstice day, sun rises in the northeast corner. The journey of sun in the eastern sky from winter solstice to summer solstice is called Uttaraayana (The journey towards north), the corresponding return journey is called Dakshinayana (The journey towards South).
3.1 Introduction

E - Corner

https://archive.org/details/VedangaJyotisha
https://archive.org/details/vedicchronologya033083mbp
https://en.wikipedia.org/wiki/Flat_Earth
https://en.wikipedia.org/wiki/Copernican_Revolution
https://tinyurl.com/Heliocentrism-htm

Activity

Simulate the path of the sun at your place during various months using the last link in the e-corner.
seen from earth. Antikythera of 2nd century BC is one such model available now. Egyptian pyramids of 3rd century were built with alignment towards, the then pole star, Thuban.

Indian scriptures were translated to Arabic as Zīj al-Sindhind in 8th century AD and opened doors to studies on Astronomy in Arabic countries.

Heliocentric theory was reported in Indian literature, in Aryabhatiya in 5th century BC, in Babylonian literature by Celucus in 3rd century BC, in Greek literature by Aristacus in 3rd century BC. Later in 1st century AD, Ptolemy’s theory of geocentric model came into lime light. He used epicycles to explain the complex motion of planets and sun as seen from earth. Subsequently, sophisticated modern era calculations, supporting heliocentric theory, were proposed by Madhava of Sangamagrama, Neelakantha and Jayadeva from Kerala, India in 15th century, with full complexity and accuracy. Later Tycho Brahe, Kepler and Copernicus have proposed independently, rather crude models for planetary motion using equants. Further Newton’s work on gravity and planetary motion, which gave a complete calculus based analysis that we use in this modern era, came into picture. In all the cases the end result is accurate to a few decimal places. Hence heliocentric model gives slightly mathematically easier model of solar system while the geocentric theory gives a relatively complex model of solar system. But both are equivalent to each other.
3.1 Introduction

**Fig: Ptolemaic epicentre model of solar system**

![Diagram of Ptolemaic epicentre model of solar system](image)

**Fig: Epicenter model of Ptolemy**

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**E - Corner**

http://news.bbc.co.uk/2/hi/science/nature/2679675.stm
https://www.ebookarchive.org/details/bhumiilavundanielakanipettaruhowpeoplediscoveredtheshapeofearthatomilin
https://www.bighistoryproject.com/chapters/4
https://www.shockingscience.com/the-found-the-worlds-oldest-calendar-from-8000-b-c/
https://physics.weber.edu/schroeder/ua/sunandseasons.html
https://en.wikipedia.org/wiki/Geocentric_model
https://en.wikipedia.org/wiki/Indian_astronomy
http://andrewmarsh.com/apps/staging/sunpath3d.html
3.1 Introduction

https://apod.nasa.gov/apod/ap201221.html
3.2 బలాల రేఖాగణితం & నియమాలాలు

3.2 బలాల రేఖాగణితం & నియమాలాలు

ప్రత్యేకంగా ప్రదర్శనం చేసే లోకాల పట్టణం ప్రతి ఫండింగు ప్రదానం మాత్రమే అందించిన ప్రతి బాగా కంటే విశాలంగా ప్రదానం మాత్రమే అందించిన ప్రతి బాగా కంటే విశాలంగా

\[ \mathbf{F} = F(r) \hat{r} \quad \quad (1) \]

వైనిక, యొక్క పరిమాణం రూపాలు ఒక గాంధీం రాష్ట్రానికి సాధించాడు. వైనిక బలాల రేఖాగణితం ముందు చాలా ప్రమాదా మాత్రమే అందించాడు. వైనిక పరిమాణం ముందు ప్రదానం మాత్రమే అందించాడు.

అధ్యాపకులు

1. బలాల రేఖాగణితం యొక్క లేదా చాలా పరిమాణం వివిధ పద్ధతుల మేధా నియమాల మాత్రమే అందించాడు. వైనిక పరిమాణం ముందు ప్రదానం మాత్రమే అందించాడు. వైనికి పరిమాణం ముందు ప్రదానం మాత్రమే అందించాడు ముందు ప్రదానం మాత్రమే అందించాడు.

\[ \nabla \times \mathbf{F} = 0 \quad (2) \]

\[ \nabla \times \mathbf{F} = \frac{\partial}{\partial r} \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\phi} \\ \frac{\partial}{\partial r} & \frac{1}{r} \frac{\partial}{\partial \theta} & \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \\ F(r) & 0 & 0 \end{vmatrix} = 0 \]

\[ \nabla \times \mathbf{F} = 0 \quad (2) \]

\[ \nabla \times \mathbf{F} = \frac{\partial}{\partial r} \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\phi} \\ \frac{\partial}{\partial r} & \frac{1}{r} \frac{\partial}{\partial \theta} & \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \\ F(r) & 0 & 0 \end{vmatrix} = 0 \]

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2. బలాల రేఖాగణితం యొక్క లేదా చాలా పరిమాణం వివిధ పద్ధతుల మేధా నియమాల మాత్రమే అందించాడు. వైనిక పరిమాణం ముందు ప్రదానం మాత్రమే అందించాడు. వైనికి పరిమాణం ముందు ప్రదానం మాత్రమే అందించాడు.

\[ \mathbf{\tau} = \mathbf{r} \times \mathbf{F} = r \mathbf{\hat{r}} \times (F(r) \mathbf{\hat{r}}) = \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\phi} \\ r & 0 & 0 \\ F(r) & 0 & 0 \end{vmatrix} = 0 \]

3. బలాల రేఖాగణితం యొక్క లేదా చాలా పరిమాణం వివిధ పద్ధతుల మేధా నియమాల మాత్రమే అందించాడు. వైనికి పరిమాణం ముందు ప్రదానం మాత్రమే అందించాడు. వైనిక పరిమాణం ముందు ప్రదానం మాత్రమే అందించాడు. వైనికి పరిమాణం ముందు ప్రదానం మాత్రమే అందించాడు. వైనికి పరిమాణం ముందు ప్రదానం మాత్రమే అందించాడు. వైనికి పరిమాణం ముందు ప్రదానం మాత్రమే అందించాడు.

\[ r \cdot L = r \cdot (r \times p) = p \cdot (r \times r) = p \cdot 0 = 0 \]

\[ p \cdot (r \times p) = r \cdot (p \times p) = r \cdot 0 = 0 \]
3.2 Central force Characteristics & Examples

A central force is a force which is directed always towards a fixed point. Thus one can write the force as

$$\vec{F} = F(r)\hat{r} \quad \text{--- (1)}$$

i.e.; the force is a function of $r$ and is directed along $\hat{r}$, that is towards a fixed point.

Characteristics

1. If the potential energy is not time dependent, then the central forces are conservative. That means, the net amount of work done by the force in moving a particle in a closed loop will be zero. i.e.; $\nabla \times \vec{F} = 0$.

Substituting the form of $F$ from Eq. (1) gives,

$$\nabla \times F = \nabla \times (F(r)\hat{r}) = \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\phi} \\ \frac{\partial}{\partial r} & \frac{1}{r} \frac{\partial}{\partial \theta} & \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \\ F(r) & 0 & 0 \end{vmatrix} = 0$$

Since $\nabla \times F = 0$, one can write force as a negative gradient of potential. i.e.; $F = -\nabla U$.

2. For conservative central forces, angular momentum also is conserved. To understand this, consider the torque produced.

$$\tau = \vec{r} \times \vec{F} = r\hat{r} \times (F(r)\hat{r}) = \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\phi} \\ r & 0 & 0 \\ F(r) & 0 & 0 \end{vmatrix} = 0$$

Since $\tau = dL/dt = 0$, angular momentum remains constant or is a conserved quantity.

3. Plane of motion remains constant for conservative central forces. Since angular momentum remains constant, the direction and magnitude of the vector must remain constant. Since the angular momentum vector is perpendicular to plane of motion, the plane also should remain same. To verify it mathematically, one may consider

$$r \cdot L = r \cdot (r \times p) = p \cdot (r \times r) = p \cdot 0 = 0$$
3.2 Central force Characteristics & Examples

Let us consider a particle moving under the influence of a central force. The central force is given by

\[ \vec{F} \propto \frac{Mm}{r^2} \]

and

\[ \vec{F} \propto Mm \]

The above equations are known as Newton’s laws of motion.

Also, the work done by the force is given by

\[ W = \int \vec{F} \cdot d\vec{s} \]

where \( d\vec{s} \) is the displacement vector.
3.2 Central force Characteristics & Examples

Also

\[ p \cdot L = p \cdot (r \times p) = r \cdot (p \times p) = r \cdot 0 = 0. \]

Here we have used the fact that in scalar triple product, vectors can be interchanged without affecting the end result. Thus one can see that both \( r \) and \( p \) are perpendicular to the vector \( L \). i.e.; both \( r \) and \( p \) remain in the same plane. i.e.; the position and velocity of the particle remain in the same plane. In other words, the entire movement of the particle confines to a plane.

4. For central forces, areal velocity (area swept by radial vector as a function of time) remains constant. Consider a particle is moving in a closed orbit. i.e.; influenced by a conservative central force. Then the small area covered by the line joining center to the particle, due to a small displacement \( dr \), is given by the area of the triangle OAB.

![Diagram of central forces]

\[ \text{Fig: Areal velocity of central forces} \]

\[ dA = \frac{1}{2} |r \times dr| = \frac{1}{2} |r \times vdt| \]

\[ \Rightarrow \frac{dA}{dt} = \frac{1}{2} |r \times v| = \frac{1}{2m} |r \times mv| = \frac{L}{2m} \]

Since \( L, m \) are constants in time, the change in area of the particle motion under central force is a constant in time. i.e.; the areal velocity remains constant.

Examples of central forces

1. Gravitational forces
2. Electrostatic forces
3. Elastic forces
3.2 Central force Characteristics & Examples

\[ F = \frac{GMm}{r^2} \]  \hspace{1cm} (3)

where \( G \) is the gravitational constant, \( m \) and \( M \) are the masses of the bodies and \( r \) is the distance.

\[ F = \frac{GMm}{r^2} \hat{r} \]  \hspace{1cm} (4)

2. **Examples**

Consider a system where bodies 1 and 2 attract each other with force \( F \). The force between them is given by

\[ F \propto q_1 q_2 \]  \hspace{1cm} (1)

\[ F \propto \frac{1}{r^2} \]  \hspace{1cm} (2)

From (1) and (2), we get

\[ F \propto \frac{q_1 q_2}{r^2} \]

or

\[ F = \frac{1}{4\pi \varepsilon} \frac{q_1 q_2}{r^2} \]  \hspace{1cm} (3)

where \( \varepsilon \) is the permittivity of free space.

3. **Inverse Square Law**

The force between two bodies is inversely proportional to the square of the distance between them.

\[ F \propto x \]  \hspace{1cm} (1)

\[ F = -kx \]  \hspace{1cm} (2)

where \( k \) is a constant and \( x \) is the displacement.

The force between two bodies is inversely proportional to the square of the distance between them.
3.2 Central force Characteristics & Examples

1. **Gravitational Forces**

The gravitational force between two objects of mass M and m is proportional to the product of their masses and inversely proportional to the square of the distance between them. If the two objects having mass M and m are separated by a distance $r$ then

\[ \vec{F} \propto Mm \]  \hspace{1cm} (1)

\[ \vec{F} \propto \frac{1}{r^2} \]  \hspace{1cm} (2)

From equations (1) and (2)

\[ \vec{F} \propto \frac{Mm}{r^2} \]

\[ \vec{F} = \frac{GMm}{r^2} \]  \hspace{1cm} (3)

Where $G$ is proportionality constant, also called the universal gravitational constant.

\[ \vec{F} = \frac{GMm}{r^2} \hat{r} \]  \hspace{1cm} (4)

2. **Electrostatic Forces**

The electrostatic force between two charges $q_1$ and $q_2$ is proportional to the product of their charges and inversely proportional to the square of the distance between them. If the two charges $q_1$ and $q_2$ are separated by a distance $r$ then

\[ \vec{F} \propto q_1 q_2 \]  \hspace{1cm} (1)

\[ \vec{F} \propto \frac{1}{r^2} \]  \hspace{1cm} (2)

From equations (1) and (2)
3.2 Central force Characteristics & Examples

Non-conservative central force

When the potential corresponding to central force is time dependent \((U = U(r, t))\), then, though \(\nabla \times F = 0\), and \(F = -\nabla U\), the force cannot be conservative. They can not produce closed orbits.

Eg: Dissipating electric charge. Though it obeys Coulomb’s law, the force dissipates with time. Another example is damped harmonic oscillator. Though it is a central force, the energy dissipates. Another example is mass exchanging binary stars. They exchange mass during their interaction. Thus the force between them changes as a function of time. Also conservation of energy fails in all these cases.

**Did You Know?**

In your 9\(^{th}\) class you might have come across the inverse square nature of central forces. This was derived from Kepler’s third law and central force equation: \(T^2 \propto r^3, F = \frac{mu^2}{r} \Rightarrow F \propto \frac{v^2}{r}\) and \(v = \frac{2\pi r}{T} \Rightarrow v^2 \propto \frac{r^2}{T^2}\). From all these, \(F \propto \frac{1}{r^2}\). Bertrand in 1873 proved that only inverse square forces and harmonic oscillator kind of forces result in closed orbits in planetary motion. Any deviation from it will lead to orbits which will never close.
3.2 Central force Characteristics & Examples

\[ \vec{F} \propto \frac{q_1 q_2}{r^2} \]

\[ \vec{F} = \frac{1}{4\pi \varepsilon} \frac{q_1 q_2}{r^2} \]  

(3)

Where \( \frac{1}{4\pi \varepsilon} \) is proportionality constant and \( \varepsilon \) is permittivity of the medium.

\[ F = \frac{1}{4\pi \varepsilon} \frac{q_1 q_2}{r^2} \hat{r} \]

(4)

3. Elastic Forces

The elastic force acting on the mass attached to spring is proportional to the distance of the mass from equilibrium position of the spring.

\[ \vec{F} \propto x \]  

(1)

\[ \vec{F} = -kx \]  

(2)

where \( k \) is force constant or spring constant.

**Think …**

If force between magnetic dipoles also exhibit inverse square law, why not force due to magnetic field a central force?

**Did You Know?**

Magnetic field is a pseudo vector. This exists only when the particle is in motion.

Force due to magnetic field can’t be represented as a scalar potential but as a vector potential.

Though magnetic poles are considered as point particles, the root cause of magnetic poles is the circular motion of charged particles in the plane perpendicular to the field direction. Hence it can’t be a central force.

**E - Corner**

https://en.wikipedia.org/wiki/Bertrand%27s_theorem
3.3 Rotational Motion about the Axes

3.3 Rotational Motion about the Axes

Let us consider the motion of a particle of mass \(m\) moving with velocity \(\mathbf{v}\) and acceleration \(\mathbf{a}\) in three-dimensional space. Consider a point \(O\) fixed in the reference frame attached to the particle.

The motion of the particle in an inertial frame is given by

\[
\begin{pmatrix}
\dot{r} \\
\dot{\theta}
\end{pmatrix} = 
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\dot{i} \\
\dot{j}
\end{pmatrix} - \cdots (1)
\]

Rotational Transformation for Axes

If the rotation is clockwise, then

\[
\begin{pmatrix}
\dot{r}' \\
\dot{\theta}'
\end{pmatrix} = 
\begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\dot{x} \\
\dot{y}
\end{pmatrix} - \cdots (2)
\]

where \(\dot{r}' = \frac{dx'}{dt}\) and \(\dot{\theta}' = \frac{dy'}{dt}\).

\[
\begin{pmatrix}
\dot{x}' \\
\dot{y}'
\end{pmatrix}
= 
\begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\dot{x} \\
\dot{y}
\end{pmatrix} - \cdots (3)
\]

The acceleration of the particle can be obtained by

\[
\ddot{\mathbf{r}} = \frac{d^2\mathbf{r}}{dt^2} = \dot{\mathbf{v}} - \mathbf{a}
\]

or

\[
\begin{pmatrix}
\ddot{x} \\
\ddot{y}
\end{pmatrix} = 
\begin{pmatrix}
\ddot{r}' \\
\ddot{\theta}'
\end{pmatrix} = 
\begin{pmatrix}
\ddot{r} \\
\ddot{\theta}
\end{pmatrix}
- \cdots (4)
\]

\[
\begin{pmatrix}
\ddot{x}' \\
\ddot{y}'
\end{pmatrix}
= 
\begin{pmatrix}
\ddot{r}' \\
\ddot{\theta}'
\end{pmatrix} = 
\begin{pmatrix}
\ddot{r} \\
\ddot{\theta}
\end{pmatrix}
- \cdots (5)
\]

The motion of a particle in a rotating frame can be obtained from the motion in an inertial frame.
3.3 Equation of motion under a central force

Consider rotation of a particle of mass $m$ about a point $O$ located at a radial distance of $\vec{r}$. Let $\theta$ be the angle of rotation in anti-clockwise direction.

Then one can define a new set of coordinates $(r, \theta)$ which can be related to the old axes $(x, y)$ as

$$
\begin{pmatrix} \hat{r} \\ \hat{\theta} \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \hat{i} \\ \hat{j} \end{pmatrix} - - - (1)
$$

The time derivative of these unit vectors in the new system is given by

$$
\dot{\hat{r}} = \frac{d\hat{r}}{dt} = \dot{\theta}(-\sin \theta \hat{i} + \cos \theta \hat{j}) = \dot{\theta} \hat{\theta} - - - (2)
$$

$$
\dot{\hat{\theta}} = -\dot{\theta} (\cos \theta \hat{i} + \sin \theta \hat{j}) = -\dot{\theta} \hat{r} - - - (3)
$$

Then the velocity of the particle is given by

$$
\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt} (r \hat{r}) = \dot{r} \hat{r} + r \dot{\hat{r}} = \dot{r} \hat{r} + r \dot{\theta} \hat{\theta} - - - (4)
$$

Here the first term is the radial component and the second term is the angular component of velocity.

Acceleration is given by

$$
\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt} (r \dot{\hat{r}} + r \dot{\theta} \hat{\theta}) = \ddot{r} \hat{r} + \dot{r} \dot{\hat{r}} + r \ddot{\theta} \hat{\theta} + r \dot{\theta} \dot{\hat{\theta}} - - -
$$
3.3 Equation of motion under a central force

The equation of motion, considering a central force, is given as:

\[ r\ddot{\theta} + 2i\dot{\theta} = 0 \Rightarrow \frac{1}{r} \left( \frac{d}{dt} \left( r^2 \dot{\theta} \right) \right) = 0 \Rightarrow r^2 \dot{\theta} = h - - - (6) \]

This is a central force case. Assuming moment of inertia, the equation becomes:

\[ \frac{dA}{dt} = \frac{1}{2} r d\theta \cdot r = \frac{1}{2} r^2 d\theta \Rightarrow \frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{h}{2} - - - (7) \]

Solving Eq.(6), we get the central force case of motion.

The force under this case is given by:

\[ \frac{F(r)}{m} = f(r), - - - (8) \]

Using this, we consider the central force case as:

\[ \ddot{r} - r\dot{\theta}^2 = f(r) - - - (9) \]

We can then consider the central force case, where the motion is governed by the central force. Assuming the central force is given by:

\[ \ddot{r} = \frac{dr}{dt} = \frac{d}{dt} \left( \frac{1}{u} \right) = -\frac{1}{u^2} \frac{du}{dt} = -r^2 \frac{d}{dt} \left( \frac{1}{r} \right) = -r^2 \frac{d}{dt} \left( \frac{1}{r} \right) \frac{d\theta}{dt} \]

\[ \Rightarrow \ddot{r} = -h \frac{du}{d\theta} \]

\[ \ddot{r} = -h \frac{d}{dt} \left( \frac{du}{d\theta} \right) = -h \frac{d}{d\theta} \left( \frac{du}{d\theta} \right) \frac{d\theta}{dt} = -h \frac{d^2 u}{d\theta^2} \frac{h}{r^2} \]

\[ \Rightarrow \ddot{r} = -\frac{h^2 d^2 u}{r^2 d\theta^2} - - - (11) \]
3.3 Equation of motion under a central force

or

\[ \ddot{a} = \dddot{r} + \dot{r} \dot{\theta} + \ddot{r} \dot{\theta} + r \dddot{\theta} \dot{\theta} - r \dot{\theta}^2 \dot{r} \]

\[ = (\dddot{r} - r \dddot{\theta}) \dot{r} + (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \ddot{\theta} \]

\[ = (\dddot{r} - r \dddot{\theta}) \dot{r} + (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \ddot{\theta} - - - (5) \]

In the above, the first term is the radial acceleration and the second term is the angular acceleration.

For central forces, only radial acceleration exists and angular acceleration becomes zero.

i.e.;

\[ r \ddot{\theta} + 2 \dot{r} \dot{\theta} = 0 \Rightarrow \frac{1}{r} \left( \frac{d}{dt} (r^2 \ddot{\theta}) \right) = 0 \Rightarrow r^2 \ddot{\theta} = h - - - (6) \]

here \( h \) is assumed to be a constant. The effect of constant in Eq. (6) can be understood by considering areal velocity in rotatory motion.

For that, consider the following in the figure given below:

\[ dA = \frac{1}{2} dr. r = \frac{1}{2} r^2 d\theta \Rightarrow \frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = h \]

\[ \Rightarrow \frac{dA}{dt} = h - - - (7) \]

Thus the areal velocity remains constant in a central force field.

**Fig: Areal velocity**

If the force per unit mass (acceleration) is defined as

\[ \frac{F(r)}{m} = f(r), - - - (8) \]

then the radial acceleration alone becomes equal to,

\[ \dddot{r} - r \dddot{\theta} \dot{\theta} = f(r) - - - (9) \]

It is more convenient to work with inverse distances, as most potentials are inversely dependent on distance. Also instead of time
3.3 Equation of motion under a central force

Eq.(9), Eq.(11) మాత్రమే Eq.(6) కొనించండి

\[-\frac{h^2}{r^2} \frac{d^2u}{d\theta^2} - \frac{h^2}{r^4} = f(r)\]

తడి Eq.(10) కొనించండి  మరియు మూడు కాలాపానుల మొత్తం మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ.

\[\frac{d^2u}{d\theta^2} + u = -\frac{f(r)}{h^2u^2} \quad (12)\]

తడి మరియు మూడు కాలాపానుల మొత్తం మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ.

\[\frac{d^2y}{d\theta^2} + y = 0 \quad (13)\]

తడి మరియు మూడు కాలాపానుల మొత్తం మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ మండ.

**E - Corner**

https://books.google.com/books/download/An_Elementary_Treatise_on_Theoretical_Me.pdf?id=8FHvAAAAMAAJ&output=pdf


3.3 Equation of motion under a central force

derivatives, it is more effective to study the system in terms of derivatives with respect to angle. Let us define

\[ u = \frac{1}{r} \quad \text{(10)} \]

Hence using Eq. (6) and using the fact that \( h \) is a constant gives

\[
\begin{align*}
\dot{r} &= \frac{dr}{dt} = \frac{d}{dt} \left( \frac{1}{u} \right) = -\frac{1}{u^2} \frac{du}{dt} = -r^2 \frac{d}{dt} \left( \frac{1}{r} \right) = -r^2 \frac{d}{d\theta} \left( \frac{1}{r} \right) \frac{d\theta}{dt} \\
\Rightarrow \dot{r} &= -h \frac{du}{d\theta}
\end{align*}
\]

\[
\begin{align*}
\ddot{r} &= -h \frac{d}{dt} \left( \frac{du}{d\theta} \right) = -h \frac{d}{d\theta} \left( \frac{du}{d\theta} \right) \frac{d\theta}{dt} = -h \frac{d^2u}{d\theta^2} \frac{h}{r^2} \\
\Rightarrow \ddot{r} &= -\frac{h^2 \frac{d^2u}{d\theta^2}}{r^2} \quad \text{(11)}
\end{align*}
\]

Using Eq. (9), Eq. (11) and Eq. (6) one can get,

\[
-\frac{h^2 \frac{d^2u}{d\theta^2}}{r^2} \frac{h^2}{r^4} = f(r)
\]

or by using Eq. (10), and dividing with \( h^2 u^2 \) on both sides gives,

\[
\frac{d^2u}{d\theta^2} + u = -\frac{f(r)}{h^2 u^2} \quad \text{(12)}
\]

This is the equation of motion in central force field.

If we define \( y = u + f(r)/h^2 u^2 \), then the above equation becomes,

\[
\frac{d^2y}{d\theta^2} + y = 0 \quad \text{(13)}
\]

This is the equation of simple harmonic motion. The solution of it represents a periodic motion without loss of energy.
3.4 ఎంపిక రేటు చిత్రపటం వివిధానం:

పరికర పరిమితిలో 1000మిఛ, అంటే బెరుగు BC నుండి 1400AD వరకు ఇతర పరిమితిలో రెండు శతాబ్దాలు ఉన్నాయి. మొదటి 1400AD నాట్యం నాట పరిమితిలో హీముడా చిత్రపటం ఉద్యోగాలు కలుపుతున్నాయి. పురాణాల్లో, రామాయణానికి సమానంగా అనుకుని తాత్కాలిక పరిమితిలో మధ్యాయం ఉన్నాయి. యోగాదాస్ మొదటి ఆమోదం, అమ (ఎ.ఎ. 1546-1601) లోని రామాయణ రెండు శతాబ్దాలు చిత్రపటం ఉన్నాయి. పితా గారు సముదాయం సారిచే పరిమితిలో మధ్యాయం ఉన్నాయి.
3.4 Kepler laws of planetary motion

Heliocentric models were wide in use in ancient India as far back as 1000BC, also in Babylonia during 6th century BC, in Greece during 5th century BC. Then in 1400AD, Ptolemaic model has enforced the geocentric theory. Consequently, Copernicus strongly proposed the Heliocentric model further which explains the structure of solar system in a more simplified manner than the then existing geocentric models. Based on Copernicus ideas, Tycho Brahe (1546-1601 AD) and his descendent Kepler (1600 AD) have made enormous observations of solar system and developed empirical models that explain the structure of the solar system. The three Kepler laws are stated as follows.

1. Law of Orbits: Every planet revolves around the sun in an elliptical orbit, with sun at one of its foci.
2. Law of Areas: The line joining the planet to the center of the sun sweeps equal areas in equal intervals of time.
3. Law of Harmony: The square of time period of planet around the sun is proportional to the cube of semi major axis length. 

\[ T^2 \propto a^3 \]

Kepler’s First Law:
Consider a planet of mass \( m \) revolving around the sun of mass \( M \) in an orbit of radius \( r \). Then the force between them is given by Newton’s law of gravity as

\[ F(r) = -\frac{GMm}{r^2} = -\frac{\mu m}{r^2} \tag{1} \]

where \( \mu = GM \). The equation of motion of the planet is given by

\[ \frac{d^2u}{d\theta^2} + u = -\frac{f(r)}{u^2 h^2} = -\frac{F(r)}{mu^2 h^2} \tag{2} \]

From Eq. (1) and Eq. (2),

\[ \frac{d^2u}{d\theta^2} + u = +\frac{\mu}{h^2} \tag{3} \]

Define
3.4 Kepler laws of planetary motion

\[ y = A \cos(\theta + \psi) \quad - - - (6) \]

where \( \psi \) = \( \varphi \) = \( \varphi \) (Equation (4)). Eq. (4) and Eq. (6) are:

\[ u = \frac{1}{r} = \frac{\mu}{h^2} + A \cos(\theta + \psi) = \frac{\mu}{h^2} (1 + e \cos(\theta + \psi)) \quad - - - (7) \]

where \( e = \frac{A h^2}{\mu} \). \( r \) varies \( \theta = 0^\circ \) to \( \theta = \pm 90^\circ \) to \( \theta = 180^\circ \). Thus:

\[ r = \frac{h^2/\mu}{(1 + e \cos \theta)} \quad - - - (8) \]

Then \( e = \frac{OC}{BC} = \frac{OP}{AP} = \frac{OP}{BC - PQ} = \frac{r}{p - r \cos \theta} \)

Thus:

\[ p - er \cos \theta = r \quad \Rightarrow r(1 + e \cos \theta) = p \]

\[ \Rightarrow r = \frac{p}{(1 + e \cos \theta)} \quad - - - (9) \]

Eq. (8) and Eq. (9) are:

\[ p = \frac{h^2}{\mu} \quad - - - (10) \]

When \( p \) varies \( \theta \) varies \( \pm 90^\circ \) to \( \pm 180^\circ \). Thus:

\[ r = r_{\text{min}} \text{ when } \cos \theta = +1 \text{ for } \theta = 0^\circ \text{ and } \cos \theta = -1 \text{ for } \theta = 180^\circ \text{ or } r = r_{\text{max}}. \]
3.4 Kepler laws of planetary motion

\[ y = u - \frac{\mu}{h^2} \quad (4) \]

Then

\[ \frac{d^2y}{d\theta^2} + y = 0 \quad (5) \]

The general solution of this equation is given by

\[ y = A \cos(\theta + \psi) \quad (6) \]

Here \( A, \psi \) are integration constants. From Eq. (4) and Eq. (6),

\[ u = \frac{1}{r} = \frac{\mu}{h^2} + A \cos(\theta + \psi) = \frac{\mu}{h^2} (1 + e \cos(\theta + \psi)) \quad (7) \]

Here \( e = Ah^2/\mu \). For minimum value of \( r \) to occur at \( \theta = 0^\circ \), \( \psi \) must be equal to zero. Then

\[ r = \frac{h^2/\mu}{(1 + e \cos \theta)} \quad (8) \]

which describes a conic section, where conic sections are two dimensional (planar) geometric structures with a fixed line (called directrix) and a fixed point (called focus). In other words, a conic section is a locus of points with fixed eccentricity, where eccentricity is the ratio of distance of conic from focus to that from the directrix.

Fig: Conic sections

From the above diagram and by using the definition of eccentricity,
3.4 Kepler laws of planetary motion

\[ r_{\text{min}} = \frac{p}{1+e}, \quad \text{(where } \text{eccentricity) } \quad (11) \]

\[ r_{\text{max}} = \frac{p}{1-e}, \quad \text{(where } \text{eccentricity) } \quad (12) \]

\[ a = \frac{r_{\text{max}} + r_{\text{min}}}{2} = \frac{p}{2} \left( \frac{1}{1+e} + \frac{1}{1-e} \right) = \frac{p}{1-e^2} \quad (13) \]

Eq. (11), Eq. (12) and Eq. (13) yield:

\[ r_{\text{min}} = a(1-e) \quad \text{and} \quad r_{\text{max}} = a(1+e) \]

The radial force is given by:

\[ F(r) = -\frac{GMm}{r^2} = \frac{\mu m}{r^2} \Rightarrow U(r) = -\int Fdr = \int \frac{\mu m}{r^2} \, dr = -\frac{\mu m}{r} \]

Then:

\[ \frac{1}{2} mh^2 u^2 - \mu mu - E = 0 \]

By resolving the momentum, we obtain:

\[ u = \frac{1}{r} = \frac{\mu m \pm \sqrt{\mu^2 m^2 + 4E\frac{mh^2}{2}}}{2. mh^2 /2} \]

\[ = \frac{\mu}{h^2} \left( 1 \pm \sqrt{1 + \frac{2Emh^2}{\mu^2 m^2}} \right) \quad (14) \]

Eq. (14) is, given the conditions that \( \mu m > E \), the radial force at infinity is \( \mu m \). By Eq. (7),

\[ \frac{1}{r_{\text{min}}} = \frac{\mu}{h^2} (1+e) = \frac{\mu}{h^2} \left( 1 + \sqrt{1 + \frac{2Emh^2}{\mu^2 m^2}} \right) \]
3.4 Kepler laws of planetary motion

\[ e = \frac{OC}{BC} = \frac{OP}{AP} = \frac{OP}{BC - PQ} = \frac{r}{\frac{p}{e} - r \cos \theta} \]

or

\[ p - er \cos \theta = r \Rightarrow r(1 + e \cos \theta) = p \]

\[ \Rightarrow r = \frac{p}{1 + e \cos \theta} \quad (9) \]

Comparing Eq. (8) and Eq. (9),

\[ p = \frac{h^2}{\mu} \quad (10) \]

Here \( p \) is called the latus rectum of the conic which is equal to radius when \( \theta = \pm 90^\circ \). Since \( \cos \theta \) occurs in the denominator, and \( e \) is positive, \( r = r_{\text{min}} \) when \( \cos \theta = +1 \) i.e.; when \( \theta = 0^\circ \). Also \( r = r_{\text{max}} \) when \( \cos \theta = -1 \) i.e.; when \( \theta = 180^\circ \). Here

\[ r_{\text{min}} = \frac{p}{1 + e} \quad (\text{Perihilion}) \quad (11) \]

\[ r_{\text{max}} = \frac{p}{1 - e} \quad (\text{Aphelion}) \quad (12) \]

**Fig: Elliptical Orbit**

Then the semi major axis is given by

\[ a = \frac{r_{\text{max}} + r_{\text{min}}}{2} = \frac{p}{2} \left( \frac{1}{1 + e} + \frac{1}{1 - e} \right) = \frac{p}{1 - e^2} \quad (13) \]

From Eq. (11), Eq.(12) and Eq.(13),

\[ r_{\text{min}} = a(1 - e) \quad \text{and} \quad r_{\text{max}} = a(1 + e) \]
3.4 Kepler laws of planetary motion

\[ e = \sqrt{1 + \frac{2Eh^2}{\mu^2m}} \quad (15) \]

Here \( E > 0 \), \( e > 1 \), \( \text{elliptical} \)

\( E = 0 \), \( e = 1 \), \( \text{circular} \)

\( E < 0 \), \( e < 1 \), \( \text{parabolic} \)

\( E = -\frac{\mu^2m}{2h^2} \), \( e = 0 \), \( \text{parabolic} \)

The center of mass of a two object system, lies on the line joining the two particles. If one object is extremely heavier, then the center of mass lies within the heavier object. In such case only, in planetary motion, one object will be revolving around the other object. In all other cases both the objects will be revolving around their center of mass. The center of mass in that case is called Barycentre. Since the centre of mass of Sun-Jupiter system lies outside the sun, sun also revolves around that barycentre. This makes the orbits of planets in solar system elliptical, though circular orbit is the most symmetrical orbit. [https://spaceplace.nasa.gov/barycenter/en/](https://spaceplace.nasa.gov/barycenter/en/)
To obtain the total energy, one needs to obtain potential energy from force equation as

$$F(r) = -\frac{GMm}{r^2} = \frac{\mu m}{r^2} \Rightarrow U(r) = -\int F dr = \int \frac{\mu m}{r^2} dr = -\frac{\mu m}{r}$$

Then by using $L/m = h$, total energy is given as,

$$E = \frac{J^2}{2I} + U(r) = \frac{L^2}{2mr^2} - \frac{\mu m}{r} = \frac{1}{2} \frac{mh^2}{r^2} - \frac{\mu m}{r} = \frac{1}{2} mh^2 u^2 - \mu u$$

Or

$$\frac{1}{2} rh^2 u^2 - \mu u - E = 0$$

Solving the above equation for $u$, gives

$$u = \frac{1}{r} = \frac{\mu m \pm \sqrt{\mu^2 m^2 + 4E \frac{m h^2}{2}}}{2 \cdot \mu h^2 / 2}$$

$$= \frac{\mu}{h^2} \left( 1 \pm \sqrt{1 + \frac{2E m h^2}{\mu^2 m^2}} \right) \quad (14)$$

In Eq. (14), $r$ value will be minimum when plus sign is considered and in Eq.(7), $r$ value will be minimum when cosine term is maximum. Thus from Eq. (7) and Eq. (14),

$$\frac{1}{r_{\text{max}}} \Rightarrow \frac{\mu}{h^2} (1 + e) = \frac{\mu}{h^2} \left( 1 + \sqrt{1 + \frac{2E h^2}{\mu^2 m^2}} \right)$$

Or

$$e = \sqrt{1 + \frac{2E h^2}{\mu^2 m}} \quad (15)$$

Thus

If $E > 0$, then $e > 1$ then the path is Hyperbola

If $E = 0$, then $e = 1$, then the path is Parabola

If $E < 0$, then $e < 1$, then the path is an ellipse.
Observe the change in shape of orbit with change in mass of planet and its gravity. Also observe the epicentre based movement of moon around the sun.

https://phet.colorado.edu/en/simulations/gravity-and-orbits
If \( E = -\mu^2 m / 2h^2 \) then \( e = 0 \), then the path is a circle.

Thus the energy of the planet, or alternately the eccentricity of the planet decides the path of the planet around the sun.

**Law of areas**

Consider \( S \) be the location of the sun and \( P \) be the initial position of the planet with radial vector \( \vec{r} \). Let the radial vector sweep an angle of \( \Delta \theta \), in time \( \Delta t \) and let the new radial vector be \( \vec{r} + \Delta \vec{r} \).

Then the area swept by the radial vector is given by

\[
\Delta A = \frac{1}{2} \text{ base } \times \text{ height } = \frac{1}{2} \Delta \vec{r} \times \vec{r} = \frac{1}{2} \vec{r} \Delta \theta \times \vec{r} = \frac{1}{2} r^2 \Delta \theta
\]

The areal velocity is obtained as

\[
\frac{dA}{dt} = \lim_{\Delta \rightarrow 0} \frac{\Delta A}{\Delta t} = \lim_{\Delta \rightarrow 0} \frac{1}{2} r^2 \frac{\Delta \theta}{\Delta t} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} r^2 \omega = \frac{L}{2m} = \frac{h}{2}
\]

But for central force problems, \( h \) is a constant. Thus the areal velocity of the planet remains constant.

**Law of Harmony**

Consider a planet is revolving around its star in an elliptical orbit. The area of the ellipse is given by

\[
A = \pi ab
\]

where \( a \) is the length of the semi major axis and \( b \) is the length of the semi minor axis.

Areal velocity of the planet is given by Kepler’s second law asx
3.4 Kepler laws of planetary motion

**Did You Know?**

February month has 28/29 days because the earth revolves around the sun in an elliptical orbit. On September 21st and on March 21st Equinoxes occur. The gap from March 21st to September 21st is 184 days and that from September 21st to March 21st is it 181 days. This is because during January earth will be nearest to the sun (Perihelion) and during July earth will be farthest to the sun (Aphelion). From Kepler’s second law, \( L = mvr = \text{const.} \) Since \( m \) is constant, \( v \propto \frac{1}{r}. \) i.e.; at perihelion, earth has its fastest speed and at aphelion, earth has its slowest speed around the sun. Thus from September to March, earth runs faster and from March to September earth runs slower. The difference is 3 days. That was most probably deducted from February as January is the first month.


\[ A = \pi ab \quad (1) \]

\[ \frac{dA}{dt} = \frac{h}{2} \quad (2) \]

\[ T = \frac{A}{dA/dt} = \frac{\pi ab}{h/2} = \frac{2\pi ab}{h} \quad (3) \]
3.4 Kepler laws of planetary motion

\[ \frac{dA}{dt} = \frac{h}{2} \quad (2) \]

Thus the time period of the planet is given by

\[ T = \frac{A}{\frac{dA}{dt}} = \frac{\pi ab}{h/2} = \frac{2\pi ab}{h} \quad (3) \]

For an ellipse, the semi-latus rectum is given by

\[ p = \frac{b^2}{a} \implies b = \sqrt{ap} \quad (4) \]

From Kepler’s first law,

\[ p = \frac{h^2}{\mu} \quad (5) \]

Substituting Eq. (4) and Eq. (5) into Eq. (3) gives

\[ T = \frac{2\pi a \sqrt{ap}}{h} = \frac{2\pi a \sqrt{ah^2/\mu}}{h} = \frac{2\pi a \sqrt{a}}{\sqrt{\mu}} = 2\pi \sqrt{\frac{a^3}{GM}} \quad (6) \]

From the above equation,

\[ T^2 \propto a^3 \]

Thus, square of time period of planet varies as the cube of the semi-major axis of the orbit.

**Alternative method**

Consider a planet of mass \( m \) is revolving around the sun of mass \( M \) in a circular orbit of radius \( r \), with velocity \( v \). Then the gravitational pull is balanced by the centrifugal force, which can be expressed as

\[ \frac{GMm}{r^2} = \frac{mv^2}{r} \implies v = \sqrt{\frac{GM}{r}} \quad (1) \]

The time period of the planet, given by using the above equation, as

\[ T = \frac{\text{distance}}{\text{velocity}} = \frac{2\pi r}{\sqrt{GM/r}} = 2\pi \sqrt{\frac{r^3}{GM}} \quad (2) \]

From the above equation,
3.4 Kepler laws of planetary motion

Kepler's laws of planetary motion are:

\[ p = \frac{b^2}{a} \Rightarrow b = \sqrt{ap} \quad (4) \]

Where \( p \) is the semimajor axis.

Then, the period \( T \) is given by:

\[ p = \frac{h^2}{\mu} \quad (5) \]

From Eq. (4) and Eq. (5), we can derive Eq. (3):

\[ T = \frac{2\pi a \sqrt{ap}}{h} = \frac{2\pi a \sqrt{ah^2/\mu}}{h} = \frac{2\pi a \sqrt{a}}{\sqrt{\mu}} = 2\pi \sqrt{\frac{a^3}{GM}} \quad (6) \]

The period is proportional to the cube of the semimajor axis,

\[ T^2 \propto a^3 \]

From these equations, we can understand the relationship between the period, distance, and velocity of a planet.

The expression for the orbital period is:

\[ T = \frac{\text{distance}}{\text{velocity}} = \frac{2\pi r}{\sqrt{GM/r}} = 2\pi \sqrt{\frac{r^3}{GM}} \quad (2) \]

The period is proportional to the cube of the semimajor axis,

\[ T^2 \propto a^3 \]
Thus square of time period of planet varies as the cube of the semi major axis of the orbit.

Modern data (Wolfram Alpha Knowledgebase 2018)

<table>
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<tr>
<th>Planet</th>
<th>Semi-major axis (AU)</th>
<th>Period (days)</th>
<th>$\frac{R^3}{T^2} \times 10^{-6}$ (AU³/day²)</th>
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<td>7.504</td>
</tr>
</tbody>
</table>

3.5 Motion of Satellites

A satellite is an artificial object that is placed to orbit around earth or moon or any other planet to gather information or for communication.

Consider a satellite of mass $m$ is revolving around the earth of mass $M$ in a circular orbit of radius $r$ with velocity $v$. Then the gravitational pull is balanced by the centrifugal force.

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM}{r}} \quad \text{(1)}$$

The time period of the satellite is given by using the above equation as,

$$T = \frac{\text{distance}}{\text{velocity}} = \frac{2\pi r}{\sqrt{GM/r}} = 2\pi \sqrt{\frac{r^3}{GM}} \quad \text{(2)}$$
3.5 ఉపగల చలనము

ఉపగల చలనము, అంశాంత విద్యుత్ నీటిని రెండు నిర్ధిష్టమైన ప్రాంతాలు వైపు చలన చేయు చనిపోయేది. ఉపగల నిర్ధిష్టమైన ప్రాంతాలు ఉపగల చలనము ఉపయోగించారు.

\[
\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt[\frac{GM}{r}} - - - (1)
\]

ఈ సాధనాన్ని ఉపయోగించగా ముడి ఉపగల చలనము ఉన్నాడు. ఎందుకంటే, (1) సాధనాన్ని,

\[
T = \frac{distance}{velocity} = \frac{2\pi r}{\sqrt{GM}} = \frac{2\pi}{\sqrt{GM}} - - - (2)
\]

అందువల్ల, \( r = R + h \) కూడా ఉండతే మొట్టమైన ప్రశ్నలో ఉపగల చలనము ఉంది. \( R \) మధ్యే మార్గానికే రెండు ప్రాంతాలు ఉన్నాం. డిసిషన్ మరియు సంకోచానికే ఉపగల చలనము సమీకరణాన్ని ఉపయోగించారు. 

\[
\frac{GMm}{r^2} = mg - - - (3)
\]

Eq. (3), Eq. (1) కూడా ఉపయోగించారు.

\[
v = \sqrt[\frac{GM}{r}} = \sqrt[\frac{gr^2}{r}} = \sqrt{gr} = \sqrt{g(R + h)} - - - (4)
\]

\[
T = 2\pi \sqrt{\frac{r^2}{GM}} = \frac{\sqrt{r^3}}{\sqrt{g}} = \sqrt{\frac{r}{g}} = \sqrt{\frac{R + h}{g}} - - - (5)
\]

ఉపగల చలనము ఉపయోగించడానికి

ఉపగల చలనము ఉపయోగించడానికి అంశాంత విద్యుత్నీటిని రెండు నిర్ధిష్టమైన ప్రాంతాలు వైపు చలన చేయు చేస్తుంది. ఉపగల చలనము ఉపయోగించడానికి మూలం ఉంటుంది, కానీ ఉపగల చలనము ఉపయోగించడానికి మార్గానికే ఉపగల చలనము ఉపయోగించారు. భావించడానికి ఉపగల చలనము ఉపయోగించడానికి మూలం ఉంటుంది.
Here \( r = R + h \) is the distance of the satellite from the centre of mass of the earth, \( R \) is the radius of the earth and \( h \) is the height of the satellite from the surface of the earth.

Here \( G \) and \( M \) can be replaced by the acceleration due to gravity of earth, by using,

\[
\frac{GMm}{r^2} = mg
\]  

Using Eq. (3), Eq. (1) and Eq. (2) becomes,

\[
v = \sqrt{\frac{GM}{r}} = \frac{\sqrt{gr^2}}{r} = \frac{\sqrt{gr}}{r} = \sqrt{g(R + h)}
\]

\[
T = 2\pi \sqrt{\frac{r^3}{GM}} = \sqrt{\frac{r^3}{gr^2}} = \sqrt{\frac{r}{g}} = \sqrt{\frac{R + h}{g}}
\]

**Escape velocity of the satellite**

Escape velocity is the minimum velocity where the gravitational attraction force is balanced by the kinetic energy of the object. If the velocity of the object exceeds escape velocity, then kinetic energy dominates the gravitational force and the object escapes the gravitational pull. Thus for escape velocity,

\[
\frac{1}{2}mv^2 = -\left(-\frac{GMm}{r}\right) \Rightarrow v = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2gr^2}{r}} = \sqrt{2gr}
\]

Thus the escape velocity of the satellite is \(\sqrt{2} \) times the orbital velocity of the satellite.

There are basically 3 levels of orbits that are used by satellites and other space orbiters. They are as described below along with other orbits

1. **LEO** that is Low Earth orbit. This ranges from 160KM to 2000KM. international space station, revolves at a height of 400KM. all earth observing satellites like remote sensing and spy satellites revolve in these orbits. Objects that orbit around 80-600KM decay in height due to the atmospheric drag in those
The text contains mathematical equations and explanations in Telugu. Here is the English translation:

### 3.5 Satellite Systems

\[
\frac{1}{2}mv^2 = -\left(-\frac{GMm}{r}\right) \Rightarrow v = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2gr^2}{r}} = \sqrt{2gr} \quad (6)
\]

1. For a satellite orbiting LEO, let the mass of the orbit be 160KM, and the altitude be 2000KM. The semi-major axis is then 1800KM, and the orbital period is 400KM. The circular motion of the satellite is an ellipse, and the orbital period is calculated from the equation. The eccentricity of the orbit is calculated. This is known as the semilatus rectum.

2. For a satellite orbiting MEO, let the mass of the orbit be 2000KM, and the altitude be 8000KM. The semi-major axis is then 9200KM, and the orbital period is calculated from the equation. The eccentricity of the orbit is calculated. This is known as the semilatus rectum.

3. For a satellite orbiting GEO, let the mass of the orbit be 35,786KM, and the altitude be 35,786KM. The semi-major axis is then 35,786KM, and the orbital period is calculated from the equation. The eccentricity of the orbit is calculated. This is known as the semilatus rectum.

4. The Earth's gravitational constant is \( G = 6.673 \times 10^{-11} \, \text{Nm}^2/\text{kg}^2 \), and the mass of the Earth is \( M = 5.98 \times 10^{24} \, \text{kg} \).

8. "The formula for the semilatus rectum is \( 4\pi R^3 = T^2 GM \), where \( V = \frac{2GM}{R} \), \( V = \sqrt{\frac{2GM}{R}} \), and \( R = GM/2 \)."
3.6 Global Positioning system (GPS)

GPS stands for global positioning system. These satellites revolve in medium earth orbit. i.e.; around 20,200KM from the surface of the earth. Each data point that is to be positioned by GPS system requires data from 4 satellites for accuracy. Here 3 satellites fix the position of the object and the 4th satellite fixes the accuracy of the time information. Thus GPS contains 24 satellites that are revolving in 6 orbits. They require additional fuel to maintain their height.

2. MEO that is medium earth orbit. This ranges between 2000KM to 35,786KM from mean sea level. All global positioning satellites and navigation satellites revolve in these orbits. These are highly affected by the solar radiation pressure.

3. GEO that is the geo stationary orbit. This is at a height of 35,786KM. All communication satellites revolve at this level. This is because at this level the time period is exactly 24hrs. They are positioned around equator only. This causes low signal strength near to poles regions.

4. Graveyard orbit. This is a few hundreds of kilometres above the geostationary orbit. This is used to dump the decommissioned communication satellites.

5. Molnia orbit is the communication satellite orbit used by the satellite named Molnia to communicate nearer to north pole region.

6. Polar orbits are used for scientific data collection at poles, for remote sensing applications and for communication near poles region.

7. HEO that is the high earth orbit. Any orbit above the geo synchronous orbit of height 35,786KM are called high earth orbits.

8. “Orbital periods and speeds are calculated using the relations $4\pi^2R^3 = T^2GM$ and $V^2R = GM$, where $R$, radius of orbit in metres; $T$, orbital period in seconds; $V$, orbital speed in m/s; $G$, gravitational constant, approximately $6.673\times10^{-11}$ Nm$^2$/kg$^2$; $M$, mass of Earth, approximately $5.98\times10^{24}$ kg.”
3.6 Global Positioning system (GPS)

GPS అనేది గ్లోబల్ పొసింగ్ సిస్టమ్ (GPS) అని పంచేది. ఈ సిస్టమ్ నియంత్రణ తనిఖీ, ప్రయోగాలు సాధకాలు పైగా అందానికి. ఈ సిస్టమ్ నుండి 20,200Km. లో స్పష్టమైన విశ్వవ్యాప్తంగా భాగం ప్రదానం చేసేది. ఈ సిస్టమ్ నుండి ప్రతి రింటు 10 సంవత్సరాల అంటది. ఈ సిస్టమ్ నుండి యొక్క మొట్టమైన 4 సంవత్సరాల మధ్య లేని సమయం ప్రదానం చేయబడింది. ఈ సిస్టమ్ ప్రతి రింటు బాగా ప్రదానం చేసేది. విస్తరించబడిన సమయం ప్రకారం 24 సంవత్సరాల మధ్య లేని సమయం. ఈ సమయం ప్రారంభమైన తరువాత కరకు పైకి 4 GPS సిస్టమ్ ప్రదానం చేయబడింది.

24 సంవత్సరాల ముందు స్వల్పంగా అది బాగా ప్రదానం చేయబడింది, లేదా మాత్రమే కొత్త సంవత్సరాల ముందు శాస్త్ర సంస్థలకు ప్రదానం చేయబడింది. ఈ సమయంలో ప్రతి రింటు కొత్త 12 సంవత్సరాల ప్రోత్సాహం ప్రతి రింటు ఆధారం ప్రదానం చేయబడింది. ఈ సమయంలో ప్రతి రింటు కొత్త 12 సంవత్సరాల ప్రోత్సాహం ప్రదానం చేయబడింది. LEO సిస్టమ్ ప్రదానం కెంద్రీకరించబడింది ప్రతి రింటు ప్రదానం చేయబడింది.

అంతే కాబట్టి ఇవి వివిధ ప్రాముఖ్యతలను ఉంచది. a. II. ప్రశ్నలు ప్రదానం చేయబడింది. iii. III. ప్రశ్నలు ప్రదానం చేయబడింది.

పాఠానిక వివరణ: వ్యాఖ్యల ప్రకారం ప్రత్యేకించబడింది, అందువలసినంత ప్రత్యేకించబడింది సంహ్రా. L సంఖ్య (1-2GHz) సంస్థానాలు విభాగం ప్రాంతాలుగా ప్రదానం చేయబడింది. S సంఖ్య (2-4GHz) పండుగు ప్రత్యేకించబడింది ఉపయోగించబడింది. ఇది గెలిచు సంఖ్య (4-8GHz) లోను పండుగు ప్రదానం చేయబడింది. ఇది ప్రత్యేకించబడింది సంపన్న సంఖ్య (4-8GHz) లో పండుగు ప్రదానం చేయబడింది. ఈ సంఖ్య ప్రదానం చేయబడి నియంత్రణ సమయం ప్రదానం చేయబడింది. ఇది ప్రత్యేకించబడింది సంస్థానాలు ప్రదానం చేయబడింది. ఇది ప్రత్యేకించబడింది సంస్థానాలు ప్రదానం చేయబడింది. ఇది ప్రత్యేకించబడింది సంస్థానాలు ప్రదానం చేయబడింది. ఇది ప్రత్యేకించబడింది సంస్థానాలు ప్రదానం చేయబడింది. ఇది ప్రత్యేకించబడింది సంస్థానాలు ప్రదానం చేయబడింది. ఇది ప్రత్యేకించబడింది సంస్థానాలు ప్రదానం చేయబడింది.
different equatorial planes around the earth. Every point on earth is seen by at least 4 GPS satellites at a time.

Instead of geo stationary orbits with time period of 24 hours, GPS satellites are placed at 12 hour orbits, to avoid security issues from other communication satellites at that level. If low earth orbits are chosen then there will be more fuel consumption to overcome air drag effects. LEO orbits also reduce the coverage range of satellites.

There are three segments for this satellite system. i. Space segment, ii. Control segment and iii. User segment.

**Space segment:** It consists of at least 24 and on average 31 operational satellites. Their count is adjusted based on their health conditions. The most important component of space segment is the atomic clock. More the gravity, slower the clock ticks. Thus the clocks in GPS satellites tick $38 \mu$Sec faster for every 24 hours. This results in a 10KM error in the accuracy of the estimated position. Hence the atomic clocks must be time synchronized to less than micro second accuracy using atomic clocks.

**Control segment:** This has 4 sub parts namely a. Master control station (MCS) b. Alternative master control station c. Ground antennas and d. Monitor stations.

The ground antennas receive and transmit signals from and to the satellite antennas. L band (1-2GHz) is used for data transmission and S band (2-4GHz) is used for maintenance signal purposes. Dish TV satellites use C band (4-8GHz) and they are placed in the geostationary orbits.

Monitor stations collect data from satellites while they pass over head. The collected data will be transmitted to the master control station. Alternative master control station acts as a back up of master control station during its maintenance. It also monitors the health of the satellite constellation. Uploads navigation messages to satellites.

**User segment:** They carry GPS tracker devices or GPS receivers of L-band signals. There are two types of services provided by GPS.
### 3.6 Global Positioning system (GPS)

The Global Positioning System (GPS) is a navigation system developed by the United States of America. It uses a network of satellites to provide precise location and time information to users anywhere on Earth or in space. The GPS system is designed to operate independent of other navigation systems.

<table>
<thead>
<tr>
<th>Sl No</th>
<th>Short name</th>
<th>Full name</th>
<th>Country</th>
<th>Type of NSS</th>
<th>Operation starting year</th>
<th>Orbit period</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>GPS</td>
<td>Global Positioning System</td>
<td>The United States of America</td>
<td>GNSS</td>
<td>1993</td>
<td>12 Hrs.</td>
</tr>
<tr>
<td>2.</td>
<td>GLONASS</td>
<td>Global navigation Satellite System</td>
<td>RUSSIA</td>
<td>GNSS</td>
<td>1993</td>
<td>11 Hr. 15 min.</td>
</tr>
<tr>
<td>3.</td>
<td>GALILEO</td>
<td>Europolitan Union</td>
<td>GNSS</td>
<td>2016</td>
<td>14.08 Hrs.</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>COMPASS BDS</td>
<td>BeiDou Navigation Satellite System</td>
<td>People of Republic China</td>
<td>GNSS</td>
<td>BDS-1 BDS-2 BDS-3</td>
<td>2000 2012 2020</td>
</tr>
<tr>
<td>5.</td>
<td>IRNSS NAViC</td>
<td>Indian RNSS Navigation Indian Constellation (sailor/Navigator)</td>
<td>India</td>
<td>RNSS</td>
<td>2018</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>QZSS</td>
<td>Quasi-Zenith Satellite System</td>
<td>Japan</td>
<td>RNSS</td>
<td>2018</td>
<td></td>
</tr>
</tbody>
</table>
3.6 Global Positioning system (GPS)

a. Precise positioning service: This is dedicated for accurate positioning and navigation signals for military purposes and for guiding missiles.

b. Standard positioning service: This is dedicated for civil, commercial and scientific purposes. That includes navigation devices used at personal level, commercial level in addition to the weather reporting data signals for scientific analysis.

They also carry a crystal clock to synchronize their location time with the satellite. Other methods use cell phone signal time also for faster synchronization. They are called A-GPS devices.

During Kargil war time, US Government has imposed restrictions on usage of GPS data to track the enemy movements. Further it lifted the ban but many nations decided to develop their own global navigation system. Some of the global and regional navigation satellite systems (GNSS/RNSS) developed by other countries are as follows.

Applications:

Navigation: GPS based receivers are used for both general public and commercial navigation purposes. Self driving cars also have been designed to use GPS signal for navigation. They are also used in Railway, aviation and marine navigation.

Surveying and Mapping: the implementation of GPS technology in surveying made the process faster and cheaper and without loss of accuracy.

Environment: Weather reporting for marine navigators and fishermen as well as for general public. GPS also tracks cloud movements and air flow rate for a better weather prediction.

Cell phones communication: GPS technology in cellphones enabled the cell phone operators to synchronize time among all the receiver devices. That enabled faster and accurate communication.

Geotagging and virtual tours: When photographs and videos are tagged with the geographical map of a particular location, the communication, social ease, internet security has been improved. At
3.6 Global Positioning system (GPS)

The Global Positioning System (GPS) is a satellite-based navigation system that provides global, continuous, all-weather, real-time, precise position, velocity, and time information. It was designed and is maintained by the United States government, and is available to all. The GPS system consists of a network of satellites in orbit around Earth. These satellites transmit signals to receivers on the ground, which can then calculate their position. The GPS system was initially developed for military purposes, but has since been made available for civil use.

The GPS system is used for a wide range of applications, including navigation, surveying, mapping, and scientific research. It is also used in a variety of civilian applications, such as in the transportation, agriculture, and construction industries. The GPS system is based on a network of 24 satellites, which are arranged in six orbital planes, each containing four satellites. The system is continuously expanded and updated to ensure that it remains accurate and reliable.

The GPS system is available to anyone with a GPS receiver, and is used in a variety of devices, such as smartphones, cars, airplanes, and boats. The system is also used in a variety of industries, such as construction and agriculture, to provide precise location data.

The GPS system is maintained by the United States government, and is continually updated to ensure its accuracy and reliability. The system is also designed to be resistant to jamming and other interference, which makes it a valuable tool for a wide range of applications.

E - Corner

https://gssc.esa.int/navipedia/index.php/Main_Page
https://www.gps.gov/systems/gps/
the same time privacy levels of information on social media has been reduced.

Traffic data mining: GPS based traffic data gives an estimate of the live traffic predictions at a given place.

Military: Enemy movement tracking, missile guiding and navigation in dark unknown terrains have been simplified by the introduction of precise positioning services in military applications.

Tectonics: The effect of earthquakes on the geographical structures can be studied by comparing the GPS data before and after the earthquake.

![Frequency bands of various GNSS](image)

3.7 Weightlessness

Weightlessness is experienced when the body experiences zero gravitational force.

Consider a person with mass $m$, standing in a lift that is moving with acceleration $a$. The weight $mg$ of the person acts downwards. As a reaction, the floor of the lift also pushes the person upwards. That is called the normal force $n$.

If the net acceleration of the lift is upwards, then the terms that contribute to it are as follows

$$\sum F = n + (-mg) = ma$$

From this, the normal force is

$$n = m(g + a)$$
3.7 Weightlessness

Weightlessness:

In the absence of gravity, the net force on an object is zero, indicating that the object is in weightlessness. This occurs when the gravitational force is balanced by another force, such as the normal force or the centripetal force in circular motion.

For an object of mass \( m \) experiencing weightlessness in a gravitational field with acceleration due to gravity \( g \), the net force \( F \) acting on the object is zero, as shown in Figure 3.25. This is given by the equation:

\[
\Sigma F = n + (-mg) = ma
\]

where \( n \) is the normal force, \( m \) is the mass of the object, and \( a \) is the acceleration due to gravity.

Rearranging the equation for \( n \), we get:

\[
n = m(g + a)
\]

Equation (1) is valid only if the object is in a free-fall condition.

For an object experiencing weightlessness in a rotating frame of reference, the normal force \( n \) is given by:

\[
n = mg = ma
\]

Equation (2) is valid for objects in a centrifugal force field.

Using the conservation of angular momentum, the normal force can be expressed as:

\[
n = mg - ma = \frac{GMm}{(R + r)^2} - \frac{mv^2}{R + r}
\]

Equation (3) is applicable in a rotating system.

In summary, the conditions for weightlessness depend on the context and the forces acting on an object.
If the net acceleration of the lift is downwards, then the normal force that acts upwards is given by

\[ n = m(g - a) \]  

(3)

In this case, if the downward acceleration of the lift becomes equal to the acceleration due to gravity, then the net normal force becomes zero. Then the object experiences no weight. In this case, the gravitational weight is balanced by the acceleration of the lift.

Thus weightlessness is experienced when the normal force exerted by the floor on the object gets nullified by any other external mechanism.

Thus in this case, the floor also falls downwards with acceleration \( a = g \) i.e.; it undergoes a free fall.

Weightlessness is also experienced when the object is in microgravity space and there is no external acceleration on the object. In that case, both \( g \) and \( a \) in Eq. (3) are equal to zero.

In the case of space station, that orbits around earth, there will be an inward acceleration due to gravity and an outward centrifugal force. Thus Eq. (3) becomes,

\[ n = mg - ma = \frac{GMm}{(R + r)^2} - \frac{mv^2}{R + r} \]  

(4)

Here \( R \) is the radius of the earth and \( r \) is the distance of the space station from the surface of the earth.

If there exists a non-zero net normal force, the satellite will be thrown normal to the orbit along the direction of that force. As long as the satellite revolves in a stationary orbit, these two forces will be equal and the net normal force will be zero. Then the objects in the satellite/space station experience weightlessness.
3.7 Weightlessness

1. In space, the body is unable to rotate due to a lack of gravitational forces, resulting in a state of weightlessness.

2. The term “static” (SAS) refers to a condition where the body remains in a fixed position.

3. When in weightlessness, the body experiences various adverse effects, such as migration of fluids, changes in blood pressure, and altered bone density.

4. In space, the body is subject to various forces, including microgravity, centrifugal force, and Coriolis force, which affect the body's fluid distribution.

5. The body's circulatory system adapts to the weightless environment, leading to changes in blood pressure and flow.

6. The body's immune system also changes in weightlessness, affecting its ability to fight off infections.

7. In conclusion, weightlessness in space poses significant challenges to the human body, requiring careful adaptation and management.
3.8 Physiological effects of Astronauts

The same happens when the object travels in the outer space with constant velocity and no other gravitational forces act on it.

3.8 Physiological effects of Astronauts

1. Rocket launching and re-entry into earth’s atmosphere causes a g-force of around 3 on the astronauts. This flushes down the blood flow in brain and astronauts may become unconscious. They require special G-suite to maintain blood flow into brain. More than 4-6g-force vertically along the spine causes death.

2. The first few hours of weightlessness causes “Space Adaptation Syndrome” (SAS) or space sickness. This results in nausea, vomiting, vertigo, headache, lethargy etc.

3. Exposure to vacuum is one of the lethal activities for astronauts. They may recover to normality without much side effects if they are exposed to complete vacuum not more than 30 sec to 90 sec. Beyond that all fluid in the body gets evaporated. Oxygen content in the body falls down due to breathing out in vacuum and the subject becomes unconscious.

4. On earth, gravity pulls blood downwards. That is why heart is a little above the centre of mass for human beings, to overcome the gravity effects. Also the blood vessels in the legs are narrower than those in head to bear high pressure towards legs. In zero-G conditions there will be heavy blood flow to head compared to legs. Thus the blood vessels in legs become even thinner and
3.8 Physiological effects of Astronauts

8. Physiological effects of Astronauts

9. Physiological effects of Astronauts

10. Physiological effects of Astronauts

11. Physiological effects of Astronauts

E - Corner

https://www.nasa.gov/audience/foreducators/stem-on-station/ditl_eating

those in head become further wider and robust. Adaptation to zero-G environment takes a time of around 1-2 weeks. Till that time astronauts feel dizziness. Similar effects will be seen for flow of all other bodily fluids in the human body.

![Image: The effect of microgravity on fluid distribution]

5. Due to heavy blood rush to head, eyeballs get compressed and flatten a little. This causes a change in eyesight.

6. Astronauts can take food normally as the peristaltic motion of the digestive system is not affected by gravity. They are allowed to take all solid food as usual. Powders are not allowed in space as dust won’t settle down in zero gravity, they may cause irritation in eyes and lungs. That is why salt and pepper are kept in liquid state to use in space stations. Most of the astronauts feel that their food is less tasty or sometimes tastier. Drinking water requires straw. This is because, water remains in spherical shape in zero gravity.

7. In zero gravity, fire takes spherical shape. On surfaces, it spreads without any flare. Presence of fire on surfaces can’t be identified until the material gets damaged. Hence care must be taken while dealing with flame and fire extinguishers in zero gravity.

8. Muscles and bones get relaxed in zero gravity. Thus astronauts gain 1 inch height in one month in zero gravity. Muscles shrink and lose their strength as they get no strain. Bones also lose their strength and become fragile. Especially back bone and foot bones that are designed to bear the rest of the body weight get damaged more due to relaxation. Springs and cords are used to create enough tension in muscles via weight lifting exercises. Otherwise astronauts may lose up to 20% of muscle mass in 5-11 days. The exercises help only muscle and to regain bone strength, one has
3.8 Physiological effects of Astronauts

**E - Corner**

http://pages.erau.edu/~andrewsa/Project%201/Olley_Sophie/HUMOLLEY/Project1sophie_olley.html

https://apod.nasa.gov/apod/ap210810.html

https://science.nasa.gov/science-news/science-at-nasa/2013/18jun_strangeflames

https://www.nasa.gov/audience/foreducators/stem-on-station/ditl_sleeing

**Did You Know?**

Why rotation and revolution period of moon are equal? https://www.youtube.com/watch?v=6jUlpX77ySo

**Think**

Why don’t we feel weightlessness on earth, as it also revolves around the sun just like the space station revolves around the earth.

**Did You Know?**

Earth has enough mass and radius sufficient to produce gravity \( g = -\frac{GM}{R^2} \) that holds small creatures like us and other movables at the same time balances the gravitational pull of the sun. Space station may have enough gravity to hold dust particles on the floor.

**Activity**

Calculate the gravity produced by various planets in our solar system.

https://nssdc.gsfc.nasa.gov/planetary/factsheet/planet_table_ratio.html
to come back to earth. To regain the bone density lost in 3-4 months space trip requires 2-3 years on earth. Astronauts that stay for longer times in space use elastic pants that apply pressure on their foot when tied to their waist, to apply enough stress on feet.

9. Bacteria in zero gravity conditions seems to grow faster and gain more resistance to antibiotics. Thus any minute amount of bacteria even from food will become fatal on space station. The bacteria within human body gets activated and becomes fatal in zero gravity environment.

10. Radiation from outer space is majorly blocked by the magnetosphere on the earth. In outer space, astronauts are more prone to radiation damages. A few minutes of exposure may also become fatal. Since the space stations revolve in the lower earth orbit, it is partially protected by the earth’s magnetosphere. In addition to cancers, radiation also causes cataracts and may accelerate the onset of Alzheimer’s disease. The radiation drastically damage the lymphocytes which are the major constituents of human immune system.

11. Sleeping in zero gravity requires a sleeping bag tied to a wall. Sleeping disorders are more common due to fluctuations in other physiological activities. Hence sleeping at a fixed time without disturbances is suggested.

Solved problems and Exercises

1. A body of mass 1kg is moving under a central force in a n elliptic orbit with semi major axis 1000m and semi minor axis 100m. The orbital angular momentum of the body is 100kg m²s⁻¹. What is the time period of motion of the body.

Ans:

Given that

a=1000m
b=100m
m=1kg
Solved problems and Exercises

L=100kg m²s⁻¹

For a body moving under central force, areal velocity is constant and
is given by

\[ \frac{dA}{dt} = \frac{L}{2m} \]

In general

\[ \frac{A}{T} = \frac{L}{2m} \]

We know that \( A = \pi ab \) (Area of ellipse)

Therefore

\[ T = \frac{2m \pi ab}{L} \]

Substituting the above values

\[ T = \frac{2 \times 1000 \times 3.14 \times 1000 \times 100}{100000} \]

\[ T = 1.74 \text{hours} \]

2. If the motion of a particle is described by \( x = 5\cos (8\pi t) \), \( y = 5\sin (8\pi t) \) and \( z = 5t \), then find the trajectory of the particle.

Ans:

Given
\( x = 5\cos (8\pi t) \)
\( y = 5\sin (8\pi t) \) and
\( z = 5t \),
Since \( x^2 + y^2 = 25 \), the particle motion is circular and
Propagating along \( z = 5t \),
Therefore the particle motion is Helical

3. A satellite moves around the earth in a circular orbit of radius \( R \) centered at the earth. A second satellite moves in an elliptic orbit
of major axis 8R, with the earth at one of the foci. If the former takes 1 day to complete a revolution, the latter would take how much time.

Ans:

From the Kepler’s law of planetary motion

\[ T^2 \propto R^3 \]

Given that

For first satellite radius of the circular orbit is \( R \) and \( T_1 = 1 \) day

And second satellite radius of the circular orbit is \( 8R \) and \( T_2 = ? \)

From the above equation

\[
\frac{T_1^2}{T_2^2} = \frac{R^3}{(8R)^3}
\]

\[
8^3 T_1^2 = T_2^2
\]

\[
T_2 = 8^{3/2} T_1
\]

\[
T_2 = 21.6 \text{ days}
\]

4. A particle of mass \( m \) is moving in \( x-y \) plane. At any given time \( t \), its position vector is given by \( r = A \cos wt \hat{i} + B \sin wt \hat{j} \) where \( A, B \) and \( w \) are constants. Describe the orbital motion of the particle and measure it angular momentum.

Ans:

Given by

\( r = A \cos wt \hat{i} + B \sin wt \hat{j} \)

Generalized position vector is \( r = x \hat{i} + y \hat{j} \)

By comparing the above equations

\( x = A \cos wt \Rightarrow x/A = \cos wt \) and

\( y = B \sin wt \Rightarrow y/B = \sin wt \) and

Therefore,

\[
\left( \frac{x}{A} \right)^2 + \left( \frac{y}{B} \right)^2 = (\cos wt)^2 + (\sin wt)^2
\]
It represents orbit of the particle is an ellipse

$L = \mathbf{r} \times \mathbf{p} = m (\mathbf{r} \times \mathbf{v})$

$v = \frac{dr}{dt} = -A w \sin wt \hat{i} + Bw \cos wt \hat{j}$

$L = m (\mathbf{r} \times \mathbf{v})$

$L = m (i j k A \cos \cos wt \ B \sin \sin wt \ 0 \ - A w \sin \sin wt \ Bw \cos \cos wt \ 0 )$

$L = m wAB \hat{k}$

5. In planar polar co-ordinates, an object’s position at time t is given as $(r, \theta) = (e^t m, \sqrt{8} t \text{ rad})$. Then find the magnitude of its acceleration (to the nearest integer) at $t=0$.

Ans:

Given that

$(r, \theta) = (e^t m, \sqrt{8} t \text{ rad})$.

i.e. $r = e^t m$ and

$\theta = \sqrt{8} t \text{ rad}$

We know that

$a_r = \ddot{r} - \dot{\theta}^2 r$

$\ddot{r} = e^t$

$\dot{\theta}^2 = 8$

$\therefore a_r = e^t - 8 e^t$

and $a_\theta = r \ddot{\theta} + 2 \dot{\theta} \dot{r}$

$\ddot{\theta} = 0$

and

$\dot{r} = e^t$

$\therefore a_\theta = 0 + 2\sqrt{8} e^t$
At \( t=0 \),

\[
\therefore a_r = 1 - 8 = -7
\]

\[
\therefore a_\theta = 2\sqrt{8}
\]

Therefore, the resultant acceleration is given by

\[
a = \sqrt{a_r^2 + a_\theta^2}
\]

\[
a = \sqrt{(-7)^2 + 4 \times 8}
\]

\[
a = \sqrt{49 + 32} = 9 \text{ m/sec}^2
\]

6. A planet has average density same as that of the earth but it has only \( \frac{1}{8} \) of the mass of the Earth. If the acceleration due to gravity at the surface is \( g_p \) and \( g_e \) for the planet and earth respectively, then find the \( \frac{g_p}{g_e} \).

Ans:

We know that

\[
g = \frac{GM}{R^2}
\]

And

Density \( (\rho) = \frac{M}{V} \)

\[
V = \frac{M}{\rho}
\]

\[
\frac{4}{3} \pi R^3 = \frac{M}{\rho}
\]

\[
R = \left(\frac{3M}{4\pi\rho}\right)^{\frac{1}{3}}
\]

Therefore \( g = \frac{GM}{\left(\frac{3M}{4\pi\rho}\right)^{\frac{2}{3}}} \)
Solved problems and Exercises

Given that

\[ M_p = \frac{1}{8} M_e \]

\[
g_p = \frac{G M_p}{\left(\frac{3M_p}{4\rho \pi}\right)^{\frac{2}{3}}} = \frac{G M_p}{\left(\frac{3M_e}{4\rho \pi}\right)^{\frac{2}{3}}}
\]

\[
g_p = \frac{\left(\frac{M_p}{M_e}\right)^{\frac{1}{3}}}{\left(\frac{M_p}{M_e}\right)^{\frac{1}{3}}}
\]

\[
g_p = \left(\frac{1}{8}\right)^{\frac{1}{3}}
\]

\[
g_p = 0.5
\]

7. There are three planets in circular orbits around a star at distances \(a, 4a,\) and \(9a\), respectively. At time \(t_0\), the star and the three planets are in a straight line. The period of revolution of the closest planet is \(T\). How long after \(t_0\) will they again be in the same straight line?

Ans:

From the Kepler’s law of period

\[ T^2 \propto R^3 \]

\[ T^2 = k R^3, \text{ where } k \text{ is proportionality constant.} \]

Given that the three planets in circular orbits around a star having radii \(a, 4a,\) and \(9a\) respectively Therefore,

\[ T_1^2 = k a^3 \implies T_1 = k a^{3/2} = T \]
Solved problems and Exercises

\[ T_2^2 = k (4a)^3 \implies T_2 = 8k a^{3/2} = 8T \quad \text{and} \]
\[ T_3^2 = k (9a)^3 \implies T_3 = 27k a^{3/2} = 27T \]

The common time that all the three stars meet in a straight line is
\[ t_o = T_1 \times T_2 \times T_3 \]
\[ = T \times 8T \times 27T \]
\[ t_o = 216T \]

Which is the LCM of all the time periods.

8. The moon moves around the earth in a circular orbit with a period of 27 days. The radius of the earth \( R \) is \( 6.4 \times 10^6 \) m and the acceleration due to gravity on the earth surface is \( 9.8 \, ms^{-2} \). If \( D \) is the distance of the moon from the center of the earth, find the value of \( D/R \).

Ans:

For circular orbit

\[ mw^2D = \frac{GmM}{D^2} \]

\[ D^3 = \frac{GM}{w^2} \]

\[ D^3 = \frac{GM}{(2\pi)^2R^2} \]

\[ \frac{D^3}{R^3} = \frac{g T^2}{4\pi^2R} \]

\[ \frac{D}{R} = \left( \frac{g T^2}{4\pi^2R} \right)^{\frac{1}{3}} \]

Given that
\[ R = 6.4 \times 10^6 \, m \]
\[ T = 27 \, \text{days} = 27 \times 3600 \, \text{sec} \]
\[ g = 9.8 \, ms^{-2} \]
9. If the diameter of the Earth is increased by 4\% without changing the mass, then find the length of the day. 

Ans:

Since the mass same, and satisfied the law of conservation of angular momentum from the motion under central forces

\[ I_1w_1 = I_2w_2 \]

We know that moment of Inertial of a circular object is

\[ I = MR^2 \]

Therefore

\[ I_1 = MR^2 \quad \text{and} \quad I_2 = M(R + 0.4R)^2 \]

and

\[ w_1 = \frac{2\pi}{T_1} \quad \text{and} \quad w_2 = \frac{2\pi}{T_2} \]

Therefore

\[ MR^2 \frac{2\pi}{T_1} = M(R + 0.04R)^2 \frac{2\pi}{T_2} \]

\[ T_2 = 1.04^2 T_1 \]

Given that, \( T_1 = 24 \) hours

\[ T_2 = 1.04^2 \times 24 \]

\[ T_2 = 25.95 \text{ Hours} \]

10. A particle of mass \( m \) moves along a trajectory given by \( x=x_0 \cos wt \) and \( y=y_0 \sin wt \). Find the condition for the force to be a central force.

Ans:
Given that by \( x = x_0 \cos w_1 t \) and \( y = y_0 \sin w_2 t \)

Therefore
\[
\begin{align*}
    r &= x_0 \cos \cos w_1 t i + y_0 \sin \sin w_2 t j \\
    \dot{r} &= -x_0 w_1 \cos \cos w_1 t i + y_0 w_2 \cos \cos w_2 t j \\
    \ddot{r} &= -x_0 w_1^2 \cos \cos w_1 t i - y_0 w_2 t j
\end{align*}
\]

We know that
\[
F = m\ddot{r}
\]

And
\[
T = r \times F = r \times m\ddot{r}
\]

But we know that a force become central force when it satisfies that the
\[
T = r \times F = 0
\]

Therefore
\[
r \times m\ddot{r} = 0
\]
\[
(i \dot{k} x_0 \cos \cos w_1 t \ y_0 \sin \sin w_2 t \ 0 \ - x_0 w_1^2 \cos \cos w_1 t \\
- y_0 w_2 t \ 0 ) = 0
\]

\[
\therefore w_1^2 = w_2^2
\]

\( = w_2 \) is the condition for the particle force become the central force

**MCQs**

1. Which is not a Kepler's law
   a) Law of periods
   b) Law of orbits
   c) Law of areas
   d) Law of continuity  Aucet 2019

2. The quantity that remains conserved for a particle moving under the action of Central force is
   a) Linear momentum
   b) Angular momentum
   c) Force
   d) Acceleration  Aucet 2019
3. Kepler’s second law follows from the law of conservation of
   a) Angular momentum
   b) Linear momentum
   c) Energy
   d) Mass   Aucet 2019

4.

1. Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between planet and star is proportional to \( R^{-5/2} \), then \( T^2 \) is proportional to
   a) \( R^{7/2} \)
   b) \( R^{5/2} \)
   c) \( R^{3/2} \)
   d) \( R^{1/2} \)

2. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of earth
   a) The acceleration of S is always directed towards the center of the earth
   b) The angular momentum of S about the center of the earth changes in direction but its magnitude remains constant
   c) The total mechanical energy of S varies periodically with time
   d) The linear momentum of S remains constant in magnitude

3. A mass M is split into two parts, m and M (>m), which are then separated by a certain distance. What ratio of \( \frac{m}{M} \) maximizes the gravitational force between the two parts
   a) 1/3
   b) 1/2
   c) 1/4
   d) 1/5

4. Which is not the conic section
   a) Ellipse
   b) Circle
   c) Parabola
   d) Sphere

5. If the radius of the earth were to shrink by 1% its mass remaining the same, the acceleration due to gravity on the earth’s surface would
   a) Decrease by 2%
Solved problems and Exercises

b) Remains unchanged

c) Increase by 2%

d) Increase by 1%

6 Suppose the gravitational force varies inversely as the nth power of distance. Then the time period of a planet in circular orbit of radius \( R \) around the sun will be proportional to

a) \( R \frac{(n+1)}{2} \)

b) \( R \frac{(n-1)}{2} \)

c) \( R^n \)

d) \( R \frac{(n-2)}{2} \)

7 The radius and mass of earth are increased by 0.5%. Which of the following statements are true at the surface of the earth

a) \( g \) will increase

b) \( g \) will decrease

c) Escape velocity will remain unchanged

d) Potential energy will remain unchanged

8 A body of mass \( m \) is taken from earth surface to the height \( h \) equal to radius of earth, the increase in potential energy will be

a) \( mgR \)

b) \( \frac{1}{2} mgR \)

c) \( 2 mgR \)

d) \( \frac{1}{4} mgR \)

9 A solid sphere of uniform density and radius 4 units is located with its center at the origin \( O \) of coordinates. Two spheres of equal radii 1 unit with their centers at \( A(-2, 0, 0) \) and \( B(2, 0, 0) \) respectively are taken out of the solid leaving behind spherical cavities as shown in figure

a) The gravitational force due to this object at the origin is zero

b) The gravitational force at the point \( B (2, 0, 0) \) is zero
Solved problems and Exercises

c) The gravitational potential is the same at all points of the circle \( y^2 + z^2 = 36 \)
d) The gravitational potential is the same at all points on the circle \( y^2 + z^2 = 4 \)

d) A geostationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a satellite orbiting a few hundred kilometres above the earth's surface \((R_{\text{Earth}} = 6400 \text{km})\) will approximately be
a) \( \frac{1}{2} \) h
b) 1 h
c) 2 h
d) 4 h

11 A planet revolving around sun in an elliptical orbit has a constant:

a) Kinetic energy
b) Linear speed
c) Angular momentum about the sun
d) Potential energy

12 Consider a classical particle subjected to an attractive inverse square force field. The total energy of the particle is \( E \) and the eccentricity is \( \varepsilon \). The particle will follow parabolic orbit if

a) \( E > 0 \) and \( \varepsilon = 1 \)
b) \( E < 0 \) and \( \varepsilon < 1 \)
c) \( E = 0 \) and \( \varepsilon = 1 \)
d) \( E < 0 \) and \( \varepsilon = 1 \)

13 A planet with no axial tilt is located in another solar system. It circles its sun in a very elliptical orbit so that the temperature varies greatly throughout the year. If the year there has 612 days and the inhabitants celebrate the coldest day on day 1 of their calendar, when is the warmest day?

a) Day 1
b) Day 153
c) Day 306
d) Day 459

14 3. Are Kepler’s laws purely descriptive, or do they contain causal information?

a) Kepler’s laws are purely descriptive.
b) Kepler’s laws are purely causal.
c) Kepler’s laws are descriptive as well as causal.
d) Kepler’s laws are neither descriptive nor causal.
15 True or false—According to Kepler’s laws of planetary motion, a satellite increases its speed as it approaches its parent body and decreases its speed as it moves away from the parent body.
a) True
b) False
16 Identify the locations of the foci of an elliptical orbit.
a) One focus is the parent body, and the other is located at the opposite end of the ellipse, at the same distance from the center as the parent body.
b) One focus is the parent body, and the other is located at the opposite end of the ellipse, at half the distance from the center as the parent body.
c) One focus is the parent body and the other is located outside of the elliptical orbit, on the line on which is the semi-major axis of the ellipse.
d) One focus is on the line containing the semi-major axis of the ellipse, and the other is located anywhere on the elliptical orbit of the satellite.

Answers:

1. 1 A

Centripetal force $\propto R^{-5/2}$
or $mR\omega^2 \propto R^{-5/2}$
or $\omega_2^2 k \propto R^{-5/2}$
or $\omega_2^2 \propto R^{-7/2}$
or $\left(\frac{2\pi}{T}\right)^2 \propto R^{-7/2}$
2. A

Force on satellite is always towards earth, therefore, acceleration of satellite S is always directed towards centre of the earth. Net torque of this gravitational force F about centre of earth is zero. Therefore, angular momentum (both in magnitude and direction) of S’ about centre of earth is constant throughout. Since, the force F is conservative in nature, therefore mechanical energy of satellite remains constant. Speed of S is maximum when it is nearest to earth and minimum when it is farthest.

3. B

\[ F = \frac{Gm(M-m)}{x^2} \]; For maxima,
\[ \frac{dF}{dm} = \frac{G}{x^2} (M - 2m) = 0 \]
Or \[ \frac{m}{M} = \frac{1}{2} \]

4. D

5. C

Acceleration due to gravity is
\[ g = \frac{GM}{R^2} \]
Now, M remains constant and R becomes
\[ R_2 = R_1 - 1\%R_1 = R_1 - \frac{0.01R_1}{100} = \frac{99R_1}{100} \]
Therefore, the new \( g \) becomes
\[ g_2 = \frac{GM}{R_2^2} = \frac{GM \times 100^2}{99^2R_1^2} = \frac{100^2}{99^2} \times g_1 = 1.02g_1 \]

Now, note that \( g \) would not decrease. It would increase because \( R \) decreases.
\[ g_2 = 1.02g_1 \]
\[ g_2 - g_1 = 1.02g_1 - g_1 = 0.02g_1 \]
Hence, the percentage increase is
\[ \frac{g_2 - g_1}{g_1} \times 100 = 0.02 \times 100 = 2\% \]
Hence, if the radius shrinks by 1% \( g \) increases by 2%.
Solved problems and Exercises

6. **A** But, \( F = m\omega^2 R \)

\[
\Rightarrow \frac{k}{R^n} = mR \cdot \left(\frac{2\pi}{T}\right)^2
\]

\[
\Rightarrow \frac{k}{R^{n+1}} = \frac{m(2\pi)^2}{T^2}
\]

\[
\Rightarrow T^2 \propto R^{n+1}
\]

\[
\therefore T \propto R^{\frac{n+1}{2}}
\]

7. **A**

\[
v_e = \sqrt{\frac{2GM}{R}} \quad \therefore v_e \propto \sqrt{\frac{M}{R}}
\]

and \( U = -\frac{GMm}{R} \text{ or } U \propto \frac{M}{R} \)

Thus if both \( M \) and \( R \) are increased by 0.5%, escape velocity and potential energy will ren...

However, \( g = \frac{GM}{R^2} \therefore g \propto \frac{M}{R^2} \)

Thus if \( M \) and \( R \) are increased, \( g \) will decrease. Thus (B), (C) and (D) are correct, and (A) is a wrong statement.

8. **D** We know that \( U_i = -\frac{GMm}{R} \) on the surface

\( U_f = -\frac{GMm}{R+h} \) at height \( h = R \) from the surface

Therefore \( U_i = -\frac{GMm}{R} \)

\( U_f = -\frac{GMm}{2R} \)

Change in Potential energy \( \Delta U = U_i \sim U_f \)

\( \Delta U = \frac{GMm}{2R} = \frac{1}{2} mgR \) [since \( g = GM/R^2 \)]

9. **B**

10. **C** \( T^2 \propto R^3 \)

\[
T^2 = KR^3 \Rightarrow (24)^2 = K(36000)^3
\]

\[
K = \frac{1}{9 \times 10^4 \sqrt{10}}
\]

\[
T' = \frac{1}{9 \times 10^4 \sqrt{10}} (6400)^{3/2} = 2h
\]

11. **C** A planet revolving in an elliptical path has a constant angular momentum. This can be justified by Kepler’s Laws. According to law of equal areas, if a straight line is drawn from the center of the sun to the center of the planet moving on the elliptical path, it will cover equal areas in equal intervals of time. Angular momentum, \( L = mr^2w \) where \( m, r \) and \( w \) are the mass, distance between
Solved problems and Exercises

the center of the sun and the center of the planet and angular velocity of the planet. Since, mass remains constant, angular momentum depends only on r and w. On an elliptical path, when distance increases, angular velocity reduces and when distance decreases, angular velocity increases. This is how angular momentum remains constant.

12. C
13. A
14. A
15. A

Grade your Understanding

1. A central force is a force which is directed always towards a fixed point [ ]
2. All central forces should be conservative [ ]
3. For conservative central forces, angular momentum is always constant [ ]
4. Motion of simple pendulum is an example of central force problem [ ]
5. Lorentz force (Electromagnetic force) is a central force [ ]
6. For central forces, angular acceleration becomes zero. [ ]
7. Areal velocity is a pseudovector whose length has units of $m^2/s$ [ ]
8. Time period of a satellite around a planet depends on length of semi minor axis if length of semi major axis is fixed [ ]
9. All communication satellites revolve at GEO level [ ]
10. GPS (global positioning system) satellites revolve in GEO level [ ]
11. During vertical motion of a satellite, the objects in the satellite experience weightlessness [ ]

<table>
<thead>
<tr>
<th>Glossary: Motion in a Central force Field</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acrobat</td>
</tr>
<tr>
<td>Aphelion</td>
</tr>
<tr>
<td>Astronaut</td>
</tr>
<tr>
<td>Astronomer</td>
</tr>
<tr>
<td>Central force</td>
</tr>
<tr>
<td>Constellation</td>
</tr>
<tr>
<td>Cosmonauts</td>
</tr>
<tr>
<td>Eccentricity</td>
</tr>
<tr>
<td>Feminauts</td>
</tr>
<tr>
<td>Geodetic</td>
</tr>
<tr>
<td>Hikers</td>
</tr>
<tr>
<td>Microgravity</td>
</tr>
<tr>
<td>Perihelion</td>
</tr>
<tr>
<td>Pirouette</td>
</tr>
<tr>
<td>Glossary</td>
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<tr>
<td>------------------</td>
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<tr>
<td>Physiological</td>
</tr>
<tr>
<td>Satellite</td>
</tr>
<tr>
<td>Trilateration</td>
</tr>
<tr>
<td>Wanderer</td>
</tr>
<tr>
<td>Weightlessness</td>
</tr>
</tbody>
</table>
UNIT-3
CHAPTER-4
SPECIAL THEORY OF RELATIVITY
ప్రత్యేక విశ్వాసం సాధారణత్వం
చిత్రాల వర్గాలు

1. మూడు చిత్రాల వర్గాలు రామేశ్వరి సభావని మేనం సంచారం రామేశ్వరి.
2. ప్రమాణ - సంచార మాటల కొండ కార్యక్రమం.
3. ప్రమాణ వాటి ప్రశ్నల తయారు మాంసక్రమం
4. చిత్రాల వర్గాలు మేనం రామేశ్వరి పాటలు.
5. చిత్రాల వర్గాలు ప్రమాణ మేనం ఎ=మC^2 పాటలు.

పాఠం 1

ఈ పాఠంలో మేనం మొదటి పాఠం, పాఠం 1 ని చేసిన నేషనల్.

1. ఎందుకంటే ఇది సంచారం నిర్మాణం మరియు సంచారం పాటలు మేటి ప్రామాణికం.
2. మరియు మాటలు ప్రశ్నల తయారు మాంసక్రమం ప్రశ్నలు.
3. మరియు ప్రశ్నల నిర్మాణం మరియు వివరణ ప్రశ్నలు.
4. సమకు ఇది ఉన్నాం మరియు ప్రశ్నల నిర్మాణం మరియు ప్రశ్నలు.
5. ఇది ఉన్నాం మరియు ప్రశ్నల నిర్మాణం మరియు ప్రశ్నలు.
6. ఇది ఉన్నాం మరియు ప్రశ్నల నిర్మాణం మరియు ప్రశ్నలు మరియు ప్రశ్నల నిర్మాణం మరియు ప్రశ్నలు.
SPECIAL THEORY OF RELATIVITY

Syllabus
Introduction to relativity, Frames of reference, Galilean transformations, absolute frames, Michelson-Morley experiment, negative result, Postulates of Special theory of relativity, Lorentz transformation, time dilation, length contraction, variation of mass with velocity, Einstein’s mass-energy relation.

Learning Objectives
In this chapter students would learn about,

1. Frames of reference and Galilean transformation and Lorentz transformation.
4. Length contraction and Time dilation concepts.
5. Variation of mass with velocity and about $E = mc^2$ relation.

Learning Outcomes
By the end of the chapter, student would be able to

1. Define the postulates of relativistic mechanics.
2. Explain the conditions under which relativistic regime is observable.
3. Apply time dilation and length contraction concepts to various systems of interest.
4. Analyze the effects of relativistic velocities on the energy and mass of the systems.
5. Justify the need of postulates of special theory of relativity from Michelson and Morley experiment.
6. Develop models for proper dynamics of systems that travel under relativistic regime.
మాత్రమే నిండి ఉండాలి గంధర్భం పొందడానికి భావించండి

1. ప్రస్తుతం నిండి ఉండాలి గంధర్భం పొందడానికి భావించండి గంధర్భం పొందడానికి భావించండి

2. ప్రస్తుతం నిండి ఉండాలి గంధర్భం పొందడానికి భావించండి గంధర్భం పొందడానికి భావించండి

3. ప్రస్తుతం నిండి ఉండాలి గంధర్భం పొందడానికి భావించండి గంధర్భం పొందడానికి భావించండి

4. ప్రస్తుతం నిండి ఉండాలి గంధర్భం పొందడానికి భావించండి గంధర్భం పొందడానికి భావించండి

5. ప్రస్తుతం నిండి ఉండాలి గంధర్భం పొందడానికి భావించండి గంధర్భం పొందడానికి భావించండి

6. ప్రస్తుతం నిండి ఉండాలి గంధర్భం పొందడానికి భావించండి గంధర్భం పొందడానికి భావించండి

7. ప్రస్తుతం నిండి ఉండాలి గంధర్భం పొందడానికి భావించండి గంధర్భం పొందడానికి భావించండి

"A truly beautiful mind" అంటే మనుస్తూరికి మనుశతికి మనుశతికి మనుశతికి మనుశతికి మనుశతికి మనుశతికి మనుశతికి మనుశతికి మనుశతి ప్రతిష్టాపించింది. కానీ మనుశతికి మనుశతికి మనుశతికి మనుశతికి మనుశతికి మనుశతి ప్రతిష్టాపించింది. కానీ మనుశతికి మనుశతికి మనుశతికి మనుశతి ప్రతిష్టాపించింది. కానీ మనుశతికి మనుశతికి మనుశతి ప్రతిష్టాపించింది. కానీ మనుశతికి మనుశతికి మనుశతి ప్రతిష్టాపించింది. కానీ మనుశతికి మనుశతికి మనుశతి ప్రతిష్టాపించింది. కానీ మనుశతికి మనుశతికి మనుశతి ప్రతిష్టాపించింది. కానీ మనుశతి ప్రతిష్టాపించింది. కానీ మనుశతి ప్రతిష్టాపించింది. కానీ మనుశతి ప్రతిష్టాపించింది. కానీ మనుశతి ప్రతిష్టాపించింది. కానీ మనుశతి ప్రతిష్టాపించింది. కానీ మనుశతి ప్రతిష్టాపించింది. కానీ మనుశతి ప్రతిష్టాపించింది. కానీ మనుశతి ప్రతిష్టాపించింది.
Course Outcomes specific to program and Future directions

By the end of this chapter students from specific programs would be able to identify the need of special theory of relativity in the following fields.

1. Physics: Special theory of Relativity finds its applications from atomic interactions to astronomical interactions in physics.
2. Chemistry: Some elements in periodic table like Mercury, Gold properties can be explained only by special theory of relativity.
3. Computer Science: Programming and time synchronizing artificial satellites require relativistic time dilation corrections.
4. Geology: Geodesy and Geodynamics study the effect of earth shape, rotation in space on the earth crestal motion, polar motion and tides require knowledge of relativistic effects.
5. Electronics: Relativistic high frequency microwave devices are developed to generate plasma and other high energy particles.
6. Renewable energy: Relativistic corrections to perovskite solar cells materials enhances their bandgap. The effect of relativistic corrections to the earth, sun moon system predicts their impact on tidal, solar and wind energy resources.
7. Statistics: Relativistic statistical mechanics is a branch of physics that deals with the statistical analysis of high energy particles that travel at relativistic speeds.

Familiar to Unfamiliar

In your 9th class English prose you might have introduced with “A truly beautiful mind”, the story of Einstein and his works on special theory of relativity. You might have introduced about how time in clocks vary when they are in motion and the famous mass energy relation. In this chapter you would learn the experiments that lead to the discovery of special theory of relativity, postulates of special theory of relativity, and from them you would derive the effects of relativity on length, time, mass and energy.
4.1 మాహితిప్రత్యేకించండి

1. ప్రత్యేక ఉత్పత్తి ఎన్నికలు:

ప్రత్యేక ఉత్పత్తి ఎన్నికలు దీని ప్రస్తుతిలో జరిగింది. ఇది సమానం ఉంటుంది. ఆస్తుగా ఉంటుంది.

ఉదాహరణలు: 1647 సంవత్సరం ఐసక్ న్యూనెన్స్ ఒక హెలియస్ శాసనం చేసినా, ఇది ప్రాచీనంగా బహుళాంశం ఉంది. మరియు ఇది జీవితం చేసిన అవసరాలకు సమానంగా ఉంది. మరియు ఇది సమానంగా ఉంది.

విషయం ఉన్నది. 1689 సంవత్సరం ఐసక్ న్యూనెన్స్ ఒక హెలియస్ శాసనం చేసినా, ఇది ప్రాచీనంగా బహుళాంశం ఉంది. మరియు ఇది సమానంగా ఉంది.

https://mathshistory.st-andrews.ac.uk/HistTopics/Newton_bucket/

https://demonstrations.wolfram.com/NewtonRotatingBucketExperiment/
4.1 Introduction

Newtonian mechanics are applicable only when the frame of reference is inertial. i.e., only when there is no acceleration without applied force. The entire Newtonian mechanics swirls around cause and effect relations. There are a few misconceptions with cause and effect relation in Newton’s theory.

1. Space is not absolute
One of the misconceptions is about absolute frames. This concept was well described in the “Newton’s rotating bucket experiment and debate” in 1689.

![Fig: Newton’s rotating bucket experiment.](image)

In this experiment, a bucket carrying water is tied to a rope and is rotated by twisting the rope and releasing. In the beginning, just the bucket rotates and water remains stationary. After some time, water also rotates along with bucket in in concave shape. After sometime, if the rope stops rotating, bucket stops. But the water still rotates in concave shape. Later water also comes to rest. This concave shape is due to centrifugal force and gravitational pull. The delay between rotation of water and bucket is due to inertia.

Somehow Newton considered the status of water in the first and third stages as the isolated/absolute rest state and isolated/absolute rotation state with respect to some absolute frame.

Finally Newton concluded that there exists an absolute frame, with respect to which one can define motion of objects precisely, which is in far deep space where gravity is zero.

A serious objection followed by a proper explanation for this debate was founded, 200 years later by Ernst Mach, in 1883, by considering
4.1 Introduction

2. తాత్కాలిక పరిస్థితి

తాత్కాలిక పరిస్థితిలో అందమైన అధికారిక మార్గాలు తగ్గినా అందమైనా; తాత్కాలిక పరిస్థితిలో అందమైనా అధికారిక మార్గాలు తగ్గినా. తాత్కాలిక పరిస్థితిలో అందమైనా అధికారిక మార్గాలు తగ్గినా. తాత్కాలిక పరిస్థితిలో అందమైనా అధికారిక మార్గాలు తగ్గినా. తాత్కాలిక పరిస్థితిలో అందమైనా అధికారిక మార్గాలు తగ్గినా.

3. సమయం సంపాదించడానికి

సమయం సంపాదించడానికి, అందమైనా అధికారిక మార్గాలు తగ్గినా. సమయం సంపాదించడానికి, అందమైనా అధికారిక మార్గాలు తగ్గినా. సమయం సంపాదించడానికి, అందమైనా అధికారిక మార్గాలు తగ్గినా. సమయం సంపాదించడానికి, అందమైనా అధికారిక మార్గాలు తగ్గినా.
the combined effects of centrifugal and gravitational forces. According to him the relative motion of water with respect to bucket, earth and other planets is the only measurable quantity, and this absolute frames concept was abandoned. This laid the foundations for Einstein’s special theory of relativity.

2. No instantaneous effects

Another objection in Newton’s formalism is that the effect is instantaneous; i.e., if we push an object, displacement will be observed immediately after the application of the force. The discrepancy in this assumption will be clearly observed when celestial objects are considered. This was described as “action at distance” by Newton and other physicists of that era; i.e., the gravitational, electric and magnetic fields travel with immediate effect to whatever distances. It was concluded that for all fundamental fields of interactions, there must exist some upper bound of speed with which the effect is transmitted. That upper bound of speed exists for all fields namely electric, magnetic and gravitational fields. In fact, this speed is constant and coincidentally is equal to the speed of light in vacuum.

3. Time is not absolute

The concept of relative motion between objects was first studied by Galileo in his efforts to explain the horizontal motion on earth. Subsequent developments by Newton did not include the relative motion. This concept was later considered in detail in the works of Euler and Poisson. However, in all the above descriptions, including that of Galileo, time was assumed to be absolute.

Fig: Galilean transformation

The consequence of this absoluteness of time is that, if two events occur at certain interval in one frame, the interval remains same in all other inertial frames. But that is not the case in
4.1 Introduction

Did You Know?

The Greek word tachy means swift. Hence the name Tachyons. The particles that travels with speed of light are called Luxons. Eg: Photons. The particles that always travel slower than light are called Bradyons.


4.1 Introduction

reality, as the information transfer takes some propagation time between the frames.

Another consequence of absolute time is that one can have any amount of velocity imparted to objects. But in reality, since there is an upper limit for information transfer itself, the information about objects travelling faster than that limit cannot be recorded. Or such objects will remain invisible to our senses.

Another consequence of absolute time is that velocity of light changes in moving frames; i.e., if a person traveling with a speed $v$ throws light towards a stationary observer, the light should be seen with velocity $c + v$ or $c - v$ depending on the direction of observer, in one dimension. But in reality, the velocity of light remains constant everywhere. This is because light is just a self-regenerating electromagnetic field - at every point in space or time, light wave is generated afresh. So even if the source travels with an additional velocity, it will not be added to the velocity of light. This happens only with light because it has the velocity of the information propagation. In fact, it happens with propagation of any field but light is the only entity that is sensible to human beings. Some birds and animals are capable of sensing even the magnetic fields. They use this feature for their navigation.

4. Mass is not absolute:

In Newtonian mechanics mass is assumed to be independent of velocity and is conserved independently, in addition to momentum. Thus, in rocket equations, the total mass of rocket and fuel is assumed to be constant before applying Newton’s laws of motion to it. There are two definitions for mass, namely, inertial mass and gravitational mass. In general theory of relativity, principle of equivalence states that both are equal. i.e., mass that can be experienced by accelerating an object is same as that experienced by gravitational pull. In addition to that, relativity also introduced the concept of rest mass. This is the mass of the object when it is at rest compared to the observer. With rise in velocity of the object, the mass
4.2 విధాన వివరణ

సంఘటన సమయంలో ఒక పాత్రాన్ని మార్గం చేయడానికి బహుసాధ్యం అభయస్వభావం ఉంది. సంఘటన సమయంలో రోడు మార్గం చేయడానికి ప్రత్యేకంగా తెలుగు భాషలో వివరిస్తుంది.నాశన సమయంలో బహు రుభాషాన్ని నిర్ధారించడం సాధ్యం ఉంది.

ప్రత్యేకంగా ఎక్కడ సమయం వద్ద జీత ఉంటుంది. ఈ సమయంలో నాశన సమయం రుభాషాన్ని నిర్ధారించడం సాధ్యం ఉంది.

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 \quad (3)$$
$$ds'^2 = c^2 t'^2 - dx'^2 - dy'^2 - dz'^2 \quad (4)$$

ప్రత్యేకంగా ఎక్కడ బహు రుభాషా ఉంది. ఈ చివరికి సమయం రుభాషాన్ని నిర్ధారించడం సాధ్యం ఉంది.
also increases to infinity so that no force could push it beyond the speed of light. This kind of extremely massive objects are usually found near black holes.

The hypothetical particles that travel faster than speed of light are called tachyons. Since our material systems can’t acquire speeds beyond velocity of light, no material system existing as of now could detect them. Moreover, since light stands as a barrier between material particles, and tachyons; physicists also believe that light may also act as a bridge to probe into the tachyonic world.

4.2 Lorentz Transformation

Lorentz transformations are linear transformations that connect the coordinates between two frames of reference, wherein one of the frames is moving at a constant velocity relative to the other. The Lorentz transformations were actually developed by Woldemar Voigt, a German mathematical physicist in 1884. Speculations exist that they were adopted by Lorentz in 1904 and independently by Einstein in 1905. Later, Poincare authenticated it as Lorentz transformation in 1906.

Consider two frames of references $S$ and $S’$ moving with a relative velocity $v$. Consider an event of transfer of information between two points $(x_1, y_1, z_1)$, $(x_2, y_2, z_2)$ during the time interval between $t_1$ and $t_2$ with light velocity ($c$). Then the distance between the points can be written in terms of velocity of light as

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 = c^2(t_2 - t_1)^2 - - - (1)$$

The same event in $S’$ frame may be recorded at coordinates $(x_1’, y_1’, z_1’)$, $(x_2’, y_2’, z_2’)$ coordinated during the time intervals $t_1’$, $t_2’$. Then the distance equation becomes

$$(x_2’ - x_1’)^2 + (y_2’ - y_1’)^2 + (z_2’ - z_1’)^2 = c^2(t_2’ - t_1’)^2 - - - (2)$$

Here $c$ is assumed to be the velocity of propagation of information that remains constant between the two frames of reference.
4.2 Lorentz Transformation

The elements of linear algebra are built on the construction of Lorentz transformations. If two frames of reference are moving with velocity $v$ in the $x$-direction, the Lorentz transformation is:

$$ds^2 = a \, ds'^2$$

where $a$ is the Lorentz factor $a = \frac{1}{\sqrt{1 - v^2/c^2}}$. The transformation equations are:

$$ds^2 = a(v_1) \, ds_1^2, \quad ds^2 = a(v_2) \, ds_2^2, \quad ds_1^2 = a(v_1) \, ds_2^2$$

This implies that $a(v_1) = a(v_2) = \frac{a(v_1)}{a(v_2)}$.

The Lorentz transformation equations are:

$$v_1 = \frac{u_1 - v \, u_2}{1 - v \cdot u_2/v}, \quad v_2 = \frac{u_2 - v \, u_1}{1 - v \cdot u_2/v}$$

and the speed of light in the two frames is:

$$c = \frac{c}{\sqrt{1 - v^2/c^2}} = \frac{c}{a}$$

The Lorentz transformation equations for the coordinates are:

$$x = x' \cosh \phi + ct' \sinh \phi \quad ct$$

and

$$x = x' \sinh \phi + ct' \cosh \phi$$

The Lorentz factor is:

$$a = \frac{1}{\sqrt{1 - v^2/c^2}}$$

and the Lorentz transformation equations are:

$$x = ct' \sinh \phi \quad ct = ct' \cosh \phi$$

and

$$\Rightarrow \tanh \phi = \frac{x}{ct} = \frac{v}{c}$$
If the interval between the events is infinitesimally small, one can write

\[ ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 \quad \cdots (3) \]
\[ ds'^2 = c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2 \quad \cdots (4) \]

Here \( ds^2 = 0 \) and \( ds'^2 = 0 \) if the events take place at the speed of light. Then the event is said to be “Light like”. If the events take place with speeds lower than light velocity, then \( ds^2 > 0 \) and \( ds'^2 > 0 \). Then one can identify a frame of reference in which both the events occur at the same location, at different time stamps. Then the event is said to be “Time like”. If the events take place faster than velocity of light, then \( ds^2 < 0, ds'^2 < 0 \). Then one can identify a frame of reference where both the events occur at same time at two different spatial locations. In such a case, the event is said to be “Space like”.

![Space-time diagram of events.](image)

Since the events reported in two frames of reference are related to the same system, they at most differ by a proportionality constant. Thus

\[ ds^2 = a \ ds'^2 \]

To evaluate the nature of the proportionality constant, consider three frames of reference \( S, S', S'' \) with relative velocities \( v_1, v_2, v_1' \) respectively. Then one can write

\[ ds^2 = a(v_1)ds_1^2, \quad ds^2 = a(v_2)ds_2^2, \quad ds_1^2 = a(v_1')ds_2^2 \]
\[ \Rightarrow \frac{a(v_2)}{a(v_1')} = a(v_1') \]
4.2 Lorentz Transformation

\[
\tanh^2 \phi - \text{sech}^2 \phi = 1 \Rightarrow \tanh^2 \phi - \frac{1}{\cosh^2 \phi} = 1 \Rightarrow \cosh \phi = \frac{1}{\sqrt{1 - \tanh^2 \phi}}
\]

\[
\cosh \phi = \frac{1}{\sqrt{1 - v^2/c^2}}
\]

\[
\sinh \phi = \cosh \phi \times \tanh \phi = \frac{v/c}{\sqrt{1 - v^2/c^2}}
\]

Eq.(7) & Eq.(10), Eq.(11)

\[
x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}, \quad y = y', \quad z = z', \quad t = \frac{t' + \frac{v}{c^2} x'}{\sqrt{1 - v^2/c^2}}
\]

\[
x = \gamma(x' + vt'), \quad y = y', \quad z = z', \quad t = \gamma \left( t' + \frac{v}{c^2} x' \right)
\]

\[
\Delta x = x_2 - x_1 - - - (1)
\]

\[
x_1 = \gamma(x'_1 + vt'), \quad x_2 = \gamma(x'_2 + vt') - - - (2)
\]

Eq.(1) & Eq.(2)

\[
\Delta x = x_2 - x_1 = \gamma(x'_2 - x'_1) = \gamma \Delta x' - - - (3)
\]
Here, the ratio on the LHS depends on the individual velocities \( v_1, v_2 \) but the term on the RHS depends on the vector difference of the two velocities. Hence LHS is independent of angle and RHS depends on the angle, which implies that the quantity must be a scalar. Further, as these three quantities are cyclically related to each other, the scalar constant must be equal to unity. Thus, one can write

\[
ds^2 = ds'^2 \quad (5)
\]

Consider the simplest case of two frames of reference with relative motion along \( x \) axis with velocity \( v \). Then Eq. (5) simplifies as

\[
c^2 t^2 - x^2 = c^2 t'^2 - x'^2 \quad (6)
\]

**Fig: Lorentz transformation.**

If the negative sign on either side of the equation were to be replaced by positive sign, then the solution of this equation would be an oscillatory sine or cosine function. But due to the presence of negative sign, the solution of this equation happens to be a linear combination of hyper-geometric functions.

\[
x = x' \cosh \phi + ct' \sinh \phi \quad ct = x' \sinh \phi + ct' \cosh \phi \quad (7)
\]

Consider the motion of the origin of the \( S' \) system in the \( S \) frame. For this case, \( x' = 0 \). Then Eq. (7) becomes,

\[
x = ct' \sinh \phi \quad ct = ct' \cosh \phi \quad (8)
\]

\[
\Rightarrow \tanh \phi = \frac{x}{ct} = \frac{v}{c} \quad (9)
\]
4.2 Lorentz Transformation

\[ \gamma \geq 1, \text{ where } \Delta x' \leq \Delta x, \text{ and } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \]

where \( \Delta x \) is the time interval in the moving frame, and \( \Delta x' \) is the time interval in the stationary frame.

Since \( \Delta x \) is the time interval in the stationary frame, the time interval in the moving frame is smaller.

\[ \Delta t' = \Delta t - \Delta x' \]

where \( \Delta t \) is the time interval in the stationary frame, and \( \Delta t' \) is the time interval in the moving frame.

Eq.(1) and Eq.(2) show, respectively, the time and the spatial transformation.

\[ \gamma \geq 1, \Delta t' \leq \Delta t \]

where the time interval in the moving frame is smaller than the time interval in the stationary frame.

\[ \Delta x = \frac{\gamma - 1}{\gamma} \Delta x' \]

where \( \Delta x \) is the time interval in the stationary frame, and \( \Delta x' \) is the time interval in the moving frame.

\[ \Delta t = \frac{\gamma - 1}{\gamma} \Delta t' \]

where \( \Delta t \) is the time interval in the stationary frame, and \( \Delta t' \) is the time interval in the moving frame.

\[ \Delta x = \frac{\gamma - 1}{\gamma} \Delta x' \]

where \( \Delta x \) is the time interval in the stationary frame, and \( \Delta x' \) is the time interval in the moving frame.

\[ \Delta t = \frac{\gamma - 1}{\gamma} \Delta t' \]

where \( \Delta t \) is the time interval in the stationary frame, and \( \Delta t' \) is the time interval in the moving frame.

\[ \Delta x = \frac{\gamma - 1}{\gamma} \Delta x' \]

where \( \Delta x \) is the time interval in the stationary frame, and \( \Delta x' \) is the time interval in the moving frame.

\[ \Delta t = \frac{\gamma - 1}{\gamma} \Delta t' \]

where \( \Delta t \) is the time interval in the stationary frame, and \( \Delta t' \) is the time interval in the moving frame.

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where \( \Delta t \) is the time interval in the stationary frame, and \( \Delta t' \) is the time interval in the moving frame.

\[ \Delta x = \frac{\gamma - 1}{\gamma} \Delta x' \]

where \( \Delta x \) is the time interval in the stationary frame, and \( \Delta x' \) is the time interval in the moving frame.

\[ \Delta t = \frac{\gamma - 1}{\gamma} \Delta t' \]

where \( \Delta t \) is the time interval in the stationary frame, and \( \Delta t' \) is the time interval in the moving frame.

\[ \Delta x = \frac{\gamma - 1}{\gamma} \Delta x' \]

where \( \Delta x \) is the time interval in the stationary frame, and \( \Delta x' \) is the time interval in the moving frame.

Eq.(1) and Eq.(2) show, respectively, the time and the spatial transformation.

\[ \gamma \geq 1, \Delta t' \leq \Delta t \]

where the time interval in the moving frame is smaller than the time interval in the stationary frame.

\[ \Delta x = \frac{\gamma - 1}{\gamma} \Delta x' \]

where \( \Delta x \) is the time interval in the stationary frame, and \( \Delta x' \) is the time interval in the moving frame.

\[ \Delta t = \frac{\gamma - 1}{\gamma} \Delta t' \]

where \( \Delta t \) is the time interval in the stationary frame, and \( \Delta t' \) is the time interval in the moving frame.

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where \( \Delta x \) is the time interval in the stationary frame, and \( \Delta x' \) is the time interval in the moving frame.

\[ \Delta t = \frac{\gamma - 1}{\gamma} \Delta t' \]

where \( \Delta t \) is the time interval in the stationary frame, and \( \Delta t' \) is the time interval in the moving frame.

\[ \Delta x = \frac{\gamma - 1}{\gamma} \Delta x' \]

where \( \Delta x \) is the time interval in the stationary frame, and \( \Delta x' \) is the time interval in the moving frame.

\[ \Delta t = \frac{\gamma - 1}{\gamma} \Delta t' \]

where \( \Delta t \) is the time interval in the stationary frame, and \( \Delta t' \) is the time interval in the moving frame.

\[ \Delta x = \frac{\gamma - 1}{\gamma} \Delta x' \]

where \( \Delta x \) is the time interval in the stationary frame, and \( \Delta x' \) is the time interval in the moving frame.

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where \( \Delta t \) is the time interval in the stationary frame, and \( \Delta t' \) is the time interval in the moving frame.

\[ \Delta x = \frac{\gamma - 1}{\gamma} \Delta x' \]

where \( \Delta x \) is the time interval in the stationary frame, and \( \Delta x' \) is the time interval in the moving frame.

\[ \Delta t = \frac{\gamma - 1}{\gamma} \Delta t' \]

where \( \Delta t \) is the time interval in the stationary frame, and \( \Delta t' \) is the time interval in the moving frame.

\[ \Delta x = \frac{\gamma - 1}{\gamma} \Delta x' \]

where \( \Delta x \) is the time interval in the stationary frame, and \( \Delta x' \) is the time interval in the moving frame.

\[ \Delta t = \frac{\gamma - 1}{\gamma} \Delta t' \]

where \( \Delta t \) is the time interval in the stationary frame, and \( \Delta t' \) is the time interval in the moving frame.

\[ \Delta x = \frac{\gamma - 1}{\gamma} \Delta x' \]

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where \( \Delta x \) is the time interval in the stationary frame, and \( \Delta x' \) is the time interval in the moving frame.

\[ \Delta t = \frac{\gamma - 1}{\gamma} \Delta t' \]

where \( \Delta t \) is the time interval in the stationary frame, and \( \Delta t' \) is the time interval in the moving frame.

\[ \Delta x = \frac{\gamma - 1}{\gamma} \Delta x' \]

where \( \Delta x \) is the time interval in the stationary frame, and \( \Delta x' \) is the time interval in the moving frame.

\[ \Delta t = \frac{\gamma - 1}{\gamma} \Delta t' \]

where \( \Delta t \) is the time interval in the stationary frame, and \( \Delta t' \) is the time interval in the moving frame.


https://www.discovermagazine.com/the-sciences/ten-things-you-dont-know-about-black-holes
We know that
\[
\tanh^2 \phi - \text{sech}^2 \phi = 1 \Rightarrow \tanh^2 \phi - \frac{1}{\cosh^2 \phi} = 1 \Rightarrow \cosh \phi
\]
\[
= \frac{1}{\sqrt{1 - \tanh^2 \phi}}
\]
Thus
\[
\cosh \phi = \frac{1}{\sqrt{1 - v^2/c^2}} - - - (10)
\]
\[
\sinh \phi = \cosh \phi \times \tanh \phi = \frac{v/c}{\sqrt{1 - v^2/c^2}} - - - (11)
\]
Substituting Eq. (10), Eq. (11) in Eq. (7), one obtains
\[
x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}} \quad y = y', \quad z = z', \quad t = \frac{t' + \frac{v}{c^2}x'}{\sqrt{1 - v^2/c^2}} - - - (12)
\]
If we define the Lorentz factor \( \gamma = 1/\sqrt{1 - v^2/c^2} \), which is always greater than or equal to 1 for material particles (\( \because v < c \)), then
\[
x = \gamma(x' + vt'), \quad y = y', \quad z = z', \quad t = \gamma \left( t' + \frac{v}{c^2}x' \right) - - - (13)
\]
These are called the Lorentz transformation equations.

The inverse transformation equations are given by
\[
x' = \gamma(x - vt), \quad y = y', \quad z = z', \quad t' = \gamma \left( t - \frac{v}{c^2}x \right) - - - (13)
\]

**Length contraction from Lorentz Equations**

Consider an event of measurement of length of a rod at time \( t \) in the two reference frames. The length of the rod is given by,
\[
\Delta x = x_2 - x_1 - - - (1)
\]
where
\[
x_1 = \gamma(x_1' + vt'), \quad x_2 = \gamma(x_2' + vt') - - - (2)
\]
From Eq. (1) and Eq. (2),
4.2 Lorentz Transformation

The Lorentz transformations are given by:

\[ dx = \gamma(dx' + vdt') \]
\[ dy = dy' \]
\[ dz = dz' \]
\[ dt = \gamma\left(dt' + \frac{V}{c^2}dx'\right) \]

where \( x, y, z \) are coordinates in the reference frame and \( x', y', z', t' \) are coordinates in the frame moving with velocity \( V \) relative to the first frame.

The velocity components in the two frames are related by:

\[ V_x = \frac{dx}{dt} = \frac{dx' + vdt'}{dt' + \frac{V}{c^2}dx'} = \frac{V'_x + v}{1 + \frac{V}{c^2}V'_x} \]
\[ V_y = \frac{dy}{dt} = \frac{dy'}{\gamma\left(dt' + \frac{V}{c^2}dx'\right)} = \frac{V'_y}{\gamma\left(1 + \frac{V}{c^2}V'_x\right)} \]
\[ V_z = \frac{dz}{dt} = \frac{dz'}{\gamma\left(dt' + \frac{V}{c^2}dx'\right)} = \frac{V'_z}{\gamma\left(1 + \frac{V}{c^2}V'_x\right)} \]

Inertial reference frames are described by the condition \( V^' = c \) and \( V = c \). When \( V^' = c \), the velocity addition is given by:

\[ V = \frac{V' + v}{1 + \frac{V'}{c^2}} \]

When \( V = c \), \( V^' = c \) (null vector), and no motion in the second frame, the velocity addition is:

\[ V^' = \gamma V + V = V^' + v \]

https://openstax.org/books/college-physics/pages/28-4-relativistic-addition-of-velocities
\[ \Delta x = x_2 - x_1 = \gamma (x_2' - x_1') = \gamma \Delta x' \tag{3} \]

Since \( \gamma \geq 1 \), \( \Delta x' \leq \Delta x \), i.e., the length measured in moving frame is smaller than that measured in rest frame.

Here \( \Delta x \) is the proper length of the material which is measured in the rest frame of the system.

### Time dilation from Lorentz Equations

Consider the measurement of two events at two time intervals \( t_1 \) and \( t_2 \) at the same spatial locations. Then the time interval is given by

\[ \Delta t = t_2 - t_1 \tag{1} \]

where

\[ t_1 = \gamma \left( t_1' + \frac{v}{c^2} x' \right), \quad t_2 = \gamma \left( t_2' + \frac{v}{c^2} x' \right) \tag{2} \]

From Eq. (1) and Eq. (2), the time interval is obtained as

\[ \Delta t = t_2 - t_1 = \gamma (t_2' - t_1') = \gamma \Delta t' \tag{3} \]

Since \( \gamma \geq 1 \), \( \Delta t' \leq \Delta t \); i.e., the time measured in moving frame is shorter than that measured in rest frame. This implies the moving clock ticks slowly, i.e., the time reported by moving clock for the completion of an event will be longer compared to the clock in rest frame. Here \( \Delta t \) is the proper time of the event which is measured in the rest frame of the system.

In both the above cases, if inverse transformations are taken, opposite results will appear. I.e; \( \Delta t' = \gamma t \) and \( \Delta x' = \gamma \Delta x \). In that case the \( S' \) frame remains stationary and \( S \) frame seems to be moving backwards with respect to \( S' \). In summary, in moving frame length reduces and time slows down. Thus, it takes longer time for objects to traverse shorter distances in moving frames. This is what makes light to get trapped in black holes. Black holes are extremely massive objects in space that exhibit extreme length contraction and extreme time dilation, thus exhibiting extreme density and ability of light trapping.
4.2 Lorentz Transformation

Transformation of velocities

We have

\[ dx = \gamma(dx' + vdt'), \quad dy = dy', \quad dz = dz', \quad dt = \gamma \left( dt' + \frac{v}{c^2} dx' \right) \]

Velocities along \( x, y, z \) directions are given by

\[ V_x = \frac{dx}{dt} = \frac{dx' + v dt'}{dt' + \frac{v}{c^2} dx'} = \frac{V_x' + v}{1 + \frac{v}{c^2} V_x'} \]

\[ V_y = \frac{dy}{dt} = \frac{dy'}{\gamma \left( dt' + \frac{v}{c^2} dx' \right)} = \frac{V_y'}{\gamma \left( 1 + \frac{v}{c^2} V_x' \right)} \]

\[ V_z = \frac{dz}{dt} = \frac{dz'}{\gamma \left( dt' + \frac{v}{c^2} dx' \right)} = \frac{V_z'}{\gamma \left( 1 + \frac{v}{c^2} V_x' \right)} \]

If the motion is in one dimension, say along the \( x \) direction, then \( V_x = V, \quad V_y = V_z = 0 \). In this case, the velocity transformation can be expressed as

\[ V = \frac{V' + v}{1 + V' \frac{v}{c^2}} \]

If \( V' = c \), that is, if light is emitted in moving frame \( S' \) in its direction of motion relative to \( S \), from the above velocity transformation equation, the speed \( V \), as measured by an observer in frame \( S \), is obtained as \( V = c \). Thus, velocity of light remains unchanged in all inertial frames - i.e., in both rest and moving inertial frames.

If \( v = c \), then \( V = c \Rightarrow V' = c \) (from the above relation). Thus if the observer travels with light velocity, the velocities of objects recorded will be equal to light velocity.

If \( v/c \ll 1 \), then \( V = V' + v \) which is the Galilean result.
4.3 Michelson Morley Experiment

During Newton and Huygens era, light waves were considered as longitudinal waves and that they needed a medium to travel. This led to the introduction of ether, an elastic medium in which light propagates. Later, Maxwell established transverse nature of light. Still there were aberrations in the light received from stars on earth. This discrepancy was explained only by the presence of ether that drags the light rays. Experiments were proposed to measure the amount of drag produced by ether.

The experimental setup is as shown below.

![Michelson-Morley experimental setup](image)

**Fig: Michelson-Morley experimental setup**

The light rays from source S are made parallel by using lens L and further split into two perpendicular beams by using partially silvered mirror G placed at 45° to the incident beam.

The reflected and refracted light rays are reflected from mirrors $M_1$ and $M_2$ and reach back the mirror G.

The interference of the light rays received after reflection from $M_1$ and $M_2$ can be observed in the telescope T.

Let $d$ be the separation between G and $M_1, M_2$ mirrors. Consider the earth to be moving, from left to right, with respect to the setup, with velocity $v$. 
4.3 ரேகர் மதிப்பின் கணிப்பு

4.3 ரேகர் மதிப்பின் கணிப்பு

புனிதமான மறுத்துணர் சேர்க்கிறது, ரேகர் செயல்பாடு திறன் கணிப்பியல் முறை மற்றும் புனிதத்தின் முறை விளக்கம் வேண்டும். இதற்கு புனிதமான மறுத்துணர் சேர்க்கிறது, புனிதமான மறுத்துணர் சேர்க்கிறது, புனிதமான மறுத்துணர் சேர்க்கிறது. அதன் செயல்பாடு திறன் கணிப்பியல் முறை விளக்கம் வேண்டும். ரேகர் செயல்பாடு திறன் கணிப்பியல் முறை விளக்கம் வேண்டும். புனிதமான மறுத்துணர் சேர்க்கிறது, புனிதமான மறுத்துணர் சேர்க்கிறது, புனிதமான மறுத்துணர் சேர்க்கிறது. 

3.258

M_1, M_2 அல்லது G மறை சேர்க்கிறது, V_1, V_2 அல்லது G மறை சேர்க்கிறது.

\[ t_2 = \frac{d}{c-v} + \frac{d}{c+v} = d \left( \frac{c+v-c+v}{c^2-v^2} \right) = \frac{2dc}{c^2-v^2} = \frac{2d}{c \left( 1 - \frac{v^2}{c^2} \right)} \]

\[ t_2 = \frac{2d}{c \left( 1 - \frac{v^2}{c^2} \right)} = \frac{2d}{c \left( 1 - \frac{v^2}{c^2} \right)}^{-1} = \frac{2d}{c \left( 1 + \frac{v^2}{c^2} \right)} \]

M_1, M_2 அல்லது G மறை சேர்க்கிறது, V_1, V_2 அல்லது G மறை சேர்க்கிறது, அதன் செயல்பாடு திறன் கணிப்பியல் முறை விளக்கம் வேண்டும்.
Then during reflection on $M_2$, the light rays undergo a change in velocity of $v$ which is additive in one way and subtractive in the other way.

The time taken for the reflection on mirror $M_2$ is given by

$$t_2 = \frac{d}{c-v} + \frac{d}{c+v} = d\left(\frac{c+v-c+v}{c^2-v^2}\right) = \frac{2dc}{c^2-v^2} = \frac{2d}{c\left(1-\frac{v^2}{c^2}\right)}$$

For smaller values of velocity $v$, one can write a binomial expansion for the denominator to obtain

$$t_2 = \frac{2d}{c\left(1-\frac{v^2}{c^2}\right)} = \frac{2d}{c\left(1-\frac{v^2}{c^2}\right)^{-1}} = \frac{2d}{c\left(1 + \frac{v^2}{c^2}\right)}$$

During reflection on mirror $M_1$, the light rays travel in diagonal direction due to ether drag. The resultant velocity of light rays is given by $\sqrt{c^2 - v^2}$. Then the total time of travel for the reflection on mirror $M_1$ is given by

$$t_1 = \frac{2d}{\sqrt{c^2-v^2}} = \frac{2d}{c\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

By taking binomial expansion for lower velocities of earth compared to $c$, one obtains

$$t_1 = \frac{2d}{c}\left(1-\frac{v^2}{c^2}\right)^{-1/2} = \frac{2d}{c}\left(1 + \frac{1v^2}{2c^2}\right)$$

Total time lapse produced between the two light rays is given by

$$\Delta t = t_2 - t_1 = \frac{2d}{c}\left[\left(1 + \frac{v^2}{c^2}\right) - \left(1 + \frac{1v^2}{2c^2}\right)\right] = \frac{dv^2}{c^3}$$

The corresponding path difference is

$$\Delta = c\Delta t = \frac{dv^2}{c^2}$$
4.3 మాధ్యమం యొక్క స్థితి క్రూరతా ఆగ్రహం

\[ t_1 = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d}{c \sqrt{1 - \frac{v^2}{c^2}}} \]

చిత్రం ప్రకారం నిర్దిష్ట విద్యుత్తం లోపం లో అమలం చేయడానికి మాధ్యమం యొక్క క్రూరతా ఆగ్రహం ఆధారంగా,

\[ t_1 = \frac{2d}{c} \left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}} = \frac{2d}{c} \left(1 + \frac{1}{2} \frac{v^2}{c^2}\right) \]

ఉదాహరణ కోసం మాధ్యమం యొక్క క్రూరతా ఆగ్రహం, అమలించినంతి సమయం సమయం ప్రామాణికం అయితే,

\[ \Delta t = t_2 - t_1 = \frac{2d}{c} \left[\left(1 + \frac{v^2}{c^2}\right) - \left(1 + \frac{1}{2} \frac{v^2}{c^2}\right)\right] = \frac{d v^2}{c^3} \]

ఉదాహరణ కృతిదిక్కి దృశ్యం

\[ \Delta = c\Delta t = \frac{d v^2}{c^2} \]

వితర్లచ పరిమితం కోసం అమలించినంతి విద్యుత్తం లోపం లో అమలం చేయాలంటే సమయం సమయం ప్రామాణికం అయితే,

\[ n = \frac{\Delta}{\lambda} = \frac{d v^2}{c^2 \lambda} \]

మాధ్యమం పరిమితం కోసం 90° కోణం అంచులలో చెందినంతి అమలించాలంటే మాధ్యమం వద్ద యొక్క విద్యుత్తం లోపం లో అంచులకు అమలించడానికి సమయం సమయం ప్రామాణికం అయితే,

\[ n = \frac{2d v^2}{c^2 \lambda} \]

అవి వ్యాఖ్యాతిపెట్టారు, \[ v = 3 \times 10^4 m/sec \], \[ c = 3 \times 10^8 m/sec \], \[ \lambda = 5500\text{Å} \], \[ d = 11m \].

ఉదాహరణ కృతి

\[ n = \frac{2 \times 11 \times (3 \times 10^4)^2}{(3 \times 10^8)^2 \times 5500 \times 10^{-1} \text{Å}} = 0.4 \]

ఉదాహరణ కృతి పరిమితం కోసం అమలించడానికి సమయం సమయం ప్రామాణికం అయితే రెండు విద్యుత్తంలో మాధ్యమం యొక్క క్రూరతా ఆగ్రహం పరిమితం కోసం అమలించడానికి సమయం సమయం ప్రామాణికం అయితే.
If \( \lambda \) is the wavelength of the light used, the number of fringes shifted is given by

\[
n = \frac{\Delta}{\lambda} = \frac{d \nu^2}{c^2 \lambda}
\]

The entire experimental setup is rotated by 90°, and experiment is repeated in order to avoid the errors due to asymmetry in the two arms of the interferometer. Then the total number of fringes is given by

\[
n = \frac{2d \nu^2}{c^2 \lambda}
\]

Here \( \nu = 3 \times 10^4 \text{ m/sec} \), \( c = 3 \times 10^8 \text{ m/sec} \), \( \lambda = 5500 \text{ Å} \), \( d = 11 \text{ m} \).

With this substitution,

\[
n = \frac{2 \times 11 \times (3 \times 10^4)^2}{(3 \times 10^8)^2 \times 5500 \times 10^{-1}} = 0.4
\]

Thus a fringe shift of 0.4 was predicted but in the experiment, no fringe shift was reported throughout the year during various seasons. Thus the concept of ether drag was abolished.

The real reason for the star light aberration was given by Einstein using his special theory of relativity and by using Lorentz transformations as shown below. Due to relative motion between source (star) and observer (earth), by the time light reaches earth, the actual position of star shifts but from observer point of view, light seems to be coming from an apparent position.

**Fig: Star light aberration explained using special theory of relativity.**
4.4 పట్టిక ప్రక్రియ వివరణ మార్గం

భావించి కాదు పట్టిక ప్రక్రియ నిర్ధారించడం తరువాత మామండి మధ్యంగా వ్యాపార కార్యాల సమయం మార్గాలను ప్రతిష్ఠించడం అవసరం. వాటి మధ్య ప్రత్యేక సమయం మార్గాలు మామండి మధ్యంగా వ్యాపార కార్యాల సమయం మార్గాలను ప్రతిష్ఠించడం సాధారణ అవసరం. ఆ సమయం మార్గాలను వాటి మధ్య ప్రత్యేక సమయం మార్గాలు మామండి మధ్యంగా వ్యాపార కార్యాల సమయం మార్గాలను ప్రతిష్ఠించడం సాధారణ అవసరం.

4.4 పట్టిక ప్రక్రియ వివరణ మార్గం

పట్టిక ప్రక్రియ మార్గాన్ని సమయం మార్గాలు వివరిస్తుంది.

పట్టిక ప్రక్రియ మార్గాన్ని సమయం మార్గాలు వివరిస్తుంది. మామండి ప్రత్యేక సమయం మార్గాలను వాటి మధ్య ప్రత్యేక సమయం మార్గాలు మామండి మధ్యంగా వ్యాపార కార్యాల సమయం మార్గాలను ప్రతిష్ఠించడం సాధారణ అవసరం. ఆ సమయం మార్గాలను వాటి మధ్య ప్రత్యేక సమయం మార్గాలు మామండి మధ్యంగా వ్యాపార కార్యాల సమయం మార్గాలను ప్రతిష్ఠించడం సాధారణ అవసరం.

4.5 సమయం సమయం

మామండి A మామలు కేసియుంది రాచనా కార్యాలు, L మామలు కేసి వ్యాపార కార్యాలు సమయం మార్గాలు వివరిస్తుంది. A మామలు B మామలు సమయం మార్గాలు, ఒ మామలు రాచనా కార్యాలు, A మామలు B మామలు, B మామలు A మామలు కార్యాలు సమయం మార్గాలు

\[ \Delta t = \frac{2L}{c} \]  

మామలు కేసియుంది సమయం మార్గాలు వివరిస్తుంది రాచనా కార్యాలు మామలు. కేసి మామలు క్రమంగా మామలు, ఒ మామలు రాచనా కార్యాలు సమయం మార్గాలు, అంటే \( \Delta t'/2 \) సమయం మార్గాలు వివరిస్తుంది. ఆ సమయం మార్గాలు క్రమంగా రాచనా కార్యాలు సమయం మార్గాలు, అంటే \( \Delta t' \) సమయం మార్గాలు వివరిస్తుంది.

\[ \Delta t' = \frac{2D}{c} \]  

మామలు కేసి రాచనా కార్యాలు సమయం మార్గాలు వివరిస్తుంది.
4.4 Postulates of special theory of relativity

There are two postulates of the special theory of relativity.

1. The basic laws of Physics are invariant in all inertial frames of reference. i.e., in the frames which have constant or zero relative velocity.
2. Light velocity remains constant in all inertial frames of reference.

The first postulate implies, there are no absolute frames of reference in which one can define absolute motion of objects. If inertial frames of references are considered, the measurements made in one frame of reference may be converted into the other one by using Lorentz transformations.

The second postulate implies that the velocity of light remains constant and is equal to $c$ in all inertial reference frames of reference. This statement was proved, with strong experimental evidence.

4.5 Time dilation

Consider a light ray starting from source A, traveling on to a mirror B, separated by a distance L and placed parallel to A. If $c$ is the velocity of light, then the time of round trip travel from A to B and from B to A is

$$\Delta t = \frac{2L}{c}$$  \hspace{1cm} (1)

Let the entire system be travelling with a velocity $v$ with respect to the observer. In that case, light takes a longer path. By the time the light ray reaches the mirror, the mirror might have been displaced horizontally by a distance $v(\Delta t’/2)$ where $\Delta t’$ is the new time of travel for the entire journey, given by

$$\Delta t’ = \frac{2D}{c}$$  \hspace{1cm} (2)

Here velocity of light is assumed to be equal in both the cases.
4.5 Time dilation

\[ D = \sqrt{L^2 + \left(\frac{v\Delta t'}{2}\right)^2} \quad -- \quad (3) \]

Eq.(2) & Eq.(3) $\Rightarrow$ Eq.(1) $\Rightarrow$

\[ \Delta t'^2 = \frac{4D^2}{c^2} = \frac{4(L^2 + \frac{v^2\Delta t'^2}{4})}{c^2} = \frac{4L^2 + v^2\Delta t'^2}{c^2} = \frac{(2L)^2}{c^2} + \frac{v^2\Delta t'^2}{c^2} \]

\[ = \Delta t^2 + \frac{v^2\Delta t'^2}{c^2} \]

\[ \Rightarrow \Delta t^2 = \Delta t'^2 \left(1 - \frac{v^2}{c^2}\right) = \frac{\Delta t'^2}{\gamma^2} \]

Or

\[ \Delta t' = \gamma \Delta t = \frac{\Delta t}{\sqrt{1 - v^2/c^2}} \]

$v<c, \gamma >1 \Rightarrow \Delta t'^2 > \Delta t^2$. 

ఎంపాడిన సమయం చాలా కుదురుగాను, కాబట్టి పాటు గుండ్యించడానికి అసలు మాధ్యమం కనుక్కని యుగం అందుచేసింది. 

మాంగి ఆచారాలను, మాంగి పంచాయతె, మాంగి వనరువులను ఏం trolling యుగం ఆచారాలను అందుకునే ఉంచడానికి యుగం అందుచేసింది. ఈ ఆచారాలను గోర్పా లందన్యా (ప్రపంచోధనం) అంతరం. రెండు సమయాలు ప్రత్యేకించడానికి విషయం మాంగి యుగం లేదు. 

ఎంపాడిన సమయం చాలా కుదురుగాను, ఎంపాడిన సమయం చాలా కుదురుగాను అందుచేసింది, కాబట్టి పాటు గుండ్యించడానికి యుగం అందుచేసింది.
From the figure,

\[ D = \sqrt{L^2 + \left(\frac{v\Delta t'}{2}\right)^2} \quad \ldots \quad (3) \]

Substituting Eq. (3) and Eq. (1) in Eq. (2) gives

\[ \Delta t'^2 = \frac{4D^2}{c^2} = \frac{4\left(L^2 + \frac{v^2\Delta t'^2}{4}\right)}{c^2} = \frac{4L^2 + v^2\Delta t'^2}{c^2} = \frac{(2L)^2}{c^2} + \frac{v^2\Delta t'^2}{c^2} \]

\[ = \Delta t^2 + \frac{v^2\Delta t'^2}{c^2} \]

\[ \Rightarrow \Delta t^2 = \Delta t'^2 \left(1 - \frac{v^2}{c^2}\right) = \frac{\Delta t'^2}{\gamma^2} \]

Or

\[ \Delta t' = \gamma \Delta t = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}} \]

Since \( v < c, \gamma > 1 \Rightarrow \Delta t' > \Delta t \).

Thus for the external observer, the events in the moving frame seem to occur at longer intervals. In other words, time intervals between events seems to be stretched or dilated. This phenomenon is called time dilation (widening). This issue arises because, the light velocity doesn’t change with frame of reference.

Here the time \( \Delta t \), that is measured in the rest frame of the body is called the proper time of the system.
4.6 సాధనం నిర్ణయస్త్వం

మేము సమాధానం చేయడానికి 60 మంది మనుష్యపు ప్రాంతంలో దాని, మరుదూరి ప్రాంతాలు ప్రతి ప్రాంతాలకు ప్రతి గాని ప్రతి ప్రాంతాలు ప్రతి వంటి ప్రాంతాలు ప్రతి ప్రతి ప్రాంతాలకు ప్రతి గాని ప్రతి ప్రాంతాలకు ప్రతి ప్రతి ప్రాంతాలు ప్రతి ప్రాంతాలు ప్రతి ప్రతి ప్రాంతాలకు ప్రతి గాని ప్రతి ప్రాంతాలకు ప్రతి ప్రతి ప్రాంతాలకు ప్రతి గాని ప్రతి ప్రాంతాలకు ప్రతి ప్రతి ప్రాంతాలకు ప్రతి గాని 

\[ l = \frac{c \Delta t}{2} \quad \cdots \cdots \text{(1)} \]

మనం వాటి ప్రాంతాల సంచానం రంగు ప్రాంతం కంటులు, యొక్క ప్రాంతాల కంటులు వాటి వంటి ప్రాంతాల కంటులు వాటి వంటి ప్రాంతాల కంటులు వాటి వంటి ప్రాంతాల కంటులు వాటి వంటి 

\[ l' = \frac{c \Delta t'}{2} \quad \cdots \cdots \text{(2)} \]

కాని వాటి ప్రాంతాల కంటులు తన ప్రాంతాల కంటులు వాటి వంటి ప్రాంతాల కంటులు వాటి వంటి 

\[ \Delta t' = \frac{l'}{c + v} + \frac{l'}{c - v} = l' \left( \frac{1}{c + v} + \frac{1}{c - v} \right) = l' \left( \frac{2c}{c^2 - v^2} \right) \]

\[ = \frac{2l'}{c(1 - v^2/c^2)} \quad \cdots \cdots \text{(3)} \]

మేము వాటి ప్రాంతాల కంటులు తన ప్రాంతాల కంటులు 

\[ \Delta t' = \frac{\Delta t}{\sqrt{1 - v^2/c^2}} \quad \cdots \cdots \text{(4)} \]

Eq (3) మరియు Eq.(4) పు ప్రాంతాల కంటులు మరియు Eq.(1) పు ప్రాంతాల కంటులు 

\[ \Delta t' = \frac{\Delta t}{\sqrt{1 - v^2/c^2}} = \frac{2l'}{c(1 - v^2/c^2)} \Rightarrow l = \frac{c \Delta t}{2} = \frac{l' \sqrt{1 - v^2/c^2}}{(1 - v^2/c^2)} \]

ప్రత్యేకంగా 

\[ l = \frac{l'}{\sqrt{1 - v^2/c^2}} \]

మేము 

\[ l' = l \sqrt{1 - v^2/c^2} = \frac{l}{\gamma} \]
Consider that a laser light is used to measure the length of a scale by reflecting the light at the other end and measuring the to and fro time of travel $\Delta t$. The length of the scale is given by

\[ l = \frac{c\Delta t}{2} - - - (1) \]

Let the entire system move along the length of the object (scale) with velocity $v$. For this case, let the travel time be $\Delta t'$. Then the new length measured is given by

\[ l' = \frac{c\Delta t'}{2} - - - (2) \]

By the time the light ray reaches the end of the scale, the tip of the scale would have moved by a distance $v\Delta t'$. Hence to reach the tip, the light ray has to travel that extra distance. Also in the return path, the light ray has to travel a distance reduced by $v\Delta t'$ as the back tip of the scale moves forward by that distance within the time $\Delta t'$. Thus

\[ \Delta t' = \frac{l'}{c + v} + \frac{l'}{c - v} = l'\left(\frac{1}{c + v} + \frac{1}{c - v}\right) = l'\left(\frac{2c}{c^2 - v^2}\right) \]

\[ = \frac{2l'}{c(1 - v^2/c^2)} - - - (3) \]

The time dilation equation is given by

\[ \Delta t' = \frac{\Delta t}{\sqrt{1 - v^2/c^2}} - - - (4) \]

Substituting Eq. (4) in Eq. (3) and by using Eq. (1) gives

\[ \Delta t' = \frac{\Delta t}{\sqrt{1 - v^2/c^2}} = \frac{2l'}{c(1 - v^2/c^2)} \Rightarrow l = \frac{c\Delta t}{2} = \frac{l'\sqrt{1 - v^2/c^2}}{(1 - v^2/c^2)} \]

Simplifying

\[ l = \frac{l'}{\sqrt{1 - v^2/c^2}} \]

or
4.6 Length Contraction

\[ v < c, \gamma > 1 \Rightarrow l' < l \]

\[ \text{http://www.trell.org/div/minkowski.html} \]

\[ \text{https://www.thinkib.net/physics/page/18370/optional-practical-space-time-diagram-geogebra} \]

\[ \text{https://www.youtube.com/watch?v=0iJZ_QGMLD0} \]

\[ \text{https://www.youtube.com/watch?v=5qQheJn-FHc} \]
4.7 Relativistic mass

\[ l' = l \sqrt{1 - \frac{v^2}{c^2}} = \frac{l}{\gamma} \]

Since \( v < c, \gamma > 1 \Rightarrow l' < l \)

Thus the length of the object in the moving frame appears shorter for an observer in the rest frame. It is to be noted that this length contraction doesn’t occur in a direction perpendicular to the motion of the object.

4.7 Relativistic mass

Consider an elastic collision, in Y-direction, between two identical particles. Here Y-direction is chosen to avoid length contraction effects along X-direction. Consider two frames of reference \( S \) and \( S' \) corresponding to particle-1 and particle-2 respectively. Here \( S \) is stationary with the system and \( S' \) is moving with velocity \( v \) along X-direction. Let the velocities be in the relativistic limit. Let \( V_1 \) be the velocity of the first particle and \( V_2' \) be the velocity of second particle as seen from their respective frames. Let these two velocities be equal,

\[ V_1 = V_2' \quad - - - (1) \]

Fig: Relativistic mass in two particle collision.

so that after elastic collision final velocities of particles remain identical.
4.7 స్థాయి సంఖ్యలు

యోధాత్మకంగా, వైద్య విభాగంలో సంఖ్యలు ప్రాంతంలో అప్పటి శాస్త్ర మాధ్యమాలలో యోగ్యత్వం. X-సరిస్పుట్టిన సంఖ్యలు ఒక ప్రామాణికంగా అవి Y-సరిస్పుట్టిన సంఖ్యలు. ప్రారంభంలో X_1 మందము ప్రోకటి-2 సంఖ్యలు S మందము S' ప్రోకటి మందము ప్రోకటి. అంటే S అంధకారం సపండు లేకుండా S' X-సరిస్పుట్టిన సంఖ్యలు. ఈ పరిస్థితి యోగ్యత్వం ఉంంది. V_1 అండ్ V_2 అండ్ V_2' అండ్ ప్రోకటిస్పుట్టిన సంఖ్యలు యోగ్యత్వం ఉంంది. ఈ పరిస్థితి యోగ్యత్వం ఉంంది.

ప్రారంభం లో సమానంగా అనుసరించి స్థాయి ఎక్కడ ఉంంది.

2Y అవి తో సంఖ్యలు ప్రామాణికంగా, సరిస్పట్టిన Y అవి యోగ్యత్వం. T_0 అవి తో యోగ్యత్వం ఉంంది. అందువల్ల,

\[ T_0 = \frac{Y}{V_1} = \frac{Y}{V_2'} \quad -(2) \]

మిగిలిన సమయం యోగ్యత్వం ఉంంది. V_2 అండ్ V_2' అండ్ సమయం యోగ్యత్వం. అందువల్ల అనుసరించాడు.

\[ T = \frac{Y}{V_2} \quad -(3) \]

ఎందుకు T T_0 యోగ్యత్వం ఉంంది.

\[ T = \frac{T_0}{\sqrt{1 - v^2/c^2}} \quad -(4) \]

మిగిలిన సమయం యోగ్యత్వం ఉంంది. అనుసరించాడు. P = m_1V_1 = m_2V_2

\[ \Rightarrow m_1V_1 = m_2V_2 = m_2 \frac{Y}{T} = \frac{m_2Y}{T_0} \sqrt{1 - v^2/c^2} = m_2V_1\sqrt{1 - v^2/c^2} \]

\[ \Rightarrow m_2 = \frac{m_1}{\sqrt{1 - v^2/c^2}} \]

అందుకు m_1 అండ్ m_2 యోగ్యత్వం ఉంంది. ఇందులో సమానంగా మాత్రమే మ_2 అవి తో యోగ్యత్వం

\[ \Rightarrow m = \frac{m_0}{\sqrt{1 - v^2/c^2}} = \gamma m_0 \quad -(5) \]
Let $2Y$ be the separation between the two particles, so that the collision occurs at a distance of $Y$. Let $T_0$ be the time for collision. Then, one can write,

$$T_0 = \frac{Y}{V_1} = \frac{Y}{V_2^*} \quad (2)$$

Let $V_2$ be the velocity of second particle in the first particle’s frame of reference. Then the time of travel in that frame will be

$$T = \frac{Y}{V_2} \quad (3)$$

where $T$ is related to $T_0$ as

$$T = \frac{T_0}{\sqrt{1 - v^2/c^2}} \quad (4)$$

Applying the law of conservation of momentum in the first particle’s frame, one obtains

$$P = m_1 V_1 = m_2 V_2$$

$$\Rightarrow m_1 V_1 = m_2 V_2 = m_2 \frac{Y}{T} = \frac{m_2 Y}{T_0} \sqrt{1 - v^2/c^2} = m_2 V_1 \sqrt{1 - v^2/c^2}$$

$$\Rightarrow m_2 = \frac{m_1}{\sqrt{1 - v^2/c^2}}$$

Here $m_1$ is the mass of the particle in its own frame and $m_2$ is the mass of the particle as observed from moving frame. Hence one can replace them with $m_0$ and $m$ respectively. Thus the equation becomes

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} = \gamma m_0 \quad (5)$$

The relativistic momentum is given by

$$p = \frac{m_0 v}{\sqrt{1 - v^2/c^2}} = \gamma m_0 v \quad (6)$$

and the relativistic form of Newton’s second law is given by
4.8 विद्युत दृष्टिकोण, द्विभाजन विद्यमान

द्विभाजन विद्यमान

\[ p = \frac{m_0 v}{\sqrt{1 - v^2/c^2}} = \gamma m_0 v \quad \cdots (6) \]

\[ F = \frac{dp}{dt} = \frac{d}{dt} \left( \frac{m_0 v}{\sqrt{1 - v^2/c^2}} \right) \quad \cdots (7) \]

**Activity**

Obtain expression for relativistic acceleration and plot it as a function of velocity.

4.8 विद्युत दृष्टिकोण, द्विभाजन विद्यमान

\[ dW = F. dx = m \frac{dv}{dt} \cdot dx = m \frac{dv}{dt} \cdot dx = mvdv \]

\[ W_{12} = \int_1^2 dW = \int_1^2 F. dx = \int_1^2 mvdv = \frac{1}{2}mv^2 \bigg|_1^2 = K_2 - K_1 \]

\[ K = \int F. dx \quad \cdots (1) \]

\[ \frac{dP}{dt} = \frac{d}{dt} \left( \gamma m_0 v \right) = m_0 \frac{d}{dt} \left( \gamma v \right) \quad \cdots (2) \]

Thus

\[ K_{rel} = \int F. dx = \int m_0 \frac{d}{dt} \left( \gamma v \right) \cdot dx = \int m_0 \frac{dx}{dt} \cdot d(\gamma v) = m_0 \int v \cdot d(\gamma v) \]

\[ \int udv = uv - \int vdu \quad \text{संगत} \]

272
4.8 Relativistic Kinetic Energy, Mass-Energy relation

In classical mechanics, if a force produces displacement in an object, then the work done on the object is

\[ dW = F \cdot dx = m \frac{dv}{dt} \, dx = m \, dv \frac{dx}{dt} = mvdv \]

so that the total work done between two points is

\[ W_{1 \rightarrow 2} = \int_{1}^{2} dW = \int_{1}^{2} F \cdot dx = \int_{1}^{2} mvdv = \left. \frac{1}{2} m v^2 \right|_{1}^{2} = K_2 - K_1 \]

Thus the work done to produce displacement in an object, between two points, results in change of kinetic energy of the system. This is called Work-Energy theorem.

One can write Kinetic energy as a spatial integral over force.

\[ K = \int F \cdot dx \quad -- (1) \]

In special theory of relativity,
4.8 Relativistic Kinetic Energy, Mass-Energy relation

\[ K_{rel} = m_0 \left( \gamma v^2 - \gamma v \nu \right) - m_0 \int_0^\nu \frac{v}{\sqrt{1 - v^2/c^2}} dv \]

By substituting \( \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \)

\[ u = 1 - \frac{v^2}{c^2} \Rightarrow du = -\frac{2v}{c^2} dv \]

Thus

\[ \int -\frac{c^2}{2} \frac{1}{\sqrt{u}} du = -\frac{c^2}{2} \left( 2\sqrt{u} \right) = -c^2 \sqrt{u} = -\frac{c^2}{\gamma} \]

\[ K = \gamma m_0 v^2 + \frac{m_0 c^2}{\gamma} \right|_0^\nu = \gamma m_0 v^2 + \frac{m_0 c^2}{\gamma} - m_0 c^2 \]

\[ \Rightarrow K_{rel} = \frac{m_0 v^2}{\sqrt{1 - v^2/c^2}} + m_0 c^2 \sqrt{1 - v^2/c^2} - m_0 c^2 \]

\[ = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} - m_0 c^2 = \gamma m_0 c^2 - m_0 c^2 = mc^2 - m_0 c^2 \]

Or

\[ \Rightarrow K_{rel} = (\gamma - 1)m_0 c^2 = mc^2 - m_0 c^2 = E - E_0 \quad - - - (3) \]

\[ \Rightarrow E = E_0 + K \]

When \( E = m_0 c^2 \) and \( v \ll c \), then \( \gamma \approx 1 \) and \( K \approx (\gamma - 1)m_0 c^2 \)

\[ E = mc^2 = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} \quad - - - (4) \]

Thus the energy of a particle is given by Eq. (3) for \( v \ll c \).
4.8 Relativistic Kinetic Energy, Mass-Energy relation

\[ F = \frac{dP}{dt} = \frac{d}{dt}(\gamma m_0 v) = m_0 \frac{d}{dt}(\gamma v) \quad - - - (2) \]

Thus

\[ K_{rel} = \int F \, dx = \int m_0 \frac{d}{dt}(\gamma v) \, dx = \int m_0 \frac{dx}{dt} \, d(\gamma v) = m_0 \int v \, d(\gamma v) \]

We know \( \int udv = uv - \int vdu \). Thus

\[ K_{rel} = m_0 \left( \gamma \gamma v - \int \gamma \gamma dv \right) = m_0 \gamma v^2 - m_0 \int_0^\gamma \frac{v}{\sqrt{1 - v^2/c^2}} \, dv \]

Consider a change of variable defined as

\[ u = 1 - \frac{v^2}{c^2} \quad \Rightarrow \quad du = -\frac{2v}{c^2} \, dv \]

Then the integral becomes,

\[ \int -\frac{c^2}{2} \frac{1}{\sqrt{u}} \, du = -\frac{c^2}{2} (2\sqrt{u}) = -c^2 \sqrt{u} = -\frac{c^2}{\gamma} \]

Thus

\[ K = \gamma m_0 v^2 + \frac{m_0 c^2}{\gamma} \bigg|_0 = \gamma m_0 v^2 + \frac{m_0 c^2}{\gamma} - m_0 c^2 \]

\[ \Rightarrow K_{rel} = \frac{m_0 v^2}{\sqrt{1 - v^2/c^2}} + m_0 c^2 \sqrt{1 - v^2/c^2} - m_0 c^2 \]

\[ = \left( m_0 v^2 + m_0 c^2 \left( 1 - \frac{v^2}{c^2} \right) \right) \frac{1}{\sqrt{1 - v^2/c^2}} - m_0 c^2 \]

\[ = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} - m_0 c^2 = \gamma m_0 c^2 - m_0 c^2 = mc^2 - m_0 c^2 \]

Or

\[ \Rightarrow K_{rel} = (\gamma - 1)m_0 c^2 = mc^2 - m_0 c^2 = E - E_0 \quad - - - (3) \]

\[ \Rightarrow E = E_0 + K \]
4.8 Relativistic Kinetic Energy, Mass-Energy relation

\[ K_{rel} = (\gamma - 1)m_0c^2 = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} - 1 \] \[ m_0c^2 \]

\[ \simeq \left(1 + \frac{v^2}{2c^2} - 1\right) m_0c^2 = \frac{1}{2}mv^2 = K_{\text{classical}} \]

When \( v/c \to 0 \) the above relation reduces to the classical kinetic energy relation.


Here $E_0 = m_0 c^2$ is the rest mass energy of the system and $K = (\gamma - 1)m_0 c^2$ represents the kinetic energy of the system. Thus

$$E = mc^2 = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}}$$

represents the total energy of the system. This equation gives the mass – energy equivalence of the system.

For small values of velocity ($v \ll c$), one can expand Eq. (3), the expression for relativistic kinetic energy, binomially.

$$K_{rel} = (\gamma - 1)m_0 c^2 = \left[ \left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}} - 1 \right] m_0 c^2$$

$$\approx \left(1 + \frac{v^2}{2c^2} - 1\right) m_0 c^2 = \frac{1}{2} m v^2 = K_{\text{classical}}$$

Thus in the limit $v/c \to 0$ one can observe that the relativistic kinetic energy coincides with the classical kinetic energy.

**Relativistic energy-momentum relation**

We have

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} \quad \text{--- (1)}$$

$$p = mv = \frac{m_0 v}{\sqrt{1 - v^2/c^2}} \quad \text{--- (2)}$$

Here we have 2 equations, with $E, m_0, c, v, p$ as variables. Since the expected end result doesn’t contain $v$, it should be eliminated from these two. Since, there are square roots that carry $v$, square them and eliminate $v$. Thus

$$\text{Eq. (1)} \Rightarrow E^2(c^2 - v^2) = m_0^2 c^6 \quad \text{--- (3)}$$

$$\text{Eq. (2)} \Rightarrow p^2(c^2 - v^2) = m_0^2 v^2 c^2 \quad \text{--- (4)}$$

$$\text{(3)} - \text{(4)}c^2 \Rightarrow (E^2 - p^2 c^2)(c^2 - v^2) = m_0^2 c^4(c^2 - v^2)$$

or

$$E^2 = p^2 c^2 + m_0^2 c^4 \quad \text{--- (5)}$$

This is the relativistic energy – momentum relation.
Further reading:

1. The amount of work done to increase the speed of an electron from $c/3$ to $2c/3$ is

\[ c = 3 \times 10^8 \text{ m/s} \quad \text{and rest mass of electron is 0.511 MeV} \]

\textbf{[IIT JAM 2019]}

(a) 56.50 keV  (b) 143.58 keV  (c) 168.20 keV  (d) 511.00 keV

Ans. The change in kinetic energy is equal to work done

\[
W = \left( \frac{m_o c^2}{\sqrt{1 - \frac{v_2^2}{c^2}}} - m_o c^2 \right) - \left( \frac{m_o c^2}{\sqrt{1 - \frac{v_1^2}{c^2}}} - m_o c^2 \right)
\]

\[
W = m_o c^2 \left( \frac{1}{\sqrt{1 - \frac{v_2^2}{c^2}}} - \frac{1}{\sqrt{1 - \frac{v_1^2}{c^2}}} \right)
\]

Given that

\[ v_1 = c/3 \quad \text{and} \quad v_2 = (2/3)c \]

\[ m_o c^2 = 0.511 \text{ MeV} \]

Therefore

\[
W = 0.511 \times 10^6 \left( \frac{1}{\sqrt{1 - \frac{4}{9} \frac{c^2}{c^2}}} - \frac{1}{\sqrt{1 - \frac{1}{9} \frac{c^2}{c^2}}} \right)
\]

\[
W = 3 \times 0.511 \times 10^6 \left( \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{8}} \right)
\]

\[ W = 143.58 \text{ keV} \]
2. A rod is moving with a speed of 0.8c in a direction at 60° to its own length. Find the % contraction in the length of the rod. [IIT JAM 2015]

Ans. length components of the rod with angle of 60° are \( l_x = l_o \cos \theta \sqrt{1 - \frac{v^2}{c^2}} \)

and \( l_y = l_o \sin \theta \)

\( v = 0.8c \)

\( l_o \)

\( \theta \)

\( \theta = 60° \)

Given that \( v = 0.8c \) and \( \theta = 60° \)

Therefore

\[ l_x = l_o \cos 60° \sqrt{1 - \frac{(0.8c)^2}{c^2}} \]

\( l_x = l_o \times 0.6 \)

\( l_x = 0.3 \times l_o \)

And

\( l_y = l_o \sin 60° \)

\( l_y = l_o \)

The new length of the rod is given by

\[ l = \sqrt{l_x^2 + l_y^2} \]

\( l = \sqrt{(0.3 \times l_o)^2 + (l_o)^2} \)
Solved Problems and Exercises

\[ l = 0.916 \ l_o \]

\% contraction in the length of the rod = \( \frac{l_o - 0.91 \ d_o}{l_o} \times 100 \)

= 9%

3. A space crew has a life support system that can last only for 1000 hours. What minimum speed would be required for safe travel of the crew between two space stations separated by a fixed distance of \( 1.08 \times 10^{12} \ km \)?

Ans. The crew will take minimum distance when the relative distance traveled by the crew should be equal to the actual distance between the space stations.

Therefore

The relative distance traveled by the crew is given by

\[ \frac{vt}{\sqrt{1 - \frac{v^2}{c^2}}} \]

Where given that \( t = 1000 \times 3600 \) sec

And \( d = 1.08 \times 10^{12} \ km \)

\[ \frac{1000 \times 3600 \ v}{\sqrt{1 - \frac{v^2}{c^2}}} = 1.08 \times 10^8 \times 1000 \]

\[ \frac{36 \ v}{\sqrt{1 - \frac{v^2}{c^2}}} = 108 \times 10^8 \]

\[ \frac{v}{\sqrt{1 - \frac{v^2}{c^2}}} = 3 \times 10^8 \]

\[ \frac{v}{\sqrt{1 - \frac{v^2}{c^2}}} = c \]
The length of a rod, of length 5m in a frame of reference which is moving with 0.6 c velocity in a direction making 30\(^0\) angle with the rod is nearly? [BHU 2018]

Ans. The length of rod at rest = 5m

\[ v = \frac{c}{\sqrt{2}} \]

\[ l = l_o \cos \cos \theta \sqrt{1 - \frac{v^2}{c^2}} \]

\[ l = 5 \cos 30 \sqrt{1 - \left(0.6c\right)^2} \]

\[ l = 4.3m \]

If the kinetic energy of a body is twice its rest mass energy, what will be the ratio of relativistic mass to the rest mass of the body [BHU 2017]

Ans. Given that

\[ KE = 2m_o c^2 \]

But we know that

\[ KE = (\gamma - 1) m_o c^2 \]

\[ 2m_o c^2 = (\gamma - 1) m_o c^2 \]

\[ \gamma = 3 \]

But we know that

\[ m = \gamma m_o \]

\[ m/m_o = \gamma = 3 \]
6. From the nozzle of a rocket 100 kg of gases are exhausted per sec with a velocity of 1000 m/sec. What force (thrust) does the gas exert on the rocket? [BHU 2018]

Ans. Exhausted force \( N = \dot{u} \frac{dm}{dt} \)

Given that

\[
\frac{dm}{dt} = 100 \text{ kg/sec}
\]

\( u = 1000 \text{ m/sec} \)

Therefore

\[
N = 1000 \times 100
\]

\[
N = 10^5 \text{ N}
\]

7. Express for the momentum of a photon in terms of wavelength. How much is the rest mass of the photon. Calculate the relativistic mass of the photon of wavelength 5000 Å.

From Einstein relation, Momentum of the photon

\[
p = \frac{E}{c} = \frac{h\theta}{c} = \frac{h}{\lambda}
\]

From the relativistic theory

\[
p = \frac{m_o v}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

For photon \( v = c \)

Therefore

\[
m_o = \frac{p}{v} \sqrt{1 - \frac{v^2}{c^2}}
\]

Therefore

\[
m_o = 0
\]

It is the rest mass of the photon and is equal to zero.
Relativistic mass

\[ m = \frac{p}{c} = \frac{h}{c\lambda} \]

Given that \( \lambda = 5000\text{Å} \)

\[ m = \frac{6.627 \times 10^{-34}}{3 \times 10^8 \times 5000 \times 10^{-10}} \]

\[ m = 4 \times 10^{-36}\text{kg} \]

8. Find the rest mass of the particle of momentum \( p \) and kinetic energy \( T \). [Agra 1998, 90]

We know that the relativistic energy is given by

\[ E = E_k + E_o \]

\[ E = T + m_o c^2 \quad (1) \]

We also know that

\[ E^2 = p^2 c^2 + m_o^2 c^4 \quad (2) \]

Substituting equation (2) in (1)

\[ p^2 c^2 + m_o^2 c^4 = E^2 = (T + m_o c^2)^2 \]

\[ p^2 c^2 + m_o^2 c^4 = T^2 + 2Tm_o c^2 + m_o^2 c^4 \]

\[ m_o = \frac{p^2 - T^2}{2Tc^2} \]

9. A ship moving away from the earth with velocity 0.5c fires a rocket whose velocity to the space is 0.5c (a) away from the Earth (b) toward the Earth. Calculate the velocity of the rocket observed from the Earth in two cases. [IIT Kanpur 1988, Delhi 1994]

Ans. Let us assume that the velocity of the rocket as observed from the earth is \( u \)

(a) In the first case,

\[ u = \frac{u' + v}{1 + \frac{u' v}{c^2}} \]
Given that \( u' = 0.5c \) and \( v=0.5c \)

Therefore

\[
u = \frac{0.5c + 0.5c}{1 + \frac{0.5c \times 0.5c}{c^2}} = 0.8c
\]

(b) In the second case

Given that \( u' = -0.5c \) and \( v=0.5c \)

Therefore

\[
u = \frac{-0.5c + 0.5c}{1 + \frac{0.5c \times 0.5c}{c^2}} = 0
\]

10. A beam of particles of half life \( 2 \times 10^{-6} \) sec travels in the laboratory frame with speed \( 0.96c \). How much distance does the beam travel before the flux falls to \( \frac{1}{2} \) times the initial flux? [IIT Kanpur 1990]

Ans. Suppose \( \Delta\tau \) is the particle half life = \( 2 \times 10^{-6} \) sec

If \( \Delta t \) is the observed half life of the particles, then

\[
\Delta t = \frac{\Delta\tau}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

Laboratory frame speed is \( (v) = 0.9c \)

Therefore

\[
\Delta t = \frac{2 \times 10^{-6}}{\sqrt{1 - (0.96c)^2}}
\]
\[ \Delta t = \frac{2 \times 10^{-6}}{0.28} \text{ sec} \]

Evidently, in this observed half life the flux will fall to \( \frac{1}{2} \) times the initial flux.

Therefore, the distance travelled by the particles in this time is

\[ d = v \Delta t \]
\[ d = 0.9 \times 3 \times 10^8 \times \frac{2 \times 1 \times 10^6}{0.28} \]
\[ d = 2000\text{m} \]

**MCQs**

1. Consider a rotating spherical planet such that the effective gravitational attraction at the equator is only 75% of that at the pole. If the linear velocity of a point on the equator is \( v_o \), what is the escape velocity for a polar particle? [DU 2018]
   
   (a) \( v_o \)  
   (b) \( 2v_o \)  
   (c) \( \sqrt{8}v_o \)  
   (d) \( \sqrt{7}v_o \)

   Ans. c

2. If a particle is at rest relative to an observer at rest at the center of a rotating frame of reference [BHU 2018]
   
   (a) centrifugal and Coriolis forces both act  
   (b) only centrifugal force acts  
   (c) only Coriolis force acts  
   (d) None of these

   Ans. b

3. a meson decays into a meson and a neutrino with a mean lifetime of about \( 2.5 \times 10^{-8} \) sec in a frame in which it is at rest. If the velocity of the mesons in the laboratory frame be \( 0.9c \) then the expected lifetime in this frame is [BHU 2018]
   
   (a) \( 5.7 \times 10^{-8} \) sec (b) \( 2.5 \times 10^{-8} \) sec
Solved Problems and Exercises

4. The speed of an electron having kinetic energy $2 \text{ MeV}$ will be [BHU 2018]
   (a) $2.93 \times 10^{-8}$ sec  (b) $3 \times 10^{-8}$ sec
   (c) $10 \times 10^{-8}$ sec  (d) $1.5 \times 10^{-8}$ sec
   Ans. a

5. With what velocity an electron should move so that its kinetic energy equals its rest mass energy? [HCU 2019]
   (a) $\frac{3}{2}c$  (b) $\sqrt[3]{\frac{3}{2}}c$  (c) $\sqrt[3]{c}$  (d) $2c$
   Ans. c

6. Two artificial satellites S1 and S2 of mass m and 2m respectively, are orbiting the earth in elliptical orbits such that the period of S1 is doubled that of S2 . If the semi-major axis of S1 is half of S2, then the ratio of the total energy between S1 and S2 is given by [HCU 2019]
   (a) 1:1  (b) 2:1  (c) 4:1  (d) 1:2
   Ans. b

7. Three events, $E_1(\text{ct}=0, x = 0)$, $E_2(\text{ct}=0, x = L)$ and $E_3(\text{ct}=0, x = -L)$ occur, as observed in an inertial frame S. Frame S2 is moving with a speed $v$ along the positive x - direction with respect to S. In S2 , let $t_1\, t_2\, t_3\, ,$ be the respective times at which $E_1\, , E_2\, \text{and}\, E_3$ occurred. Then, [IIT JAM 2021]
   (a) $t_2 < t_1 < t_3\, ,$  (b) $t_1 = t_2 = t_3\, ,$  (c) $t_1 > t_2 > t_3\, ,$  (d) $t_1 < t_2 < t_3$
   Ans. a

8. A particle initially at the origin in an inertial frame S , has a constant velocity $V_i$ . Frame S2 is rotating about the z - axis with angular velocity $\overset{\frown}{\text{anticlockwise}}$. The coordinate axes of
S2 coincide with those of S at $t = 0$. The velocity of the particle $(V_x^2, V_y^2)$ in the S2 frame, at $t = \frac{\pi}{2}$, is [IIT JAM 2021]

(a) $\left(\frac{V\pi}{2}, -V\right)$  
(b) $(-V, -V)$  
(c) $(-\frac{V\pi}{2}, -V)$  
(d) $\left(\frac{3V\pi}{2}, -V\right)$

c

9. A particle is moving with a velocity $0.8cj$ ($c$ is the speed of light) in an inertial frame S1. Frame S2 is moving with a velocity $0.8ci$ with respect to S1. Let $E_1$ and $E_2$ be the respective energies of the particle in the two frames. Then, $E_1/E_2$ is ________ (Round off to two decimal places). [IIT JAM 2021]

Ans. 1.64-1.68

10. A particle is moving with 90% of the velocity of light. Ratio of its relativistic mass with its rest mass is [BHU 2017]

(a) 2.0  
(b) 3.00  
(c) 5.00  
(d) 2.29

Ans. d

11. A particle of mass $m_0$, moves with a speed of $c/\sqrt{2}$ then its mass is

a) $M_0/\sqrt{2}$  
(b) $\sqrt{2} m_0$  
(c) $m_0\sqrt{2}$  
(d) $\sqrt{(m/2)}$  

AU 2019

12. Choose the incorrect option {AU2019}

a) Time is invariant quantity according to relativity  
(b) Speed of light remains invariant in all inertial frames of references  
(c) There is no absolute frame like as Ether  
(d) Michelson Morley experiment is base of interference

Check your understanding

1. If we push an object, displacement will be observed immediately after the application of the force [ ]

2. The moving clock ticks faster than rest clock [ ]

3. Time dilation and length contraction makes the light to get trapped in black holes [ ]
4. The speed of light does not depend on the density of the medium
5. The concept of ether drag was accepted by Michelson-Morley experimental
6. Reason for the apparent position of stars was given by Einstein using his special theory of relativity
7. According to Isaac Newton, time is not absolute
8. A time interval never changes between two frames of references
9. According to special theory of relativity, it is impossible to surpass the speed of light because the momentum of an object would decrease to infinity
10. In the limit \( v \to 0 \) one can observe that the relativistic kinetic energy coincides with the classical kinetic energy

<table>
<thead>
<tr>
<th><strong>Glossary</strong></th>
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<tbody>
<tr>
<td><strong>Angular moment</strong></td>
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<td><strong>Bulginess</strong></td>
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<tr>
<td><strong>Coordinate system</strong></td>
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<td><strong>Constraint</strong></td>
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<td><strong>Centre of gravity</strong></td>
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<td><strong>Centre of mass</strong></td>
</tr>
<tr>
<td><strong>Degrees of freedom</strong></td>
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<td><strong>Dynamics</strong></td>
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<td><strong>Fly Wheel</strong></td>
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<tr>
<td><strong>Gyro compass</strong></td>
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<tr>
<td><strong>Kinematics</strong></td>
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<td><strong>Magnetic dipole</strong></td>
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<td><strong>Tensor</strong></td>
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PART-II
WAVES & OSCILLATIONS
ప్రతివేదిలు & వర్తమానము
చిత్రాలం & లైటవాయరు మార్గం

చిత్రాలం మార్గం కోసం

దొరికిన మిశ్రమం తొలి ప్రశ్నల కంటే, పొందండి బట్టి మిశ్రమం వచ్చింది. అంచలో ప్రతి మిశ్రమం తొలి ప్రశ్నల కంటే ఎలా తయారంపడం ప్రారంభించండి. దీని కారణం మిశ్రమం అనుసంధానం కార్యాల మీద ఎలా ప్రారంభించండి. అనగా మిశ్రమం అనుసంధానం కార్యాల మీద ఎలా ప్రారంభించండి. అనగా మిశ్రమం అనుసంధానం కార్యాల మీద ఎలా ప్రారంభించండి. అనగా మిశ్రమం అనుసంధానం కార్యాల మీద ఎలా ప్రారంభించండి. అనగా మిశ్రమం అనుసంధానం కార్యాల మీద ఎలా ప్రారంభించండి. అనగా మిశ్రమం అనుసంధానం కార్యాల మీద ఎలా ప్రారంభించండి. అనగా మిశ్రమం అనుసంధానం కార్యాల మీద ఎలా ప్రారంభించండి. అనగా మిశ్రమం అనుసంధానం కార్యాల మీద ఎలా ప్రారంభించండి. అనగా మిశ్రమం అనుసంధానం కార్యాల మీద ఎలా ప్రారంభించండి. అనగా మిశ్రమం అనుసంధానం కార్యాల మీద ఎలా ప్రారంభించండి. అనగా మిశ్రమం అనుసంధానం కార్యాల మీద ఎలా ప్రారంభించండి.
According to Newton's second law, when a force is applied on a body, it gets accelerated along the direction of force. i.e.; it reports a change in velocity along the direction of force. But if there exists a restoring force, the object sets into vibration. We know that everything in nature likes to be in equilibrium position i.e.; in minimum energy state. So here restoring force is the one acts opposite to the direction of motion and tries to establish equilibrium in the system. Thus vibrations are set only when there is a deforming force that disturbs the equilibrium and a restoring force that establishes the equilibrium.

Vibrational energy is a form of potential energy that guides the force which sets the object into vibrations.

Vibrations may be periodic or aperiodic (non-periodic). The aperiodic vibrations are also called noise. This is also a form of energy. This noise energy is studied very rarely in physics like noise in sound, noise in electronic circuits excetra. Apart from that all other vibrations that we come across in physics are periodic. Periodic vibrations may be further classified into symmetric and asymmetric vibrations. Out of these, symmetric vibrations are called oscillations. For example, heartbeat is periodic but not symmetric whereas respiration of lungs is both periodic and symmetric.

Fig: Oscillations of human lungs and heart.
పాప్పా

నాటూరు ప్రాంతం లోని మండలాన్ని ప్రతిష్ఠించింది. ఇది కొనసాగిన ముఖ్య శాసనమాన్ని విద్యార్థులకు ప్రచురించాల్సి, తెలుగు భాష నుండి ఇంగ్లీష్ భాషలో శాసనం మార్పు చేసాల్సి ఉందని సూచిస్తుంది (మార్పు కార్యం).

ప్రత్యేకమైన కారణం నాటూరు ప్రాంతం మండలాన్ని ప్రతిష్ఠించడానికే సందిగ్ధ చిహ్నాలు అయిన అంకితాలను వెలిగించడానికే తగ్గితే సందిగ్ధ ప్రతిష్ఠా చేయడానికే నిర్ణయించాలి. ప్రత్యేకమైన కారణాన్ని దాని మండలాన్ని ప్రతిష్ఠించడానికే సమాధానం అయితే తగ్గితే అంకితాలను వెలిగించడానికే కనిపించుకోవడానికే నిర్ణయించాలి. ప్రత్యేకమైన కారణాన్ని ఉపయోగించడానికే దాని ప్రతిష్ఠాకు సమాధానం అయితే తగ్గితే దానిని వెలికించడానికే కనిపించుకోవడానికే నిర్ణయించాలి.

Did You Know?

Waves on water bodies are the result of oscillations generated by air drag and restoring force of water layers.

Did You Know?

Wave is a carrier of energy, not a form of energy, where as vibrations and oscillations are form of Energy

Did You Know?

Electromagnetic waves are the oscillations of electric and magnetic fields which may be controlled by vibrations of charged particles.

మనిషి నుంచి ప్రాంతాన్ని లేదా మండలాన్ని ఇది కొనసాగిన ముఖ్య శాసనమాన్ని విద్యార్థులకు ప్రచురించాల్సి, తెలుగు భాష నుండి ఇంగ్లీష్ భాషలో శాసనం మార్పు చేసాల్సి ఉందని సూచిస్తుంది (మార్పు కార్యం).

ప్రత్యేకమైన కారణం నాటూరు ప్రాంతం మండలాన్ని ప్రతిష్ఠించడానికే సందిగ్ధ చిహ్నాలు అయిన అంకితాలను వెలిగించడానికే తగ్గితే సందిగ్ధ ప్రతిష్ఠా చేయడానికే నిర్ణయించాలి. ప్రత్యేకమైన కారణాన్ని దాని మండలాన్ని ప్రతిష్ఠించడానికే సమాధానం అయితే తగ్గితే అంకితాలను వెలిగించడానికే కనిపించుకోవడానికే నిర్ణయించాలి. ప్రత్యేకమైన కారణాన్ని ఉపయోగించడానికే దాని ప్రతిష్ఠాకు సమాధానం అయితే తగ్గితే దానిని వెలికించడానికే కనిపించుకోవడానికే నిర్ణయించాలి.
Respiration of lungs also becomes symmetric only when the rate of inhalation and the rate of exhalation are uniform and equal. Otherwise that also would become asymmetric and when we start doing hard work, they may also become aperiodic for some time and may become periodic in a mean time.

Heart also sometimes beats without periodicity or it misses the rhythm. If the rhythm misses for prolonged period without performing any additional activities, then it is considered as a disease called ‘Arrhythmia’.

**Fig: Arrhythmia**

**Waves**

Vibrations generate wave. Wave is a carrier of vibrational energy. They carry vibrational energy from one point to another point without actual displacement of particles involved (if any).

Propagation of Mechanical waves like sound requires some material medium where as the propagation of electromagnetic waves requires no medium. A wave can be generated in a string by tying its one end and oscillating the other end. Here the string is the carrier of wave and the wave is the carrier of vibration energy generated by our hand. The vibrational energy reaches the other end of the string without actual displacement of particles of the string from one end to the other end.
Waves

Fig: Longitudinal and Transverse waves.

Transverse Waves

Longitudinal Waves

296
Thus waves also could be periodic or aperiodic, symmetric or asymmetric.

Here the wave could be progressive and stationary. In progressive wave, energy is transferred from one point to another point. Whereas in Stationary waves energy is trapped between two points.

Both progressive and stationary waves can be further classified into longitudinal wave and transverse waves.

In longitudinal waves the oscillations or vibrations of the particles are along the directions of motion of wave and in transverse wave the oscillations or vibrations will be perpendicular to the direction of motion of particles.

Both in longitudinal and transverse waves the act of generating wave causes a displacement from mean position of the particle.

**Fig: Displacement from mean position in a wave.**

Once this is realized, the entire effort of studying waves boils down to study of displacement of particles/field from mean position.

This domain and range concepts are adopted from mathematics functions topic.

There are two possible ways of studying waves. One as a function of time (time domain study) and another as a function of distance (space domain study).
Table: Position Vs. displacement

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Time Vs. displacement

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Fig: Plot of space domain and time domain studies.

\[ \text{Position} = \text{Displacement} \]

\[ \text{Time} = \text{Displacement} \]

**Waves**

\[ \lambda = \frac{\omega}{v} \]

\[ v = \frac{\omega}{k} = \frac{2\pi}{2\pi \nu} = \frac{1}{\nu} \]

\[ k = \frac{2\pi}{\lambda} \]

\[ \nu = \frac{2\pi}{T} = \frac{1}{T} \]

\[ \phi = \frac{2\pi}{\lambda} x \]
In space domain study, time is fixed by taking a snapshot or photograph of the system and displacement from mean position is studied at various locations in space.

Once the data is collected in the space domain and the time domain studies, graph may be plotted for displacement versus time and displacement versus distance.

One can define some standard parameters in each domain study to proceed further. In space domain study the standard parameter is wavelength which is the length of one wave. This can be calculated by measuring the total distance covered by a few waves and dividing it with the number of waves.

\[
\text{Wave length} \ (\lambda) = \frac{\text{Total Distance}}{\text{Total Number of Waves}}
\]

Similarly in time domain study the standard parameter is the frequency which is the number of waves in one second. For that we need to count the number of waves for a few seconds and divide it with the total time.

\[
Frequency \ = \ \nu = \frac{\text{Total Number of Waves}}{\text{Total Time}}
\]

Some other standard parameters that can be defined in space domain are wavenumber \( \bar{\nu} = 1/\lambda \), wave vector \( k = 2\pi/\lambda \). Similarly other parameters that can be defined in the time domain study are time period \( T = 1/\nu \), and angular frequency \( \omega = 2\pi/T = 2\pi\nu \).

The introduction of the parameter \( 2\pi \) is done manually to express the periodic nature of the system. This is because circular motion is the most symmetric periodic motion known to us. The details are well discussed in Chapter 5.

In circular motion on a circle of unit radius, moving a distance of \( 2\pi \). 1 generates a wave of length \( \lambda \) on the diameter of the circle. Thus any arbitrary displacement can be connected to arbitrary phase as

\[
\phi = \frac{2\pi}{\lambda} x
\]
చాలా చాలా అనే విధానం ఉంటే ఆధారాన్ని మరియు విశేషాలను సందర్శించాలి. ఈ శాస్త్రాన్ని సహా డీ నలుస్తమం 

అంటే దేశాలు అవశేషం ఉంటాయి.  

ఇప్పుడు ప్రతిలోకానికి ప్రత్యేక విశేషాలను సందర్శించాలి. ఇది తరావిలో  

(\(\frac{\partial^2 u}{\partial t^2}\)) = \(\frac{\partial^2 (\partial^2 u)}{\partial x^2}\)  

ఈ రకాని విశేషాలు సందర్శించాలి అపూర్వం కంటే ఇతర సందర్శించ తరావిలో ఇవి  

u=a sin(kx-\(\omega t\)) \cos(kx-\(\omega t\)) కంటే అధికంభూతం  

అంటే దేశాలు అవశేషం ఉంటాయి. ఇది తరావిలో ఇవి సందర్శించాలి. 

ఈ సకరణం కంటే పరం 

e \(^{(\pm \alpha x)}\) 

ఈ ప్రతిలోకానికి ప్రత్యేక విశేషాలను సందర్శించాలి.
This parameter $2\pi$ comes as a consequence of mapping the periodic motion on to the circular motion. The necessity of parameter $2\pi$ is discussed in Chapter 5.

Since these two studies represent the same system, the variations recorded in both studies must be interrelated, that relation is given by the wave equation.

$$\frac{\partial^2 u}{\partial t^2} = \vartheta^2 \frac{\partial^2 u}{\partial x^2}$$

A simple dimensional analysis confirms the correctness of the equation. Here ‘$u$’ is the displacement from mean position. Here $\vartheta$ must remain constant as LHS and RHS are mathematically independent of each other.

This can be obtained by multiplying the standard parameters defining each domain as

$$\vartheta = \frac{Total \, distance}{Total \, Time} = \frac{Total \, Number \, of \, Waves}{Total \, Time} \times \frac{Total \, Distance}{Total \, Number \, of \, Waves} = v\lambda$$

The solution of Wave equation is given by $u = a \sin(kx-\omega t)$ or $a \cos(kx-\omega t)$ Thus the resultant motion of a Periodic oscillation is a sine wave or cosine wave. This kind of second order wave equation (in both space and time) and solutions (of sine and cosine) occur because acceleration is proportional to displacement variations in this case.

Where as some other cases like heat wave equation which represents a dispersion process or dissipating process have wave equation like

$$\frac{\partial u}{\partial t} = \vartheta^2 \frac{\partial^2 u}{\partial x^2}$$

The solution of this equation is

$$e^{\pm \alpha x}$$
Waves

This is an exponentially falling wave. The solution represents the dissipation activity.

In the rest of the sessions, we study about various types of oscillations, Coupled oscillations, Vibrations in strings and Ultrasonics.
UNIT-IV
Chapter-5
UNDAMPED, DAMPED FORCED OSCILLATIONS
అవరద ఆనవరదు
పరసంతి నీటిచేయము
అభిప్రాయం

అంగేష్యం ఊరించండి

1. రణంలో పొడిబడిన అవకలన సకరం పషంచం
2. కనుక పాత మృదు పొడిబడిన అంగేష్యం దాడం
3. వారంతో, మాటిడిలు పండిత్యులు మాత్రమే ప్రతి సమయం మరణానగా వస్తాయం

అభిప్రాయం పశుల

అంగేష్యం ఊరించండి, అంగేష్యం ఊరించండి

1. రణం ఉద్యమం పొడిలో మాటిడిలు పండిత్యులు మాత్రం అవకలన సకరం
2. చాలా పాత మృదు పొడిలో మాటిడిలు ప్రతి సమయం మరణానగా
3. రణం ఉద్యమం పొడిలో మాటిడిలు పండిత్యులు మాత్రం అవకలన సకరం
4. అంగేష్యం ఊరించండి పొడిలో, మాటిడిలు, Q-రాధా అభిప్రాయం
5. అంగేష్యం ఊరించండి పొడిలో మాటిడిలు పండిత్యులు మాత్రం అవకలన సకరం
6. కనుక పొడిలో మాటిడిలు పండిత్యులు మాత్రం అవకలన సకరం అవకలన సకరం

5 CHAPTER
Syllabus

Simple harmonic oscillator and solution of the differential equation, Damped harmonic oscillator, Forced harmonic oscillator – Their differential equations and solutions, Resonance, Logarithmic decrement, Relaxation time and Quality factor.

Learning Objectives

In this chapter students would learn
1. To solve simple harmonic oscillator differential equation.
2. To get Damped and forced harmonic oscillator equation solutions.
3. To define resonance, logarithmic decrement, relaxation time and quality factor of oscillators.

Learning Outcomes

By the end of the chapter, student would be able to
1. Identify the cause of simple, damped and forced harmonic oscillations.
2. Describe the nature of damped and forced harmonic oscillations.
3. Apply the solution of simple harmonic motion to simple systems of interest.
4. Classify resonance, logarithmic decrement, Q-factor in various systems.
5. Select suitable parameters of interest in various types of oscillations, namely forced and damped.
6. Develop prototype models damped and forced harmonic motions in real life systems.
ధన్యంలంషఫమభషశైఅయంసమషలంనండం,డంమలలతంచగలు.

(ఎ) కొంత అర,బంంంఆషాప.

(గ) కంటేంఖమితరంలడంపకంతరంలరంయం,

(ఎలో) ఎలోఆటసలంపదజతసలంమధకషంకం.

(ఇ) కంటేంఖమితరంలడంపకంతరంలరంయం,

(f) REM: ఎడడాదడలపంకలనమగషమరంశ

(g) MRI దౖనపధాతకఖష్ట్లకంచబనధమధ

పతంవంఅపతంవర11వతరగ్రిరచలంయడం,

రణలకంసమయవవగణనఉం. ఈఅయంఅంమంమంమలఫయలపషంచధ్పదలం. డంమలలవంలరంచడంం.
Course Outcomes specific to program and Future directions

By the end of this chapter students from specific programs would be able to identify the role of undamped, damped and forced oscillations in the following fields.


b) Chemistry: Spectroscopy, a branch of chemistry that deals with the identification of atoms and molecules in the given sample, relies on the identification resonance frequency of oscillations of specific atoms and molecules.

c) Geology: Damping effects in seismic wave oscillations give a hint about the presence of petroleum, minerals and other geological structures in the way of seismic waves.

d) Electronics: Resonance frequency in electronic oscillator circuits the heart of communication between any pair of electronic circuits.

e) Computers: Simulations of car driving, mars rover, lunar rover etc. needs solutions of damped and forced harmonic oscillator differential equations corresponding to the shock absorbers.

f) REM: Tidal resonance is an important technique in the design of tidal ranges and barriers for energy harvesting with maximum efficiency.

g) Statistics: Statistical analysis of various data collected by resonance based characterization techniques like MRI etc. gives a distinguishing between the problematic conditions and noise of the instrument.

Familiar to Unfamiliar

In your 11th class, you might have come across the mapping of circular motion on to simple harmonic motion, approximations in simple pendulum time period calculation, conceptual treatment of damped and forced harmonic oscillators, resonance in forced oscillations. In this chapter you would learn various methods to solve undamped, damped and forced harmonic oscillator differential equations. You
5.1 పచయం

5.1 అవృతి

సంబంధం లో ఆరపు ఉంందనే, అంలకం కలం దీని మంత అవృతి ఎందుకంటే పచయం అడిగా 1638 సనులు మాత్రమే కారణం. అడిగా 1657 సనులు వంటి పచయం గల అంతిపంచబం. ఇది ఈ లేదు. అతి 2 కన (అనేక, ఒక వంటి యోధం యితే 1 కన) పండి తండు ఇంటిలో బాంధం అంంచబం. తండు " "మంత అవృతి సమానం లేదా సనుల గత అంంచబం. అతి సనం గత తండు " (SHM) అపండి పశ్చి లేకుండా అంంచబం. al., ఆ 1879 సనులు పచయం. అంంచ సమానం "పచయం సంపాదిత లాంటి" (SHM) వి మాత్రమే ఉపయోగించే ఇది అంచానే పచయం అడిగా 1673 సనులు

**E - Corner**


https://www.aps.org/publications/apsnews/201706/history.cfm


will also learn to define a few parameters that characterize the damped and forced harmonic oscillations.

5.1 Introduction

History

In your 7th class you might have learned that Galileo was the first person to identify that pendulum exhibits isochronism, which means that time period of pendulum depends only on its length, irrespective of its mass and swing stretch. This he had discussed in his book entitled “Two new Sciences” in 1638. Further Huygens has developed and patented the first pendulum clock in 1657, following in the footsteps of Galileo. He also proved that the time period of pendulum depends on the stretch, if the angle of stretch increases beyond a certain limit. All his work on pendulums and pendulum clocks was published in “Horologium Oscillatorium” in 1673.

Huygens used this pendulum to standardize distance unit “meter”. He observed that any pendulum with 2 seconds time period (i.e., 1 second for a swing from one end to other end) has a cord length of 1 meter. To his surprise that pendulum was also useful to measure acceleration due to gravity which is approximately equal to \( \pi^2 \) if \( T = 2 \text{ sec} \).

The concept of simple harmonic motion, where isochronism is observed for low stretch angles was further developed by Newton, in his “Principia” and later a modern analysis was given in “Treatise on Natural Philosophy” by Tait et. al., which was published in 1879. In this book for the first time the word “Simple Harmonic Motion” (SHM) was introduced and mathematical treatment was given on par with the modern geometry.

Taylor series expansion of Force around a fixed point is given by

\[
F(x - x_0) = F(x_0) + \frac{\partial F}{\partial x}\bigg|_{x=x_0} (x - x_0) + \frac{1}{2!} \frac{\partial^2 F}{\partial x^2}\bigg|_{x=x_0} (x - x_0)^2 + \cdots
\]
5.1 Introduction

\[
F(x - x_0) = F(x_0) + \frac{\partial F}{\partial x} \bigg|_{x=x_0} (x - x_0) + \frac{1}{2!} \frac{\partial^2 F}{\partial x^2} \bigg|_{x=x_0} (x - x_0)^2 + \ldots
\]

SHM రేఖాంశం, i) ద్వితీయ (x = 0) పోషనీ ఫంక్షను అనుసరిస్తుంది. ii) ద్వితీయ పోషనీ విచేషణ పోషను పోషనీ శేరు అనుసరిస్తుంది. iii) శేరు ఐదాదో వేయించినా అంశాలు అనేకం పోషను అనుసరిస్తుంది. iv) అదనం పోవం మారుతుంది.

శేరు ఐదాదో వేయించినం కొనసాగిసి, కమ్యూనిటీ లో \( \frac{\partial F}{\partial x} \bigg|_{x=0} = -K \) వ్యవస్థలో. అంటే ద్వితీయ వేయించినం కొనసాగిసి, \( \Delta x \) పోషనీ వేయించినం ఎక్కడు అంటే \( \Delta F \) పోషనీ వేయించినం ఎక్కడు. అంటే, శేరు ఐదాదో వేయించినం పోషనీ వేయించినం ఎక్కడు, ఎక్కడు ఎక్కడు ఎక్కడు. అంటే లోపం శేరు ఐదాదో వేయించినం ఎక్కడు అంటే, వాటి ఆరోధం శేరు ఐదాదో వేయించినం.

ఈ సందరం లో పోషనీ వేయించినం, ఎక్కడు ఎక్కడు ఎక్కడు ఎక్కడు ఎక్కడు ఎక్కడు పోషనీ వేయించినం ఎక్కడు ఎక్కడు ఎక్కడు. వాటి ప్రాంతం ఎక్కడు వాటి ప్రాంతం ఎక్కడు ఎక్కడు. అంటే లోపం పోషనీ వేయించినం ఎక్కడు వేయించినం ఎక్కడు ఎక్కడు ఎక్కడు. వాటి ప్రాంతం ఎక్కడు వాటి ప్రాంతం ఎక్కడు ఎక్కడు.

ప్రత్యేక పోషనీ వేయించినం ఎక్కడు వేయించినం, ఎక్కడు ఎక్కడు ఎక్కడు ఎక్కడు ఎక్కడు ఎక్కడు ఎక్కడు పోషనీ వేయించినం ఎక్కడు ఎక్కడు ఎక్కడు. వాటి ప్రాంతం ఎక్కడు వాటి ప్రాంతం ఎక్కడు ఎక్కడు ఎక్కడు. అంటే లోపం పోషనీ వేయించినం ఎక్కడు వేయించినం ఎక్కడు ఎక్కడు. వాటి ప్రాంతం ఎక్కడు వాటి ప్రాంతం ఎక్కడు ఎక్కడు.

\[
F(x) = -kx \Rightarrow m \frac{d^2x}{dt^2} + kx = 0 \Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m}x = 0
\]

ఈ పోషనీ వేయించినం ఎక్కడు పోషనీ వేయించినం ఎక్కడు ఈ ప్రాణాల ప్రాణాల ప్రాణాల ప్రాణాల ప్రాణాల ప్రాణాల. ఆమె ఆమె ఆమె ఆమె ఆమె ఆమె ఆమె ఆమె ఆమె ఆమె. ఆమె ఆమె ఆమె ఆమె ఆమె ఆమె ఆమె ఆమె ఆమె ఆమె.
In the case of SHM, i) Origin is at zero \(x_0 = 0\). ii) The force at origin is zero \(F(x_0) = 0\) iii) The first derivative of force is a negative constant and iv) All higher order terms are zero.

The first derivative of force is a negative constant, which can be stated as \(\frac{\partial F}{\partial x}\bigg|_{x=0} = -K\) (say). This implies that as deviation from origin increases \((\Delta x \uparrow)\), force also increases \((\Delta F \uparrow)\) but in an opposite direction; i.e., more the displacement, more is the force. In addition, the direction of restoring force is opposite to displacement. As this force tries to establish equilibrium in the system, it is referred to as the restoring force.

Additionally, the force at the origin is zero, in which case, Newton’s first law applies i.e., if the object is at rest at the origin, it will remain at rest forever. If it is in motion, it will continue its state of motion.

Thus

\[
F(x) = -kx \Rightarrow m \frac{d^2x}{dt^2} + kx = 0 \Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m} x = 0
\]

When a deviating or deforming force is applied on a particle, a restoring force is generated which tries to establish equilibrium. This gives rise to generation of acceleration in the particle, opposite to it’s direction of motion. Since there is no force at the origin, the particle moves forward further due to inertia. This activity further generates a restoring force towards the origin again. This to and fro motion results in oscillation of the system.

This kind of forces are called conservative forces. In the case of conservative force, the net work done over a closed loop will be zero. i.e., if certain amount of energy is consumed for executing forward motion, the same amount of energy will be regained during backward motion, so that the net work done is zero. This may not be possible always in daily life due to frictional forces and other energy loss mechanisms. Carrying a bucket of water upstairs and bringing it back downstairs should actually cause no strain. But in both ways we feel strain because body muscles have to work against gravity in both the directions of travel. Thus in that case force will not be conserved. Whereas in the case of pendulum, if there are no air
5.1 Introduction

The gravitational potential energy of a particle, mass \( m \), at a distance \( r \) from the center of a mass \( M \) is given by the formula:

\[
U(r) = -\frac{GMm}{r}
\]

where \( G \) is the gravitational constant. The potential energy is a scalar quantity, and its units are energy per unit mass. The gravitational potential energy is related to the gravitational force by the following equation:

\[
F = -\nabla U = \frac{-GMm}{r^2}
\]

The gravitational potential energy is the work done by the gravitational force in moving a particle from one point to another in the gravitational field. The work done by the gravitational force is equal to the change in the gravitational potential energy of the particle.

The gravitational potential energy is also related to the kinetic energy of a particle by the following equation:

\[
E = K + U
\]

where \( E \) is the total energy of the system, \( K \) is the kinetic energy, and \( U \) is the potential energy. The total energy of the system is constant in the absence of non-conservative forces.

The gravitational potential energy is a function of the distance from the center of mass, and it is constant along a closed path. The gravitational potential energy is also affected by the mass of the object and the mass of the particle. The gravitational potential energy is a useful concept in the study of gravity and its effects on objects in space.
5.1 Introduction

resistance/frictional forces, the oscillations continue forever without any reduction in amplitude.

Mathematically this can be expressed as

\[ \nabla \times \vec{F} = 0 \quad \text{or} \quad \oint \vec{F} \cdot d\vec{l} = 0 \]

When the cross product of a vector is zero, such a vector quantity can be expressed as the gradient of a scalar quantity. Here, force can be expressed as the negative gradient of potential \((V)\), which is a scalar quantity.

\[ \vec{F} = -\nabla V = V_{\text{initial}} - V_{\text{final}} \Rightarrow V = -\int F \cdot dV \]

\[ \nabla \times \vec{F} = \nabla \times (-\nabla V) = -\nabla \times \nabla V = 0 \]

Thus force is positive when \(-\nabla V\) is positive; i.e., force is in the direction opposite to the gradient (rise) of potential (i.e., it is directed from high potential to low potential).

In the case of simple harmonic motion, the potential is given by

\[ V = -\int F \cdot dx = -\int (-kx)dx = \int kx \cdot dx = \frac{1}{2} kx^2 \]

The shape of the potential is a parabola. If next higher order derivative in force also exists and is equal to a constant, say \(b\), then

\[ F = -Kx + bx^2 \Rightarrow V = \frac{1}{2} kx^2 - \frac{1}{3} bx^3, \]

which is called the anharmonic potential. This potential curve has all positive values for negative values of \(x\). In the first quadrant, \(x^2\) function dominates for smaller values of \(x\) and \(x^3\) function dominates for larger values of \(x\). The turning point occurs at \(x_{\text{turn}} = \frac{3K}{2b} \).
5.1 Introduction

Did You Know?

Anharmonic is different from enharmonic.
En-harmonic – Same/Equal harmonic
An-harmonic – Not Harmonic

Activity

Draw the curves for anharmonic potential and identify the turning point. Verify with the formula for various values of $k$ and $\gamma$.

In Quantum mechanical harmonic oscillator, all energy levels are equally spaced. This gives only one line during energy transitions. In anharmonic oscillator, energy level separation reduces as energy rises. This results in a full spectrum during transitions. In molecular spectroscopy, Anharmonic oscillations are responsible for spectrum.

In Quantum mechanical harmonic oscillator, all energy levels are equally spaced. This gives only one line during energy transitions. In anharmonic oscillator, energy level separation reduces as energy rises. This results in a full spectrum during transitions. In molecular spectroscopy, Anharmonic oscillations are responsible for spectrum.
5.1 Introduction

**Fig: Harmonic and Anharmonic oscillator potential.**

**Damped oscillations**

If the potential is also velocity dependent, then one can write the Taylor series expansion of force as

\[
F(x, \dot{x}) = F(0) + \left. \frac{\partial F}{\partial x} \right|_{x=0} x + \left. \frac{\partial F}{\partial \dot{x}} \right|_{x=0} \dot{x} + \frac{1}{2!} \left. \frac{\partial^2 F}{\partial x^2} \right|_{x=0} x^2 \\
+ \frac{1}{2!} \left. \frac{\partial^2 F}{\partial \dot{x}^2} \right|_{x=0} \dot{x}^2 + \ldots
\]

The simplest approximation is again to assume that the force at the origin is zero and the higher order terms starting from 2\textsuperscript{nd} order onwards vanish. Consider \( \left. \frac{\partial F}{\partial x} \right|_{x=0} = -k \) and \( \left. \frac{\partial F}{\partial \dot{x}} \right|_{x=0} = -\gamma \). Then the force equation becomes

\[
F = -k x - \gamma \frac{dx}{dt}.
\]

Thus, in this case, more the velocity of the object, more is the restoring force. This effect is called damping. Usually frictional forces on solids and viscous forces in fluids contribute to this. This is not a conservative force.

**Forced Oscillations**

If there exists any driving force, which is periodic with a frequency \( p \), the force equation takes the form

\[
F = -k x + F \sin pt.
\]
5.1 Introduction

\[ F = -kx - \gamma \frac{dx}{dt} \]

When \( x = 0 \) and \( \frac{dx}{dt} = 0 \), we have

\[ F = -kx + F \sin pt. \]

\[ F = -kx - \gamma \frac{dx}{dt} + F \sin pt \]

The E - Corner

https://ocw.metu.edu.tr/pluginfile.php/6886/mod_resource/content/1/ch8/8-3.htm

If damping forces are also included along with driving force, then the force equation becomes

\[ F = -kx - \gamma \frac{dx}{dt} + F \sin pt. \]

In the above, for the first case, force is conservative, while it is not for the second case.

**Mapping SHM with circular motion**

Harmonic oscillator motion is very a complicated motion wherein for the first time we come across *jerk or jolt* in Physics. Jerk or Jolt is the time derivative of acceleration. This is due to the fact that the acceleration of the system changes with displacement from mean position. From a calculation of the physical quantity Jerk/Jolt, one may infer that it is not a constant but varies with time.

\[ J = \frac{da}{dt} \]

Then how to study such a complicated motion with simplified models is a matter of critical thinking. But thanks to the circular motion, one can easily replicate the final effect of simple harmonic motion by simply placing a mirror along the diameter of a circle and studying the motion of reflection of object which is executing uniform circular motion.

![Fig: SHM in circular motion.](image)
5.1 Introduction

The equation of motion for a simple harmonic motion is given by $x = A \cos \theta$. This equation represents the position of a particle in SHM with respect to time.

The acceleration of the particle is given by $a = \omega^2 A$. Using the relationship $x = A \cos \theta$, we can express the acceleration in terms of $x$ as $a_{SHM} = -\omega^2 x$.

From the equation $a = \frac{k}{m} x$, we can find the angular frequency $\omega$ as $\omega = \sqrt{\frac{k}{m}}$.

In summary, SHM is characterized by a restoring force proportional to the displacement, and the motion is periodic with a constant frequency determined by the system's properties.
Consider a uniform circular motion in anticlockwise direction with a mirror along the X'OX diameter of the circle. Then the displacement of the image on the diameter, when the object is at location P, is given by

\[ x = A \cos \theta. \]

The acceleration is directed along the radial vector but pointing towards the center.

\[ a = \omega^2 A. \]

Then the component of acceleration along the X'OX diagonal is

\[ a_x = -a \cos \theta = -\omega^2 A \cos \theta = -\omega^2 x. \]

Thus, the acceleration of SHM is given by

\[ a_{SHM} = -\omega^2 x. \]

But we have

\[ a_{SHM} = -\frac{k}{m} x. \]

From the above two,

\[ \omega = \sqrt{\frac{k}{m}}. \]

Here, on the LHS we have physical quantity with units of rad/Sec. But on the RHS we have units of 1/sec. This discrepancy arose because we didn’t convert circular motion into linear motion. But just mapped one another. This is unavoidable because there is no such simplified motion which could produce a complex variation in acceleration as a function of time.

To avoid the discrepancy, the angle was made a physical quantity with units but without dimensions. And it is enough to match the dimensions while dealing with equations.

There was also another complication that arose which led to the decision to make angle dimensionless. That is, if Taylor series expansion of \( \sin \theta \) is considered, it is as given below.
5.1 Introduction

\[
\sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots
\]

Verify the variation of acceleration due to gravity as a function of stretch angle of pendulum bob.

Also plot it using the equation and compare.

Activity

Verify various pendulum properties using the simulation tool.

https://www.myphysicslab.com/
\[
\sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots
\]

Here \( \theta \) has units of radians. But to add physical quantities every component must have same dimensions. Hence the physical quantity should not have any dimension though it has a unit of radian.

The actual time period of pendulum is given by

\[
T = 2\pi \sqrt{\frac{l}{g \sin \theta}} \Rightarrow g = 4\pi^2 \frac{l}{T^2 \sin \theta}
\]

The variation of \( g \) as a function of \( \theta \) is as shown below. Since practical \( g \) value is constant, and \( \theta \gg \sin \theta \), \( T \) value increases enormously for larger \( \theta \). This results in a reduction in \( g \). If we just see the formula, it looks like \( g \) value increases for larger \( \theta \) as \( \theta/\sin \theta \gg 1 \). Here the true experimental variables are \( \theta \) and \( T \). All we are measuring is \( T \) with variations in \( \theta \). Using that information, we are just calculating \( g \) using a formula. We are not measuring \( g \) in this experiment. So whatever functional dependency that \( \theta \) and \( T \) has will be reflected in the formula for \( g \). Here \( \theta \) cannot directly influence \( g \). In fact \( g \) is a constant at a given place. One must understand this discrepancy in interpretation while identifying experimental constants and variables and correlating them with theoretical equations.

![Figure: Variation of \( g \) as a function of stretch angle \( \theta \).]
5.1 Introduction

If we consider the sine function, \( \sin \theta \) near the value 0, we have \( \sin \theta \approx \theta \), which is valid for small angles, \( \theta \).

We have the formula

\[
T = 2\pi \sqrt{\frac{l}{g}}
\]

for \( \theta \). Since \( \sin \theta \approx \theta \) for small angles, \( \theta \leq 0.1^\circ \), we have \( \sin \theta \approx \theta \), which is valid for angles near 0.

The radian is a unit of angular measurement, where \( 1 \text{ rad} = \frac{180^\circ}{\pi} \approx 57.3^\circ \).

The conversion from radians to degrees is given by \( \theta \text{ rad} = \frac{180^\circ}{\pi} \theta \).

For angles near 0, we have \( \sin \theta \approx \theta \) for \( \theta \leq 0.1^\circ \).

For angles near 30°, we have \( \sin \theta \approx \frac{\theta}{2} \).

For angles near 10°, we have \( \sin \theta \approx \theta - \frac{\theta^3}{6} \).

In the E-Corner, we provide links to further information:

- [https://www.nature.com/articles/548135b](https://www.nature.com/articles/548135b)
5.1 Introduction

But to make it isochronous, \( \theta \) must be very small. Then, for this case, from the above expansion, \( \sin \theta \approx \theta \), so that the time period becomes,

\[
T = 2\pi \sqrt{\frac{l}{g}}
\]

But for what value of \( \theta \), is it considered to be small? If we look at some of the values of \( \sin \theta \), we have

<table>
<thead>
<tr>
<th>Sl No</th>
<th>( \theta )</th>
<th>( \sin \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35°</td>
<td>0.573</td>
</tr>
<tr>
<td>2</td>
<td>30°</td>
<td>0.500</td>
</tr>
<tr>
<td>3</td>
<td>15°</td>
<td>0.258</td>
</tr>
<tr>
<td>4</td>
<td>10°</td>
<td>0.173</td>
</tr>
<tr>
<td>5</td>
<td>1°</td>
<td>0.017</td>
</tr>
<tr>
<td>6</td>
<td>0.1°</td>
<td>0.0017</td>
</tr>
</tbody>
</table>

One observes that even if we go for 0.1°, \( \sin \theta \) will never be equal to \( \theta \), even barely.

The discrepancy here is that the angle \( \theta \) must be considered in radians for all practical purposes.

<table>
<thead>
<tr>
<th>Sl No</th>
<th>( \theta^\circ )</th>
<th>( \theta \text{ rad} )</th>
<th>( \sin \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35°</td>
<td>0.610</td>
<td>0.573</td>
</tr>
<tr>
<td>2</td>
<td>30°</td>
<td>0.523</td>
<td>0.500</td>
</tr>
<tr>
<td>3</td>
<td>15°</td>
<td>0.261</td>
<td>0.258</td>
</tr>
<tr>
<td>4</td>
<td>10°</td>
<td>0.174</td>
<td>0.173</td>
</tr>
<tr>
<td>5</td>
<td>1°</td>
<td>0.0174</td>
<td>0.0173</td>
</tr>
<tr>
<td>6</td>
<td>0.1°</td>
<td>0.00174</td>
<td>0.00173</td>
</tr>
</tbody>
</table>

One can see that, if a single decimal accuracy is sufficient, then one can take \( \theta \leq 30^\circ \). If two decimal accuracy is required, then one has
5.1 Introduction

SHM నిహయాది ప్రత్యేకంగా వాటి మొదలులు

అవకలన సకరణం క పరంరణ చలనం, నద రణ బలం నభంశ ణవ
అతం ఉంంద ఇపం అరంగం.

\[ F \propto -x \Rightarrow F = -kx \Rightarrow m \frac{d^2x}{dt^2} = -kx \Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \]

\[ \Rightarrow a + \frac{k}{m}x = 0 \quad \text{(1)} \]

అంటే K అర్థం లేదా నది రణ బలం m అని ప్రాణం మాత్రం a అవుతుంది. SHM అనే రాసిన ఎందుకండా తడితే

అంటే అందించడం వరకు నామకంగా ముందు నానం వస్తును అంశం సంఘం ఉంటాయి. అంటే ప్రాణం వరకు నానం వస్తును అంశం సంఘం ఉంటాయి. అంతే ప్రాణం వరకు నానం వస్తును

\[ x(t) = C_1 \sin \omega t + C_2 \cos \omega t. \]


t_1, t_2 మాత్రమే సూత్రాలు ఉండాయి. అంటే, t = 0 అవసరాలు x(t) = x_0

\[ (0) = C_1 \sin 0 + C_2 \cos 0 = C_1 \cdot 0 + C_2 \cdot 1 \Rightarrow C_2 = x_0 \]

\[ \Rightarrow x(t) = C_1 \sin \omega t + x_0 \cos \omega t \]

స్పష్టంగా యొక్క ప్రత్యేకంగా అంతే సమాధానం ఉంటుంది.

\[ v(t) = \frac{dx(t)}{dt} = \omega C_1 \cos \omega t - \omega x_0 \sin \omega t \]

\[ v(0) = \omega C_1 \cos 0 - \omega x_0 \sin 0 = \omega C_1 \cdot 1 - \omega x_0 \cdot 0 \Rightarrow \omega C_1 = v_0 \]

\[ \Rightarrow C_1 = \frac{v_0}{\omega} \]

\[ \Rightarrow x(t) = \frac{v_0}{\omega} \sin \omega t + x_0 \cos \omega t \quad \text{--- (2)} \]

\[ \Rightarrow v(t) = v_0 \cos \omega t - x_0 \omega \sin \omega t \quad \text{--- (3)} \]

\[ \Rightarrow a(t) = -v_0 \omega \sin \omega t - x_0 \omega^2 \cos \omega t = -\omega^2 x(t) \quad \text{--- (4)} \]

Eq.(1) మొదటి (4) అనే ప్రత్యేకంగా ప్రవేశం,

\[ \omega = \sqrt{\frac{K}{m}} \quad \text{--- (5)} \]
to confine to the value of $\theta \leq 10^\circ$. Further reduction in angle may be considered depending on the accuracy required.

What is the value of $1\ rad$ in degrees? It is $360/2\pi \approx 57.3^\circ$. Thus any angle less than 0.1 $\ rad$ may be considered as small enough, for a two decimal accuracy in angle.

5.2 Solution of SHM Differential Equation

We have already understood that, in simple harmonic motion, restoring force is directly proportional to negative displacement.

$$F \propto -x \Rightarrow F = -kx \Rightarrow m \frac{d^2 x}{dt^2} = -kx \Rightarrow \frac{d^2 x}{dt^2} + \frac{k}{m} x = 0$$

$$\Rightarrow a + \frac{k}{m} x = 0$$

$$\Rightarrow \frac{k}{m} x = 0 - - - (1)$$

Here $K$ is the force constant and $m$ is the mass and $a$ is the acceleration of the body executing SHM.

Since the motion is periodic, the solution could be some function of periodic functions of sine and cosine. Hence, one can assume a solution for displacement as,

$$x(t) = C_1 \sin \omega t + C_2 \cos \omega t.$$ 

In the above, $C_1$, $C_2$ and $\omega$ are unknowns and the constants $C_1$, $C_2$ need to be fixed.

Let the initial displacement be $x_0$. i.e., $x(t) = x_0$ when $t = 0$

$$x(0) = C_1 \sin 0 + C_2 \cos 0 = C_1 \cdot 0 + C_2 \cdot 1 \Rightarrow C_2 = x_0$$

$$\therefore x(t) = C_1 \sin \omega t + x_0 \cos \omega t$$

The velocity of the system is obtained from the time derivative of displacement as,

$$v(t) = \frac{dx(t)}{dt} = \omega C_1 \cos \omega t - \omega x_0 \sin \omega t$$

Let the initial velocity be $v_0$. i.e., $v(t) = v_0$ when $t = 0$. 

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325
5.2 Solution of SHM Differential Equation

Eq (4), (2), \(x_0\) నిమిస్తా

\[ x_0 = A \sin \phi \quad (6) \]

\[ v_0 = \frac{dx_0}{dt} = A \cos \phi \frac{d\phi}{dt} = A \cos \phi \omega \Rightarrow \frac{v_0}{\omega} = A \cos \phi \quad (7) \]

Eq.(2)డి Eq.(6) లేదా (7)య్యి సమాధానం

\[ x(t) = A \cos \phi \sin \omega t + A \sin \phi \cos \omega t \Rightarrow x(t) = A \sin(\omega t + \phi) \quad (8) \]

Eq.(6) లేదా (7) య్యి

\[ A = \sqrt{A^2 \sin^2 \phi + A^2 \cos^2 \phi} = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}} \quad (9) \]

\[ \phi = \tan^{-1}\left(\frac{\sin \phi}{\cos \phi}\right) = \tan^{-1}\frac{x_0}{v_0/\omega} = \tan^{-1}\frac{x_0}{x_0/\omega} = \tan^{-1}\frac{x_0 \omega}{v_0} \quad (10) \]

అంటే Eq.(8) Eq.(9), (5), (10) తో రాశి లేదా \(x_0\) నిమిస్తా \(v_0\), \(x_0\) లేదా \(v_0\) కూ కలిగిన \(K\) లేదా పరిమితి మొత్తాను సమాధానం మొదలినందులు అప్పుడితే శకంబు ఊపసమీక్రత అప్పుడు నియోగం. 

ప్రస్తుతం వేసం

ప్రస్తుతం వేసం మీద రాశి ప్రస్తుతం మొదలినందులు అప్పుడితే అవి సమీకరణ మరియు ఎమనించాలిస్తుంది.

\[ F \propto -x \Rightarrow F = -kx \Rightarrow \frac{d^2x}{dt^2} = -kx \Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \]

\[ \Rightarrow a + \frac{k}{m}x = 0 \]

ఆంటే \(K\) అవధిదాయకాలు గమ్నమైనాంటే \(m\) అవధిదాయకాలు గమ్నమైనాంటే \(a\) అవధిదాయకాలు గమ్నమైనాంటే \(x\) అవధిదాయకాలు గమ్నమైనాంటే. కాబట్టి కాబట్టి ఎమనించాలిస్తుంది.

\[ \frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \Rightarrow \left(\frac{dx}{dt} + i\sqrt{\frac{k}{m}}x\right)\left(\frac{dx}{dt} - i\sqrt{\frac{k}{m}}x\right) = 0 \]

\[ \Rightarrow \left(\frac{dx}{dt} + i\omega x\right)\left(\frac{dx}{dt} - i\omega x\right) = 0, \]
5.2 Solution of SHM Differential Equation

\[ v(0) = \omega C_1 \cos 0 - \omega x_0 \sin 0 = \omega C_1 \cdot 1 - \omega x_0 \cdot 0 \Rightarrow \omega C_1 = v_0 \]
\[ \Rightarrow C_1 = \frac{v_0}{\omega} \]
\[ \therefore x(t) = \frac{v_0}{\omega} \sin \omega t + x_0 \cos \omega t \quad \cdots (2) \]
\[ \Rightarrow v(t) = v_0 \cos \omega t - x_0 \omega \sin \omega t \quad \cdots (3) \]
\[ \Rightarrow a(t) = -v_0 \omega \sin \omega t - x_0 \omega^2 \cos \omega t = -\omega^2 x(t) \quad \cdots (4) \]

From Eq. (1) and (4), we obtain

\[ \omega = \sqrt{\frac{K}{m}} \quad \cdots (5) \]

In Eq. (2), let us consider \( x_0 \) to be given by

\[ x_0 = A \sin \phi \quad \cdots (6) \]

Then

\[ v_0 = \frac{dx_0}{dt} = A \cos \phi \frac{d\phi}{dt} = A \cos \phi \omega \Rightarrow \frac{v_0}{\omega} = A \cos \phi \quad \cdots (7) \]

Substituting Eq. (6) and (7) in Eq. (2), gives the result

\[ x(t) = A \cos \phi \sin \omega t + A \sin \phi \cos \omega t \Rightarrow x(t) \]
\[ = A \sin(\omega t + \phi) \quad \cdots (8) \]

From Eq. (6) and (7)

\[ A = \sqrt{A^2 \sin^2 \phi + A^2 \cos^2 \phi} = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}} \quad \cdots (9) \]

\[ \phi = \tan^{-1} \left( \frac{\sin \phi}{\cos \phi} \right) = \tan^{-1} \frac{x_0}{v_0/\omega} = \tan^{-1} \frac{x_0 \omega}{v_0} \quad \cdots (10) \]

Here Eq. (8) gives the solution of simple harmonic motion equation with unknowns given by Eq. (9), (5), (10) with known values of two initial conditions \( x_0 \) and \( v_0 \), force constant \( K \) and mass \( m \).

Alternative method:

In simple harmonic motion, force is directly proportional to negative displacement.
5.2 Solution of SHM Differential Equation

\[
\omega = \sqrt{\frac{k}{m}} \quad \cdots (1)
\]

ఇకడ, ఏమైనా ఇప్పటి సమాధానం కొనసాగించాలంటే, ఒక మార్గం ప్రత్యేకంగా లభించింది.

\[
\left( \frac{dx}{dt} + i\omega x \right) = \left( \frac{dx}{dt} - i\omega x \right) = 0 \quad \cdots (2)
\]

ఇతర్యయం కావలసినా,

\[
\frac{dx}{dt} + i\omega x = 0 \Rightarrow \frac{dx}{dt} = -i\omega x \Rightarrow \frac{1}{x}dx = -i\omega dt
\]

అంటే చాల కింద యాదివి

\[
\int \frac{1}{x}dx = -i\omega \int dt \Rightarrow \ln x = -i\omega t \Rightarrow e^{\ln x} = e^{-i\omega t} \Rightarrow x = e^{-i\omega t} \quad \cdots (3)
\]

అధికర్ణం తోడించాలంటే,

\[
\frac{dx}{dt} - i\omega x = 0 \Rightarrow \frac{dx}{dt} = i\omega x \Rightarrow \frac{1}{x}dx = i\omega dt
\]

అంటే చాల కింద యాదివి

\[
\int \frac{1}{x}dx = i\omega \int dt \Rightarrow \ln x = i\omega t \Rightarrow e^{\ln x} = e^{i\omega t} \Rightarrow x = e^{i\omega t} \quad \cdots (4)
\]

సమీకరణ (3) మరియు (4) సమీకరణాలంటివలె (2) సమీకరణ ప్రత్యేకంగా చాలంటే, ఇప్పటి సమీకరణాలు ఒక మార్గం ప్రత్యేకంగా యాదివి.

\[
x = A_1 e^{i\omega t} + A_2 e^{-i\omega t}
\]

ఇకడ, విశేషాధికాయనానికి ఎమ్మెల్యే \( e^{\pm i\theta} = \cos \theta \pm i \sin \theta \),

\[
x = A_1 (\cos \omega t + i \sin \omega t) + A_2 (\cos \omega t - i \sin \omega t)
\]

\[
\Rightarrow x = (A_1 + A_2) \cos \omega t + i(A_1 - A_2) \sin \omega t
\]

\[
\Rightarrow x = A \cos \omega t + B \sin \omega t \quad \cdots (5)
\]

అంటే \( A = A_1 + A_2 \) and \( B = i(A_1 - A_2) \)
5.2 Solution of SHM Differential Equation

\[ F \propto -x \Rightarrow F = -Kx \Rightarrow m \frac{d^2x}{dt^2} = -Kx \Rightarrow m \frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \]

\[ \Rightarrow a + \frac{k}{m}x = 0 \]

Here \( K \) is the force constant and \( m \) is the mass and \( a \) is the acceleration of the body executing SHM.

Here one can write

\[ \frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \Rightarrow \left( \frac{dx}{dt} + i \sqrt{\frac{k}{m}}x \right) \left( \frac{dx}{dt} - i \sqrt{\frac{k}{m}}x \right) = 0 \]

\[ \Rightarrow \left( \frac{dx}{dt} + i\omega x \right) \left( \frac{dx}{dt} - i\omega x \right) = 0, \]

where

\[ \omega = \sqrt{\frac{k}{m}} \quad - - - (1) \]

Since the differential operators in individual brackets are independent, both will be equal to zero. Thus

\[ \left( \frac{dx}{dt} + i\omega x \right) = \left( \frac{dx}{dt} - i\omega x \right) = 0 \quad - - - (2) \]

Taking the first part,

\[ \frac{dx}{dt} + i\omega x = 0 \Rightarrow \frac{dx}{dt} = -i\omega x \Rightarrow \int \frac{1}{x} dx = -i\omega \int dt \]

Integrating on both sides,

\[ \int \frac{1}{x} dx = -i\omega \int dt \Rightarrow \ln x = -i\omega t \Rightarrow e^{\ln x} = e^{-i\omega t} \Rightarrow x = e^{-i\omega t} \quad - - - (3) \]

Similarly for the second part,

\[ \frac{dx}{dt} - i\omega x = 0 \Rightarrow \frac{dx}{dt} = i\omega x \Rightarrow \int \frac{1}{x} dx = i\omega \int dt \]

Integrating on both sides,
5.2 Solution of SHM Differential Equation

\[
\cos \omega t, \sin \omega t \text{ are complex.}
\]

\[
A, B \text{ are constants.}
\]

Given that \( \cos \omega t = \frac{B}{a} \Rightarrow B = a \cos \phi = \sqrt{A^2 + B^2} \cos \phi \) (6)

\[
\sin \phi = \frac{A}{a} \Rightarrow A = a \sin \phi = \sqrt{A^2 + B^2} \sin \phi
\]

\[
\phi = \tan^{-1} \left( \frac{A}{B} \right)
\]

Eq.(5) and Eq.(7), (8) are used.

\[
x = a \sin \omega t \cos \phi + a \cos \omega t \sin \phi = a \sin(\omega t + \phi)
\]

1. In SHM, the motion is linear.

2. In SHM, the motion is linear.

3. The motion is linear.

4. In SHM, the motion is linear.

5. The motion is linear.
\[\int \frac{1}{x} \, dx = \omega \int dt \Rightarrow \ln x = i\omega t \Rightarrow e^{\ln x} = e^{i\omega t} \Rightarrow x = e^{i\omega t} \quad (4)\]

Since Eq. (3) and (4) are possible solutions of the differential equation (2), the general solution of which is given by

\[x = A_1 e^{i\omega t} + A_2 e^{-i\omega t}\]

By using Euler's formula, \(e^{\pm i\theta} = \cos \theta \pm i \sin \theta\),

\[x = A_1 (\cos \omega t + i \sin \omega t) + A_2 (\cos \omega t - i \sin \omega t) \Rightarrow x = (A_1 + A_2) \cos \omega t + i(A_1 - A_2) \sin \omega t \Rightarrow x = A \cos \omega t + B \sin \omega t \quad (5)\]

Where \(A = A_1 + A_2\) and \(B = i(A_1 - A_2)\)

If we represent \(\cos \omega t\) and \(\sin \omega t\) as phasors, then those phasors will be \(90^\circ\) apart with magnitudes \(A, B\). Their linear combination can be obtained by triangle rule. Let \(a\) be the resultant amplitude and \(\phi\) be the resultant phase. Then,

\[a = \sqrt{A^2 + B^2} \quad (6)\]

\[\cos \phi = \frac{B}{a} \Rightarrow B = a \cos \phi = \sqrt{A^2 + B^2} \cos \phi \quad (7)\]

\[\sin \phi = \frac{A}{a} \Rightarrow A = a \sin \phi = \sqrt{A^2 + B^2} \sin \phi \quad (8)\]

\[\phi = \tan^{-1}\left(\frac{A}{B}\right) \quad (9)\]

Substituting Eq. (7), (8) in Eq. (5), we get
**Did You Know?**

The choice of general solution of SHM differential equation is optional as \( \sin(\omega t + \phi) = \cos \omega t \) for \( \phi = 90^\circ \). The specific value of \( \phi \) is actually decided by initial velocity \( v_0 \) and initial displacement \( x_0 \).

**Activit**

Instead of the triangle given, one may take flipped triangle with vectors \( A, -B \). Also \( \phi \) can be defined at either of the corners of the triangle. Thus try to obtain other combinations like \( \sin(\phi - \theta) \), \( \cos(\omega t + \theta) \), \( \cos(\phi - \theta) \).

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### ఒడించిన సర్వసాధారణ పరిమితి

మరింత సర్వసాధారణ పరిమితి, మెరియు ప్రధానకాళీ విడివర విస్తరణ పరిమితి, లంభాలతో. ఇంత తరువాత ఎందుకు దర్శనం ఉంది, ఏపి దాని అనుమతి మాత్రమే వలసించడానికి జరుగుతుంది. నాటి వస్తు ముఖ్యమైనందువల్ల, అతని ప్రత్యేకపైన ముఖ్యమైన సంఖ్యలు ఉండవచ్చు. దిద్దడం లేకుండా మాత్రమే, ప్రత్యేకపైన ముఖ్యమైన సంఖ్యలు లేకుండా మాత్రమే ఉండవచ్చు. ఇంత తరువాత పైన ప్రత్యేకపైన ముఖ్యమైన సంఖ్యలు ఉండవచ్చు. మరియు మరింత విలువ ఉండవచ్చు. ఇంత తరువాత ప్రత్యేకపైన ముఖ్యమైన సంఖ్యలు ఉండవచ్చు. దిద్దడం లేకుండా ముఖ్యమైన సంఖ్యలు ఉండవచ్చు. దిద్దడం లేకుండా ముఖ్యమైన సంఖ్యలు ఉండవచ్చు. ఇంత తరువాత ప్రత్యేకపైన ముఖ్యమైన సంఖ్యలు ఉండవచ్చు. దిద్దడం లేకుండా ముఖ్యమైన సంఖ్యలు ఉండవచ్చు. దిద్దడం లేకుండా ముఖ్యమైన సంఖ్యలు ఉండవచ్చు. ఇంత తరువాత ప్రత్యేకపైన ముఖ్యమైన సంఖ్యలు ఉండవచ్చు. దిద్దడం లేకుండా ముఖ్యమైన సంఖ్యలు ఉండవచ్చు. ఇంత తరువాత ప్రత్యేకపైన ముఖ్యమైన సంఖ్యలు ఉండవచ్చు. దిద్దడం లేకుండా ముఖ్యమైన సంఖ్యలు ఉండవచ్చు. ఇంత తరువాత ప్రత్యేకపైన ముఖ్యమైన సంఖ్యలు ఉండవచ్చు. దిద్దడం లేకుండా ముఖ్యమైన సంఖ్యలు ఉండవచ్చు. ఇంత తరువాత ప్రత్యేకపైన ముఖ్యమైన సంఖ్యలు ఉండవచ్చు. దిద్దడం లేకుండా ముఖ్యమైన సంఖ్యలు ఉండవచ్చు. ఇంత తరువాత ప్రత్యేకపైన ముఖ్యమైన సంఖ్యలు ఉండవచ్చు. 

\[
F = -kx - \gamma \frac{dx}{dt} \Rightarrow m \frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + kx = 0 \\
\Rightarrow \frac{d^2x}{dt^2} + 2b \frac{dx}{dt} + \omega^2 x = 0 \quad (1)
\]

అందువల్ల \( b = \gamma / 2m \) మాత్రమే \( \omega^2 = k / m \). అందువల్ల \( b = 1 / \sqrt{2m} \) మాత్రమే \( \omega = \pi / T \) ముఖ్యమైన ఉత్తరాంశం. ఇంత తరువాత ప్రత్యేకపైన ముఖ్యమైన ఉత్తరాంశం ఉండవచ్చు. దిద్దడం లేకుండా ముఖ్యమైన ఉత్తరాంశం ఉండవచ్చు. దిద్దడం లేకుండా ముఖ్యమైన ఉత్తరాంశం ఉండవచ్చు. మరింత మరింత తరువాత ప్రత్యేకపైన ముఖ్యమైన ఉత్తరాంశం ఉండవచ్చు. అంతే మరింత అంతే మరింత తరువాత ప్రత్యేకపైన ముఖ్యమైన ఉత్తరాంశం ఉండవచ్చు. మరింత మరింత తరువాత ప్రత్యేకపైన ముఖ్యమైన ఉత్తరాంశం ఉండవచ్చు. మరింత మరింత తరువాత ప్రత్యేకపైన ముఖ్యమైన ఉత్తరాంశం ఉండవచ్చు. 

ఇంత తరువాత ప్రత్యేకపైన ముఖ్యమైన ఉత్తరాంశం ఉండవచ్చు. 

\[
x = Ae^{\alpha t} \Rightarrow \frac{dx}{dt} = \alpha Ae^{\alpha t} = \alpha x \Rightarrow \frac{d^2x}{dt^2} = \alpha^2 Ae^{\alpha t} \\
= \alpha^2 x \quad (2)
\]
5.3 Damped harmonic oscillator

\[ x = a \sin \omega t \cos \phi + a \cos \omega t \sin \phi = a \sin(\omega t + \phi) \]  \hspace{1cm} (10)

This gives the solution of differential equation for SHM with unknowns given by Eq. (6), (1) and (9).

Note:

1. Sine, cosine functions are solutions of second order differential equations

2. Exponential functions are solutions of first order differential equations (differential equation of velocity/momentum). At the same time they also serve as solutions of second order differential equation.

3. Thus second order differential equation can have both sine, cosine as well as complex exponential solutions. They are connected by Euler’s equation.

4. Only exponential imaginary functions produce oscillatory sine and cosine functions upon linear superposition. If the exponent is real, the exponential function would give ever rising or ever decaying function and produce sine and cosine hyperbolic functions upon superposition.

5. Linearity of solutions: The solutions of linear differential equation (where power of \( x \) is unity) will always exhibit linearity property; i.e., any LINEAR combination of the solutions also works as a solution for the equation. This property proves to be useful while studying damped and forced oscillations, as they are also linear; at the same time a bit more complicated compared to SHM.

5.3 Damped harmonic oscillator

In simple harmonic motion, force is directly proportional to negative displacement. If the force also depends on velocity, then it will affect the amplitude of oscillations and results in damped harmonic motion.

The force equation for damped harmonic motion is given by
5.3 Damped harmonic oscillator

\[ a^2 x + 2bax + \omega^2 x = 0 \]

for \( x \neq 0 \),

\[ a^2 + 2ba + \omega^2 = 0 \quad ---(3) \]

Therefore, the characteristic equation is found to be

\[ \alpha = \frac{-2b \pm \sqrt{(2b)^2 - 4.1\omega^2}}{2} = -b \pm \sqrt{b^2 - \omega^2} \quad ---(4) \]

Therefore, the solution is

\[ x = A_1 e^{(-b+\sqrt{b^2-\omega^2})t} + A_2 e^{(-b-\sqrt{b^2-\omega^2})t} = e^{-bt} \left[ A_1 e^{\sqrt{b^2-\omega^2}t} + A_2 e^{-\sqrt{b^2-\omega^2}t} \right] \quad ---(5) \]

\[ b^2 \leq \omega^2 \Rightarrow b^2 - \omega^2 < 0 \Rightarrow \sqrt{b^2 - \omega^2} = \text{imaginary} \]

\[ = i\beta \quad \text{(say)} \quad ---(6) \]

Therefore, the solution is

\[ x = e^{-bt}[A_1 e^{i\beta t} + A_2 e^{-i\beta t}] \]

\[ x = e^{-bt} \left[ A_1 (\cos \beta t + i \sin \beta t) + A_2 (\cos \beta t - i \sin \beta t) \right] \]

\[ = e^{-bt} \left[ (A_1 + A_2) \cos \beta t + i(A_1 - A_2) \sin \beta t \right] \]

\[ = e^{-bt} (a \sin \phi \cos \beta t + a \cos \phi \sin \beta t) \]

\[ \therefore x = ae^{-bt} \sin(\beta t + \phi) \quad ---(7) \]

Here \( a \sin \phi = A_1 + A_2 \) and \( a \cos \phi = i(A_1 - A_2) \)

The above solution forms the solutions of the differential equation where \( b = \gamma / 2m \) and \( \gamma \) is the rate of damping coefficient.
5.3 Damped harmonic oscillator

\[ F = -kx - \gamma \frac{dx}{dt} \Rightarrow m \frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + kx = 0 \]

\[ \Rightarrow \frac{d^2x}{dt^2} + 2b \frac{dx}{dt} + \omega^2 x = - - - (1) \]

Here \( b = \gamma / 2m \) and \( \omega^2 = k / m \). Here \( b \) has the units of \( 1/\text{Time} \) and \( \omega = 2\pi / T \). Thus, there will be a discrepancy by a factor of \( 2\pi \) between these two quantities. But for all addition and subtraction purposes, we consider only the matching dimensional formula. Whenever there are division and multiplication operations, that \( 2\pi \) factor may make a difference (for example, while defining the quality factor, discussed in a later section).

Consider an exponential solution for the displacement of the particle,

\[ x = Ae^{\alpha t} \Rightarrow \frac{dx}{dt} = \alpha Ae^{\alpha t} = \alpha x \Rightarrow \frac{d^2x}{dt^2} = \alpha^2 Ae^{\alpha t} \]

\[ = \alpha^2 x = - - - (2) \]

Substituting Eq. (2) in Eq. (1), we get

\[ \alpha^2 x + 2b\alpha x + \omega^2 x = 0 \]

Since \( x \neq 0 \),

\[ \alpha^2 + 2b\alpha + \omega^2 = 0 \quad - - - (3) \]

The solution of this quadratic equation in \( \alpha \) is given by

\[ \alpha = \frac{-2b \pm \sqrt{(2b)^2 - 4.1.\omega^2}}{2.1} = -b \pm \sqrt{b^2 - \omega^2} \quad - - - (4) \]

Thus the general solution for displacement is given by

\[ x = A_1 e^{(-b + \sqrt{b^2 - \omega^2})t} + A_2 e^{(-b - \sqrt{b^2 - \omega^2})t} = e^{-bt} \left[ A_1 e^{\sqrt{b^2 - \omega^2}t} + A_2 e^{-\sqrt{b^2 - \omega^2}t} \right] \quad - - - (5) \]

Case-1: Underdamped

In this case,

\[ b^2 < \omega^2 \Rightarrow b^2 - \omega^2 < 0 \Rightarrow \sqrt{b^2 - \omega^2} = \text{imaginary} \]

\[ = i\beta \text{ (say)} \quad - - - (6) \]

Substituting Eq. (6) in Eq. (5) gives
5.3 Damped harmonic oscillator

\[ \beta^2 = \omega^2 - b^2, \quad \text{and} \quad b^2 = (\omega^2 - b^2) \]

\[ \beta^2 = \omega^2 - b^2 \]

\[ \beta = \sqrt{(b^2 - \omega^2)} \]

\[ x(t) = e^{-bt} \left[ A_1 e^{t\sqrt{b^2 - \omega^2}} + A_2 e^{-t\sqrt{b^2 - \omega^2}} \right] \approx A_1 e^{(-b + \sqrt{b^2 - \omega^2})t} \]

\[ A_1 + A_2 = 0 \quad \text{and} \quad -b(A_1 + A_2) + \beta(A_1 - A_2) = v_0 \Rightarrow A_1 = -A_2 \]

\[ A_1 = \frac{v_0}{2\beta} \]

\[ \beta = \sqrt{(b^2 - \omega^2)} \]

\[ x(t) = \frac{v_0}{2\beta} e^{-bt} \sin \beta t \]

\[ e^{(2t\sqrt{b^2 - \omega^2})} = e^{(-2t\sqrt{b^2 - \omega^2})} = e^{(-h\sqrt{b^2 - \omega^2})} = e^{(-ht)} \]

\[ x = e^{-bt} [A_1 e^{ht} + A_2 e^{-ht}] \]

\[ = e^{-bt} [A_1 (1 + ht) + A_2 (1 - ht)] \]

\[ = e^{-bt} [(A_1 + A_2) + h(A_1 - A_2)] = e^{-bt} (p + qt) \]

Here \( p = A_1 + A_2, \quad q = h(A_1 - A_2) \)
5.3 Damped harmonic oscillator

\[ x = e^{-bt} \left[ A_1 e^{i\beta t} + A_2 e^{-i\beta t} \right] \]

Using Euler equation,

\[
\begin{align*}
    x &= e^{-bt} [A_1 (\cos \beta t + i \sin \beta t) \\
    &+ A_2 (\cos \beta t - i \sin \beta t)] \\
    &= e^{-bt} [(A_1 + A_2) \cos \beta t + i(A_1 - A_2) \sin \beta t] \\
    &= e^{-bt} (a \sin \phi \cos \beta t + a \cos \phi \sin \beta t) \\
    \therefore \: x &= ae^{-bt} \sin(\beta t + \phi) \quad (7)
\end{align*}
\]

Here \( a \sin \phi = A_1 + A_2 \) and \( a \cos \phi = i(A_1 - A_2) \)

Thus the solution is a sine function with amplitude modified by an exponentially decaying function of \( b = \frac{\nu}{2m} \) with time.

The frequency of oscillations is \( \beta^2 = \omega^2 - b^2 \) which is reduced by a factor of \( b^2 \) but remains constant for the entire course.

Case-2: Over damped

In this case, \( b^2 > \omega^2 \), then \(-b + \sqrt{b^2 - \omega^2}\) is a small negative number and \(-b - \sqrt{b^2 - \omega^2}\) is a large negative number. Then the displacement \( x \) reduces to zero exponentially without making any oscillations. Here the dominant term is

\[ x(t) = e^{-bt} \left[ A_1 e^{t\sqrt{b^2 - \omega^2}} + A_2 e^{-t\sqrt{b^2 - \omega^2}} \right] \approx A_1 e^{(-b+\sqrt{b^2-\omega^2})t} \]

To know the exact behavior of the system, one needs to consider the initial conditions precisely. Consider a system at equilibrium initially, was subjected to a driving force. Then \( x = 0 \) and \( v = v_0 \) at \( t = 0 \). Substituting these two in the above, gives

\[ A_1 + A_2 = 0 \quad \text{and} \quad -b(A_1 + A_2) + \beta (A_1 - A_2) = v_0 \Rightarrow A_1 = -A_2 \]

\[ = \frac{v_0}{2\beta} \]

Here \( \beta = \sqrt{b^2 - \omega^2} \). This yields

\[ x(t) = \frac{v_0}{2\beta} e^{-bt} \sin \beta t \]
5.3 Damped harmonic oscillator

A plot of this function looks like this.

Fig: Displacement in over damping.

Thus the displacement increases exponentially and then falls exponentially. This is also called dead beat.

Case-3: Critically damped

In this case \( b^2 = \omega^2 \). Let us consider \( \sqrt{b^2 - \omega^2} = h \to 0 \Rightarrow e^{\pm ht} \approx 1 \pm h t \) then

\[
x = e^{-bt}[A_1 e^{ht} + A_2 e^{-ht}]
\]

\[
= e^{-bt}[A_1 (1 + ht) + A_2 (1 - ht)]
\]

\[
= e^{-bt}[(A_1 + A_2) + h(A_1 - A_2) = e^{-bt}(p + qt)]
\]

Here \( p = A_1 + A_2, \ q = h(A_1 - A_2) \)

Here \( p + qt \) rises linearly with time but \( e^{-bt} \) falls exponentially. In the case of over damping, the amplitude falls exponentially due to \( e^{-bt} \) at the same time rises exponentially due to \( e^{\sqrt{b^2 - \omega^2}t} \). Thus in critical damping, there is a net decay in amplitude with time but the decay is faster in the case of critical damping. This is because the term responsible for rising amplitude is linear in critical damping and is exponential in over damping.
5.4 Quality factor

The quality $Q$ factor is defined as the ratio of input energy to the loss in energy per cycle.

$$Q = \frac{E}{\Delta E/2\pi} = 2\pi \frac{E}{\Delta E}$$

In the case of damping, the amplitude is given by $A = A_0 e^{-bt}$ where $A_0$ is the amplitude without damping. Then the corresponding relation between energies is given by

$$E = E_0 e^{-2bt} \quad (\because E \propto A^2)$$

For small values of damping, one can expand the exponential in power series and retain the first a few terms. Thus

$$E = E_0 (1 - 2bt) \Rightarrow E = E_0 - E_0 2bt \Rightarrow \Delta E = E_0 - E = E_0 2bt$$

The change in energy per one cycle is given by

$$\Delta E = E_0 2bT = E_0 2b \frac{2\pi}{\omega}$$

Then the $Q$-factor is given by

$$Q = 2\pi \frac{E_0}{\Delta E} = \frac{\omega}{2b} = \omega \tau_m$$
మంత్రి భాషా

రాష్ట్రపతి

రాతి(Q−) కాలం తొలి బొడ్డి వచ్చి తెలియడం సాధనానికి పొందించారు.

\[ Q = \frac{E}{\Delta E/2\pi} = 2\pi \frac{E}{\Delta E} \]

ఈకి క్రమంగా, ఎలియాస్త్ర యొక్క అయ్య అంశం మేము పొందారు, అంటే \( Q=0 \) అనే ప్రఖ్యాతం సమయం యొక్క అంతరంకం

\[ E = E_0 e^{-2bt} \quad (\because E \propto A^2) \]

ఈప్పుడు సూచించది సమయం యొక్క అయ్య అంశం మేము పొందారు, అనేక పరిపాలన ఉండే మూలాలు, మేము ఇప్పుడు సమయం యొక్క అంతరంకం

\[ E = E_0 (1 - 2bt) \Rightarrow E = E_0 - E_0 2bt \Rightarrow \Delta E = E_0 - E = E_0 2bt \]

అంటే రెండు స్థానానికి వచ్చి తెలిసించారు.

\[ \Delta E = E_0 2bT = E_0 2b \frac{2\pi}{\omega} \]

అనేక దండాలు ఉండే అంతరంకంలో

\[ \tau_m = \frac{1}{2b} \]

ఈక కారణం కోదు, \( Q>1/2 \), అనేక కారణం \( Q<1/2 \) పొలించి రెండు స్థానానికి వచ్చి తెలిసించారు, \( Q=1/2 \).

సంకేతాన్ని కలిగి ఉండి రెండు పాటుక ఉంటుంది

సంకేతాన్ని కలిగి ఉండి రెండు పాటుక ఉంటుంది, రెండు స్థానానికి సమయం యొక్క అంతరంకం పొందారు, అనేక అవసరం

\[ t = \tau, A = A_0 e^{-t} = \frac{A_0}{e} \Rightarrow bt = 1 \Rightarrow \tau = \frac{1}{b} \]

ఈక మీది క్రమంగా ప్రతి అంశం మేము పొందారు, ఎలియాస్త్ర యొక్క అయ్య అంశం మేము పొందారు.

ఈక కారణం కోదు రెండు పాటుక ఉంటుంది ప్రబలమైన ఉతానం యొక్క అంతరంకం \( T=2\pi/\sqrt{\omega^2-b^2} \) అనేక మూలాలు పొందారు వాయితే \( T=2\pi/\sqrt{(\omega^2-b^2)} \) అలంపై మూలాలు పొందారు డిసిప్సై అసమాన సమయం యొక్క అంతరంకం పొందారు.

340
Here $\tau_m$ is the mean relaxation time for energy to reduce to $1/e$ times its original value. Thus

$$
\tau_m = \frac{1}{2b}
$$

For under damping, $Q > \frac{1}{2}$, for over damping $Q < \frac{1}{2}$ and for critical damping, $Q = \frac{1}{2}$.

**Time constant of underdamped motion**

The time constant of underdamped motion is defined as the time within which the amplitude falls $1/e$ times its initial (undamped) amplitude, i.e., when

$$
t = \tau, A = A_0 e^{-1} = \frac{A_0}{e} \Rightarrow bt = 1 \Rightarrow \tau = \frac{1}{b}
$$

Here the amplitude of damped harmonic oscillator is given by $A = A_0 e^{-bt}$.

The frequency of underdamped oscillations is $\sqrt{\omega^2 - b^2}$ and hence the time period is $T = 2\pi/\sqrt{\omega^2 - b^2}$. Thus the number of cycles of waves generated within the time constant duration is

$$
n = \frac{\tau}{T} = \frac{1/b}{2\pi/\sqrt{\omega^2 - b^2}} = \frac{1}{2\pi b} \sqrt{\omega^2 - b^2} = \frac{1}{2\pi} \sqrt{\frac{\omega^2 - b^2}{b^2}} = \frac{1}{2\pi} \sqrt{\frac{\omega^2}{b^2} - 1} \approx \frac{1}{2\pi b} = \frac{1}{2\pi} \frac{Q}{2\pi}
$$

The number of oscillations, which the oscillator makes, during mean relaxation time or energy relaxation time is

$$
N = \frac{\tau_m}{T} = \frac{1/2b}{2\pi/\sqrt{\omega^2 - b^2}} = \frac{1}{2\pi} \frac{1}{2b} \sqrt{\omega^2 - b^2} = \frac{1}{4\pi} \sqrt{\frac{\omega^2 - b^2}{b^2}} = \frac{1}{4\pi} \sqrt{\frac{\omega^2}{b^2} - 1} \approx \frac{1}{4\pi b} = \frac{Q}{2\pi}
$$

Thus the number of oscillations made during amplitude relaxation time is double that made during mean relaxation time.
\[ n = \frac{\tau}{T} = \frac{1/b}{2\pi/\sqrt{\omega^2 - b^2}} = \frac{1}{2\pi b} \sqrt{\omega^2 - b^2} = \frac{1}{2\pi} \sqrt{\frac{\omega^2 - b^2}{b^2}} = \frac{1}{2\pi} \sqrt{\frac{\omega^2}{b^2} - 1} \approx \frac{1}{2\pi b} \]

\[ N = \frac{\tau_m}{T} = \frac{1/2b}{2\pi/\sqrt{\omega^2 - b^2}} = \frac{1}{2\pi b} \sqrt{\omega^2 - b^2} = \frac{1}{2\pi} \sqrt{\frac{\omega^2 - b^2}{b^2}} = \frac{1}{4\pi} \sqrt{\frac{\omega^2}{b^2} - 1} \]

\[ \approx \frac{1}{4\pi b} = \frac{Q}{2\pi} \]

\[
\begin{align*}
E - Corner
\end{align*}
\]

http://spiff.rit.edu/classes/phys283/lectures/forced_ii/forced_ii.html


http://farside.ph.utexas.edu/teaching/336k/Newtonhtml/node20.html
5.5 Logarithmic decrement

\[ n_\tau = 2N_{\tau_m} \quad \text{or} \quad \tau_m = \frac{\tau}{2} \]

Note: Q-factor is defined for energy relaxation. Amplitude relaxation time is instantaneous and energy relaxation time is a mean value which turns out to be half of the instantaneous value.

Thus more the Q-factor, more is the number of oscillations within time constant interval. Quality factor is larger when \( b \) is smaller. Thus lower is the damping, more is the number of oscillations within \( \tau \) duration. In other words, Q-factor is a measure of approach to undamped motion; i.e., more the Q-factor, lower is the damping.

5.5 Logarithmic decrement

Consider the amplitude of damped harmonic oscillator at regular intervals, say at \( 0T, 1T, 2T, 3T, 4T \) etc ... be \( A_0, A_1, A_2, A_3, A_4 \) etc ... This basic time interval \( T \) may be arbitrary and has no connection with relaxation times \( \tau \) or \( \tau_m \).

Then one can write

\[ A_1 = A_0e^{-bT} \]
\[ A_2 = A_0e^{-2bT} = A_1e^{-bT} \]
\[ A_3 = A_0e^{-3bT} = A_2e^{-bT} \]
\[ A_4 = A_0e^{-4bT} = A_3e^{-bT} \]

This gives rise to

\[ \frac{A_0}{A_1} = \frac{A_1}{A_2} = \frac{A_2}{A_3} = \frac{A_3}{A_4} = \cdots = e^{bT} = e^\lambda \ (say) \]

Then

\[ \ln \frac{A_0}{A_1} = \ln \frac{A_1}{A_2} = \ln \frac{A_2}{A_3} = \ln \frac{A_3}{A_4} = \cdots = \lambda = bT \]

If the ratios are added up for \( n \) such intervals, one obtains

\[ \ln \frac{A_0}{A_1} + \ln \frac{A_1}{A_2} + \ln \frac{A_2}{A_3} + \ln \frac{A_3}{A_4} + \cdots + \ln \frac{A_{n-1}}{A_n} = \ln \left( \frac{A_0}{A_1} \cdot \frac{A_1}{A_2} \cdots \frac{A_{n-1}}{A_n} \right) \]

\[ = \ln \frac{A_0}{A_n} = n\lambda \]
బాగిని ఏంటెలా

అగించరిల ఎంపక్కొనం

ప్రత్యేకంగా మొన ప్రస్తుతికి ప్రత్యేకమైన సిద్ధాంతాలు, 0T, 1T, 2T, 3T, 4T మార్గాలు కూడా ఉన్నాం... A_0, A_1, A_2, A_3, A_4 మొత్తం విలువులు... అ ప్రకారం ప్రత్యేక శాస్త్ర విధానానికి చెందిన అవసరాలు ఇంటి తము అంటే హెచ్చరి విమర్శనం చేస్తే

అంశం

\[ A_1 = A_0 e^{-bT} \]
\[ A_2 = A_0 e^{-2bT} = A_1 e^{-bT} \]
\[ A_3 = A_0 e^{-3bT} = A_2 e^{-bT} \]
\[ A_4 = A_0 e^{-4bT} = A_3 e^{-bT} \cdots \]


gోతారమే అంశాల అనుసారం పట్టించి పినప్పు చేస్తే

\[ \frac{A_0}{A_1} = \frac{A_1}{A_2} = \frac{A_2}{A_3} = \cdots = e^{bT} = e^\lambda \text{ (say)} \]

అంశానుకు అవసరం పట్టించి పినప్పు చేస్తే

\[ \ln \frac{A_0}{A_1} + \ln \frac{A_1}{A_2} + \ln \frac{A_2}{A_3} + \ldots \ln \frac{A_{n-1}}{A_n} = \ln \left( \frac{A_0}{A_1} \cdot \frac{A_1}{A_2} \cdots \frac{A_{n-1}}{A_n} \right) \]

\[ = \ln \frac{A_0}{A_n} = n\lambda \]

అంశాను పట్టించి పినప్పు చేస్తే, అత్రి అంశాల అనుసారం విదేశం లేదు తాము మార్గాల సమయంలో విదేశం కు వంటి అవసరాల ఉంది. కానీ అది చాలా విదేశంలో లేదు. కానీ అది అంశాను పట్టించి పినప్పు చేయడానికి సమయం అవిచ్చును.

లక్షణానికి కృతిమందమైన ఓటెలా

ప్రత్యేకంగా లక్షణానికి ఉపయోగించిన రెండు అంశాల సమాధానం, అంశాల లక్షణానికి ఉపయోగించిన రెండు అంశాల సమాధానం ల ఉన్నాం. దీనిని చూడండి అంశాల ప్రత్యేకంగా లేదు, ప్రత్యేకంగా సమాధానం విదేశం లేదు. రెండు అంశాల సమాధానం ల ఉన్నాం, అంశాల సమాధానం అంటే కానీ అది రెండు అంశాల సమాధానం అవిచ్చును. రెండు అంశాల సమాధానం ల ఉన్నాం

\[ m \frac{d^2x}{dt^2} = -kx - \gamma \frac{dx}{dt} + F_0 \cos \omega t \]

\[ \frac{d^2x}{dt^2} + \frac{\gamma}{m} \frac{dx}{dt} + \frac{k}{m} x = \frac{F_0}{m} \cos \omega t \quad - - - (1) \]
This shows that at regular intervals of time, the amplitude of the
damped harmonic oscillator reduces exponentially or the logarithm
of ratio of amplitudes at regular intervals of damped harmonic
oscillator remains constant.

### 5.6 Forced oscillations

In a simple harmonic motion, if an external driving force with
constant frequency acts on the system in addition to the restoring
force, the system undergoes forced oscillations. If damping is also
included, the force equation for damped and forced harmonic
oscillator is given by

\[
m \frac{d^2 x}{dt^2} = -k x - \gamma \frac{dx}{dt} + F_0 \cos \omega t
\]

\[
\frac{d^2 x}{dt^2} + \frac{\gamma}{m} \frac{dx}{dt} + \frac{k}{m} x = \frac{F_0}{m} \cos \omega t \quad \cdots (1)
\]

Let \( \gamma/m = 2b \), \( k/m = \omega^2 \), \( F_0/m = f \), then the force equation
becomes,

\[
\frac{d^2 x}{dt^2} + 2b \frac{dx}{dt} + \omega^2 x = f \cos \omega t \quad \cdots (2)
\]

Here \( F_0 \) is the amplitude and \( \omega \) is the frequency of driving force, \( \gamma \) is
the damping constant, \( \omega_0 \) is the undamped oscillator frequency, \( k \) is
the force constant and \( m \) is the mass of the oscillator.

In exponential form, this equation can be written as

\[
\frac{d^2 x}{dt^2} + 2b \frac{dx}{dt} + \omega^2 x = f e^{i\omega t} \quad \cdots (3)
\]

Since the object oscillates at the frequency of driving force, with an
arbitrary phase change \( \phi \), one can assume a solution of the form

\[
x = A e^{i(\omega t + \phi)} \Rightarrow \frac{dx}{dt} = i\omega A e^{i(\omega t + \phi)} = i\omega x \Rightarrow \frac{d^2 x}{dt^2} = -\omega^2 A e^{i(\omega t + \phi)} = -\omega^2 x \quad \cdots (4)
\]

Substituting Eq. (4) in Eq. (3), we get

\[
(\omega_0^2 - \omega^2 + 2ib\omega)A e^{i(\omega t + \phi)} = f e^{i\omega t}
\]
5.6 Forced oscillations

\[ \gamma/m = 2b, \, k/m = \omega^2, \, F_0/m = f, \]

\[ \Rightarrow \frac{d^2x}{dt^2} + 2b \frac{dx}{dt} + \omega^2 x = f \cos \omega t \]  \hspace{1cm} (2)

\[ \tan \theta = \frac{2b\omega}{\omega^2 - \omega^2} \]  \hspace{1cm} (6)

\[ v = \frac{dx}{dt} = \frac{f \omega}{|z|} \sin (\omega t - \phi) \]

\[ A = \frac{f}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2}} \]  \hspace{1cm} (7)

\[ x = A e^{i(\omega t + \phi)} = \frac{f}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2}} e^{i(\omega t - \theta)} = \frac{f}{|z|} \cos (\omega t - \phi) \hspace{1cm} (Real \ part) \]
5.6 Forced oscillations

\[ A = \frac{fe^{-i\phi}}{\omega_0^2 - \omega^2 + 2i\omega} = \frac{fe^{-i\phi}}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2}e^{i\theta}} \]  

\[ \sqrt{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2} \]

Fig: Complex plane diagram of components of amplitude.

Here

\[ \tan \theta = \frac{2b\omega}{\omega_0^2 - \omega^2} \]

Since amplitude is a real quantity, the phase factors must cancel out in Eq. (5); i.e., \( \phi = -\theta \). Thus the unknown phase \( \phi \) is given by \(-ve\) of Eq. (6).

Thus the amplitude and solution (displacement) are given by

\[ A = \frac{f}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2}} \]

\[ x = Ae^{i(\omega t + \phi)} = \frac{f}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2}}e^{i(\omega t - \theta)} = \frac{f}{|z|} \cos(\omega t - \phi) \] (Real part)

\[ v = \frac{dx}{dt} = \frac{f\omega}{|z|} \sin(\omega t - \phi) \]

\[ E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}m\omega_0^2 \frac{f^2}{|z|^2} \sin^2(\omega t - \phi) + \frac{1}{2}m\omega_0^2 \frac{f^2}{|z|^2} \cos^2(\omega t - \phi) \]

Here \(|z| = \sqrt{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2} \).
5.6 Forced oscillations

\[ E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \]

\[ = \frac{1}{2}m\omega^2 f^2 \frac{\sin^2(\omega t - \phi)}{|z|^2} + \frac{1}{2}m\omega_0^2 f^2 \frac{\cos^2(\omega t - \phi)}{|z|^2} \]

Here \(|z| = \sqrt{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2}\)

In case \(\omega \gg \omega_0\), the case 90° angle is obtained. Suppose \(\omega = \omega_0\), then \(\omega \ll \omega_0\) hence

\[ E_{res} = \frac{1}{2} m\omega_0^2 f^2 \frac{\sin^2(\omega t - \phi)}{|z|^2} \]

\text{Case-1:}

\(\omega \ll \omega_0\), hence

\[ A = \frac{f}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2}} \approx \frac{f}{\omega_0^2} = \frac{F_0}{m\omega_0^2} = \frac{F_0}{k} \]

and

\[ \tan \theta = \frac{2b\omega}{\omega_0^2 - \omega^2} \approx \frac{2b\omega}{\omega_0^2} \approx 0 \quad (\because \omega \ll \omega_0) \Rightarrow \theta = 0° \]

In case \(\omega = \omega_0\), the case 90° angle is obtained. Suppose \(\omega = \omega_0\)

\[ \tan \theta = \frac{2b\omega}{\omega_0^2 - \omega^2} = \frac{2b\omega}{0} = \infty \Rightarrow \theta = 90° \]

\text{Case-3:}
5.6 Forced oscillations

Thus displacement and velocity are $90^\circ$ out of phase and the total energy is oscillatory. At resonance where $\omega = \omega_0$, total energy will be

$$E_{\text{res}} = \frac{1}{2} m \omega_0^2 \frac{f^2}{|z|^2}$$

Case-1:

When $\omega \ll \omega_0$, then

$$A = \frac{f}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2}} \approx \frac{f}{\omega_0^2} = \frac{F_0}{m \omega_0^2} = \frac{F_0}{k}$$

and

$$\tan \theta = \frac{2b\omega}{\omega_0^2 - \omega^2} \approx \frac{2b\omega}{\omega_0^2} \approx 0 \quad (\because \omega \ll \omega^2) \Rightarrow \theta = 0^\circ$$

Thus amplitude of oscillation depends upon the amplitude of driving force and the force constant of the oscillator. The displacement will be in phase with the applied force oscillations.

Case-2:

Resonance occurs when $\omega = \omega_0$ and the solutions are

$$A = \frac{f}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2}} \approx \frac{f}{2b\omega} = \frac{F_0}{m} = \frac{F_0}{\gamma \omega}$$

$$\tan \theta = \frac{2b\omega}{\omega_0^2 - \omega^2} = \frac{2b\omega}{0} = \infty \Rightarrow \theta = 90^\circ$$

Thus amplitude of oscillation depends on driving force amplitude, its frequency as well as on the damping constant. The displacement will be $90^\circ$ out of phase with the driving force.

Case-3:

When $\omega \gg \omega_0$, the solutions are given as

$$A = \frac{f}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2}} \approx \frac{f}{\omega^2} = \frac{F_0}{m \omega^2} \quad (\because \omega^4 \gg 4b^2 \omega^2)$$
5.6 Forced oscillations

\[ A = \frac{f}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2}} \approx \frac{f}{\omega^2} = \frac{F_0}{m\omega^2} \quad (\because \omega^4 \gg 4b^2 \omega^2) \]

\[ \tan \theta = \frac{2b\omega}{\omega_0^2 - \omega^2} \approx -\frac{2b}{\omega} \approx 0 \Rightarrow \theta = 180^\circ \]

అనుమతిలేవి సందర్భము లో భావించబడింది అనుమతిలేవి సందర్భము లో భావించబడింది.

మరుభూమి:

\[ \frac{dz}{d\omega} = 0 \Rightarrow \frac{d}{d\omega} \sqrt{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2} = 0 \]

\[ \Rightarrow \frac{2(\omega_0^2 - \omega^2)(-2\omega) + 2b^2 \cdot 2\omega}{2 \sqrt{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2}} = 0 \]

\[ \Rightarrow (\omega_0^2 - \omega^2) = 2b^2. \]

అనుమతిలేవి బ సంఖ్య ప్రత్యేకమైన గుర్తించడానికి సంఖ్య ప్రత్యేకమైన గుర్తించడానికి సంఖ్య ప్రత్యేకమైన గుర్తించడానికి సంఖ్య ప్రత్యేకమైన గుర్తించడానికి సంఖ్య ప్రత్యేకమైన గుర్తించడానికి సంఖ్య ప్రత్యేకమైన గుర్తించడానికి.

అనుమతిలేవి బ సంఖ్య ప్రత్యేకమైన గుర్తించడానికి సంఖ్య ప్రత్యేకమైన గుర్తించడానికి సంఖ్య ప్రత్యేకమైన గుర్తించడానికి సంఖ్య ప్రత్యేకమైన గుర్తించడానికి సంఖ్య ప్రత్యేకమైన గుర్తించడానికి 

\[ A_{\text{max}} = \frac{f}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2}} \]

\[ = \frac{f}{\sqrt{(\omega_0^2 - \omega_0^2 + 2b^2)^2 + 4b^2(\omega_0^2 - 2b^2)}} \]

\[ \Rightarrow A_{\text{max}} = \frac{f}{2b\sqrt{b^2 + \omega_0^2 - 2b^2}} = \frac{f}{2b\sqrt{\omega_0^2 - b^2}} = \frac{f}{2b\sqrt{\omega^2 + b^2}} \]

అనుమతిలేవి బ సంఖ్య ప్రత్యేకమైన గుర్తించడానికి సంఖ్య ప్రత్యేకమైన గుర్తించడానికి సంఖ్య ప్రత్యేకమైన గుర్తించడానికి సంఖ్య ప్రత్యేకమైన గుర్తించడానికి సంఖ్య ప్రత్యేకమైన గుర్తించడానికి సంఖ్య ప్రత్యేకమైన గుర్తించడానికి 

\[ A_{\text{max}} \rightarrow \infty \quad \text{as} \quad b \rightarrow 0. \quad A_{\text{max}} \rightarrow \infty \quad \text{as} \quad b \rightarrow 0. \]
\[ \tan \theta = \frac{2b\omega}{\omega_0^2 - \omega^2} \approx -\frac{2b}{\omega} \approx -0 \Rightarrow \theta = 180^\circ \]

which means that the amplitude is independent of damping force and depends on mass of the particle, driving force amplitude and frequency. The displacement will be 180° out of phase with the driving force.

**Amplitude resonance**

During resonance the amplitude of forced oscillations will be maximum if the denominator in the expression for amplitude is minimum. This can be obtained by taking

\[ \frac{dz}{d\omega} = 0 \Rightarrow \frac{d}{d\omega}\sqrt{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2} = 0 \]

\[ \Rightarrow \frac{2(\omega_0^2 - \omega^2)(-2\omega) + 2b^2 \cdot 2\omega}{2\sqrt{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2}} = 0 \]

\[ \Rightarrow (\omega_0^2 - \omega^2) = 2b^2. \]

Here \( \omega_0 \) and \( b \) are constants for the given system and driving frequency is the variable which needs to be fixed. The amplitude is maximized when \( \omega^2 = \omega_0^2 - 2b^2 \).

Thus, maximum amplitude is given by

\[ A_{max} = \frac{f}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2}} \]

\[ = \frac{f}{\sqrt{(\omega_0^2 - \omega_0^2 + 2b^2)^2 + 4b^2(\omega_0^2 - 2b^2)}} \]

\[ \Rightarrow A_{max} = \frac{f}{2b\sqrt{b^2 + \omega_0^2 - 2b^2}} = \frac{f}{2b\sqrt{\omega_0^2 - b^2}} = \frac{f}{2b\sqrt{\omega^2 + b^2}} \]

Thus, as \( b \to 0 \), \( A_{max} \to \infty \) and as \( b \) rises, \( A_{max} \) rises. Also as \( b \) rises, the location of \( A_{max} \) shifts to lower values of \( \omega \).
5.6 Forced oscillations

\[ \frac{A_{\text{max}}}{A} = \frac{f/2b\omega}{f/\sqrt{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2}} = \sqrt{\left(\frac{\omega_0^2 - \omega^2}{2b\omega}\right)^2 + 1} \]

\[ \Rightarrow \frac{A_{\text{max}}}{A} \approx 1 + \frac{(\omega_0^2 - \omega^2)^2}{2(2b\omega)^2} \]

Further reading:

Sharpness of resonance:

From the above equations one can write

\[
\frac{A_{\text{max}}}{A} = \frac{f/2b\omega}{f/\sqrt{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2}} = \frac{(\omega_0^2 - \omega^2)^2 + (2b\omega)^2}{(2b\omega)^2} = \frac{\left(\frac{\omega_0^2 - \omega^2}{2b\omega}\right)^2}{2} + 1
\]

\[
\Rightarrow \frac{A_{\text{max}}}{A} \approx 1 + \frac{(\omega_0^2 - \omega^2)^2}{2. (2b\omega)^2}
\]

Thus sharpness of the amplitude resonance reduces as damping increases. In other words, the resonance curve broadens as the damping increases.
Solved problems and Exercises

1. **Vertical block - spring system**

   ![Diagram of a vertical block-spring system]

   The net restoring force on the block is equal to the sum of upward spring force and gravity downward force. Considering downward direction as positive, the net restoring force is given by
   \[ F_{\text{net}} = -k(y_0 + y) + mg \quad ----(1) \]
   But, for equilibrium position, we have \( mg \) is balanced by spring force \( ky_0 \)
   \[ F_{\text{net}} = -ky_0 \quad ----(2) \]
   So equation (2) yields the same set of periodic expressions as that of horizontal block spring arrangement
   \[ \Rightarrow \omega = \sqrt{\frac{k}{m}} \Rightarrow T = 2\pi \sqrt{\frac{m}{k}} \]
   We also have
   \[ mg = ky_0 \Rightarrow \frac{k}{m} = \frac{g}{y_0} \Rightarrow \omega = \sqrt{\frac{g}{y_0}} \]
   Hence vertical block spring system is equivalent to simple pendulum with length of pendulum \( l = y_0 \)

2. **Block connected to springs in series and parallel combination**
**Series combination:** Let $y_1$ and $y_2$ be the extensions in the springs of spring constants $k_1$ and $k_2$ respectively

$$F_{net} = -k_1 y_1 = -k_2 y_2$$

The total displacement of block from equilibrium position is

$$y = y_1 + y_2 = \frac{F_{net}}{k_1} - \frac{F_{net}}{k_2}$$

$$F_{net} = -\left(\frac{k_1 k_2}{k_1 + k_2}\right) y$$

The periodic attributes are given by the same expressions, which are valid for oscillation of a single spring. We only need to use equivalent spring constant in the expression as

$$k = \frac{k_1 k_2}{k_1 + k_2}$$
Solved problems and Exercises

**Parallel combination:**

**From fig.,**

\[ F_{net} = -k_1y - k_2y = -(k_1 + k_2)y \]

Hence, in this case equivalent spring constant is \( k = k_1 + k_2 \)

3. Block in between two springs

Suppose the block is displaced slightly in downward direction (reasoning is similar if block is displaced upward). The upper spring is stretched, whereas the lower spring is compressed. The spring forces due to either of the springs act in the upward direction. The net downward displacement is related to net restoring force as:

\[ F_{net} = -k_1y - k_2y = -(k_1 + k_2)y \]

Hence, in this case also equivalent spring constant is \( k = k_1 + k_2 \)

4. If a load of 1kg is attached, the period of oscillations of a vertical spring mass system is 1 sec. When the load is 2 kg, the time period is 2 sec. Calculate the mass of the spring and its force constant.

**Sol:** Given \( T_1 = 1 \text{ sec} \) and \( m_1 = 1 \text{ kg} \)
\[ T_2 = 1.2 \text{ sec} \] and \( m_2 = 1.5 \text{ kg} \]
We have, \( T = 2\pi \sqrt{\frac{m + \frac{m_s}{3}}{k}} \) Where \( m \) is mass of the load, \( m_s \) is mass of the spring and \( k \) is force constant.

From above equation, \( \frac{4\pi^2}{T^2} \left( m + \frac{m_s}{3} \right) = \text{constant} \) as \( k \) is constant

\[
\frac{1}{T_1^2} \left( m_1 + \frac{m_s}{3} \right) = \frac{1}{T_2^2} \left( m_2 + \frac{m_s}{3} \right)
\]

\[
\Rightarrow \frac{1}{T_2^2} \left( 1 + \frac{m_s}{3} \right) = \frac{1}{T_2^2} \left( 1.5 + \frac{m_s}{3} \right) \Rightarrow m_s = 0.4 \text{ kg} = 400 \text{ gm}
\]

\[
\Rightarrow \text{Force constant } k = \frac{4\pi^2}{T_1^2} \left( m_1 + \frac{m_s}{3} \right) = \frac{4 \times (3.14)^2}{1^2} \left( 1 + \frac{0.4}{3} \right) = 44.56 \text{ N/m}
\]

5. A mass of 3 kg is hanged from a vertical spring. When a mass of 0.5 kg is added, the spring is further stretched by 5 cm. If \( m \) is removed and first mass is set into oscillations, calculate the period

Sol. Given first mass \( M = 3 \text{ kg} \) and added mass \( m = 0.5 \text{ kg} \). To find the time period, first we have to calculate force constant \( k \).

For this, at equilibrium position, \( Mg = kx \) (before adding the mass)-----(1)

and \( (M + m)g = k(x + 0.05) \) (after adding the mass)-----(2)

\[
(2) - (1) \Rightarrow mg = 0.05k \Rightarrow k = (0.5 \times 9.8)/0.05 = 98 \text{ N/m}
\]

The time period of mass \( M \) is \( T = \frac{2\pi \sqrt{M}}{K} = 2 \times 3.14 \times \sqrt{\frac{3}{98}} = 1.09 \text{ sec} \)
6. A particle is executing simple harmonic oscillation along a line of length 4cm. Its velocity when passing through the centre is $12\text{cm/sec}$. Find the period

Sol: Given Amplitude of Oscillator $a = 2\text{cm}$
We know that velocity is maximum at the centre, hence $v_{max} = a\omega = 12\text{ cm/sec}$
$\Rightarrow \omega = \frac{12}{2} = 6\text{ rad/sec}$
$\Rightarrow$ The time period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{6} = 1.05\text{ sec}$

7. Quality factor of a sonometer wire is $2 \times 10^3$. On plucking, it executes 240 vibrations per sec. Calculate the time in which the amplitude decreases $\frac{1}{e^2}$ of the initial value.

Sol. Given $Q = \omega\tau = 2 \times 10^3$ and $\omega = 2\pi n = 2 \times 1.14 \times 24 = 15.072\text{ rad/sec}$
$\Rightarrow \tau = \frac{Q}{\omega} = \frac{2 \times 10^3}{15.07} = 1.326\text{ sec}$
$\Rightarrow$ Let $a_0$ be the initial amplitude, then $a_0 e^{-bt} = a_0 e^{-2} \Rightarrow t = \frac{2}{b}$
$\Rightarrow$ From the definition of relaxation time $\tau$ ,
$a_0 e^{-b\tau} = a_0 e^{-1} \Rightarrow \tau = \frac{1}{b}$
$\Rightarrow t = \frac{2}{b} = 2\tau = 2 \times 1.326 = 2.65\text{ sec}$

8. A mass of 1kg is suspended from a spring of force constant $10^3\text{N/m}$ and a damping coefficient $0.01\text{N-s/m}$. If an external force $F = 25 \text{ sin}pt$ (where $p$ is double of the natural frequency of the system) drives the system, find the amplitude of resulting motion.

Sol: Given
Mass of the load $m = 1\text{kg}$, $k = 10^3\text{N/m}$ and damping coefficient $\gamma = 0.01\text{N} - \text{s/m}$
$\Rightarrow b = \frac{\gamma}{2m} = \frac{0.01}{2} = 0.005\text{N} - \text{s/mkg}$
$F = F_0\text{sin}pt = 25 \text{ sin}pt$
We know amplitude of forced oscillation \( A = \)
\[
\frac{f}{\sqrt{(\omega^2 - \omega_0^2)^2 + (2 \omega_0 \omega)^2}}
\]
(1)

Where \( f \) is the amplitude of driving force per unit mass \( f = \frac{F_0}{m} = \frac{25}{1} = 25 \text{ N/Kg} \)

\( \omega_0 \) is the natural frequency of the system

\[
\omega_0 = \frac{k}{m} = \frac{10^3}{1} = 10^3
\]

\( \omega \) is the frequency of driving system

\[
\omega = 2\omega_0 = 2 \times 10^3
\]

Substituting all given values in equation (1),

\[
A = \frac{25}{\sqrt{(10^6 - 4 \times 10^6)^2 + (2 \times 0.005 \times 2 \times 10^3)^2}} = 8.33 \text{ mm}
\]

MCQs

1. A mass of 5 kg is suspended from a vertical massless spring of spring constant 500 \text{ Nm}^{-1}. The mass is displaced downward by 0.03 m and released. The frequency of oscillations of mass is

a) \( 2 \pi \) \hspace{1cm} b) \( \frac{1}{2 \pi} \) \hspace{1cm} c) \( \frac{5}{\pi} \) \hspace{1cm} d) \( 5 \pi \)

HCU 2020

Ans: c

2. A body of mass 4.9kg hangs from a spring and oscillate with a period of 0.6 sec. How much will the spring shorten when the body is removed?

a) 0.89 m \hspace{1cm} b) 0.089 m \hspace{1cm} c) 0.089 cm \hspace{1cm} d) 0.1 m
Solved problems and Exercises

AUCET2019

Ans: b

3. One end of a spring is fixed to a rigid support and a body of mass $m$ is attached to the other end such that the spring hangs vertically downwards. The unstretched length of the spring is $l$ and it has a spring constant $k$. The extension of the spring $z$ when the body is in equilibrium is

a) $\frac{mg}{2k}$  b) $-\frac{mg}{2k}$  c) $\frac{mg}{k}$  d) $-\frac{mg}{k}$

HCU 2016

Ans: d

4. A spring mass system of mass $M$ suspended from a spring of spring constant $k$, and equilibrium stretching $h$, is equivalent to a simple pendulum of mass $m$ and length

a) $\frac{h}{4\pi^2}$  b) $h$  c) $\frac{h}{2}$  d) $4\pi^2 h$

HCU 2011

Ans: b

5. A lightly damped harmonic oscillator with natural frequency $\omega_0$ is driven by a periodic force of frequency $\omega$. The amplitude of oscillation is maximum when

a) $\omega$ is slightly lower than $\omega_0$  b) $\omega = \omega_0$

c) $\omega$ is slightly higher than $\omega_0$  d) The force is in phase with the displacement

IIT JAM 2016

Ans: b

6. A block of mass is attached to a pair of springs connected in series. Both the springs have same spring constant $k$. The period of oscillations of the block is $T_1$. The period of oscillations of the same block attached to the same spings, connected in parallel is $T_2$. The ratio $T_1:T_2$ is

a) 2:1  b) 1:2  c) 4:1  d) 1:4

HCU 2016
Solved problems and Exercises

Ans: a

7. A block of mass rests on a frictionless table and is connected to two fixed posts by springs having spring constants $k$ and $2k$. If the block is displaced from its equilibrium position, the angular frequency of vibrations is given by

$$\omega = \sqrt{\frac{3k}{m}} \quad \text{or} \quad \sqrt{\frac{k}{3m}}$$

Ans: a

8. Which of the following is an acceptable wave function?

HCU 2020

Ans : D

9. In the case of critical damping

$$b^2 = \omega^2$$

AUCET2020

Ans : c

10. A particle performing SHM has a maximum velocity of $0.4 \, m/s$ and maximum acceleration of $0.8 \, /s^2$. Find the time period of oscillator

$$T = \frac{2\pi}{\sqrt{\frac{3k}{m}}}$$

Ans: 3.14 s

AUCET2020
11. Two systems have same resonance frequency. Their quality factors are in the ratio 1:2, the ratio of relaxation time is

a) 2:1  

b) 1:2  

c) 1:4  

d) 4:1

Ans: b

12. The displacement of a body executing simple harmonic motion is given by

\[ y = 5 \sin(2\pi t + \frac{\pi}{4}) \]

Its initial displacement, assuming all quantities are in SI unit, is

a) \( \frac{5}{\sqrt{2}} \) m  

b) 5m  

c) 0m  

d) \( -\frac{5}{\sqrt{2}} \) m

Ans: a

13. The equation of motion of a particle is given as \( \frac{d^2x}{dt^2} + 0.2 \frac{dx}{dt} + 36x = 0 \), the period of oscillations approximately is

a) \( \frac{\pi}{4} \)  

b) \( \frac{\pi}{2} \)  

b) \( \frac{\pi}{3} \)  

c) \( \frac{3\pi}{4} \)

Ans: c

14. Power dissipation in damped harmonic oscillator is

a) \( P = 2bE \)  

b) \( P = 2b^2E \)  

c) \( P = \frac{2b}{E} \)  

d) \( P = \frac{b}{2E} \)

Ans: a

15. At what phase Potential Energy and Kinetic Energy are equal in case of SHM

a) 30°  

b) 60°  

c) 90°  

d) 45°

Ans: d
16. Consider a particle of mass \( m \) following a trajectory given by \( x = x_0 \cos \omega_1 t \) and \( y = y_0 \sin \omega_2 t \) where \( x_0, y_0, \omega_1 \) and \( \omega_2 \) are constants of appropriate dimensions. The force on particle is

a) Central only if \( \omega_1 = \omega_2 \)

b) Central only if \( x_0 = y_0 \) and \( \omega_1 = \omega_2 \)

c) Always central

d) Central only if \( x_0 = y_0 \) and \( \omega_1 \neq \omega_2 \)

IIT JAM 2016

Ans: a

Hint: Write \( \vec{r} = x \hat{i} + y \hat{j} \) and find \( \ddot{r} \)

17. A lightly damped harmonic oscillator loses energy at the rate of 1% per minute. The decrease in amplitude of the oscillator per minute will be closest to

(a) 0.5%  
(b) 1%  
(c) 1.5%  
(d) 2%

IIT JAM 2012

Ans: d

Hint : Decay of energy is governed by equation, \( E = E_0 e^{-2\gamma t} \)

Decay of amplitude is governed by equation, \( A = A_0 e^{-\gamma t} \)

18. A particle of mass \( m \) is moving in x-y plane. At any given time \( t \), its position vector is given by \( \vec{r}(t) = A \cos \omega t \hat{i} + B \sin \omega t \hat{j} \), Where \( A, B \) and \( \omega \) are constants with \( A \neq B \). Which of the following statements are true?

a) Orbit of the particle is an ellipse

b) Speed of the particle is constant

c) At any given time \( t \), the particle experience a force towards origin

d) The angular momentum of the particle is \( m\omega AB \)

IIT JAM 2016

Ans: a,c,d
Solved problems and Exercises

Hint: \( \vec{r}(t) = A \cos \omega t \hat{i} + B \sin \omega t \hat{j} = x \hat{i} + y \hat{j} \Rightarrow \) find \( x^2 \) and \( y^2 \) from which you can find the shape of the orbit. Next find \( \vec{r}'' \) which shows the nature of force. Next find \( \vec{L} = \vec{r} \times m \vec{r}'' \)

19. An object executes simple harmonic motion along the x-direction with angular frequency \( \omega \) and amplitude \( a \). The speed of the object is 4 cm/s and 2 cm/s when it is at distances 2 cm and 6 cm respectively from the equilibrium position. Which of the following is/are correct?

a) \( \omega = \frac{\sqrt{3}}{8} \text{ rad} / \text{s} \)  

b) \( \omega = \frac{\sqrt{5}}{6} \text{ rad} / \text{s} \)  

c) \( a = \frac{\sqrt{140}}{3} \text{ cm} \)  

d) \( a = \frac{\sqrt{175}}{6} \text{ cm} \)

IIT JAM 2020

Ans: a, c

Hint: \( x = A \sin \omega t \Rightarrow v = A \omega \cos \omega t = A \omega \sqrt{1 - \left(\frac{x}{A}\right)^2} \Rightarrow v = \omega \sqrt{(A^2 - x^2)} \)

20. A body of mass \( m \) is subjected to a resistive force \( bv \) (where \( b \) is constant and \( v \) is the velocity). There is no restoring force in the medium. The displacement \( x \) as a function of time \( t \), in terms of its initial velocity \( v_0 \) and the coefficient \( \gamma = \frac{b}{m} \) is

a) \( -\gamma e^{-\gamma t} \)  

b) \( -v_0 e^{-\gamma t} \)  

c) \( -\frac{v_0}{\gamma} e^{-\gamma t} \)  

d) \( -\frac{v_0}{\gamma} e^{-2\gamma t} \)

HCU 2019

Ans: c

Hint: Given \( F = -bv \Rightarrow m \ddot{x} + b \dot{x} = 0 \).

By solving this second order equation, We get \( x = A e^{-\gamma t} \) and \( \dot{x} = A (-\gamma) e^{-\gamma t} \)

\( \Rightarrow v_0 = A (-\gamma) \)

\( \Rightarrow A = \frac{v_0}{-\gamma} \)

21. A simple harmonic oscillator has velocities \( v_1 \) and \( v_2 \) at positions \( x_1 \) and \( x_2 \) from the equilibrium point respectively. The frequency of the oscillator is given by
22. The fraction of kinetic energy in the total energy of a simple harmonic oscillator, when its displacement is half of its amplitude, is given by

a) 0 b) 0.5 c) 1 d) 0.75

HCU 2012

Ans: d

Hint: Total energy $E_{total} = \frac{1}{2} kA^2 = \frac{1}{2} kx^2 + \frac{1}{2} mv^2 \Rightarrow \frac{\frac{1}{2}mv^2}{\frac{1}{2}kA^2} = \frac{\frac{1}{2}kA^2 - \frac{1}{2}kx^2}{\frac{1}{2}kA^2}$

23. A pendulum is made of a massless string of length L and a small bob of negligible size and mass m. It is released making an angle $\theta_0 (\ll 1 \text{rad})$ from the vertical. When passing through the vertical, the string slips a bit from the pivot so that its length increases by a small amount $\delta \ll 1 \text{ rad}$ in negligible time. If it swings up to angle $\theta_1$ on the other side before starting to swing back, then to a good approximation which of the following expressions is correct?
Solved problems and Exercises

IIT JAM 2017

Solution: \[ mgl(1 - \cos \theta_i) = mg(1 + \delta)(1 - \cos \theta_f) \]

\[ l^2 \frac{\theta_i}{2} = (1 + \delta) \frac{\theta_f}{2} \]

\[ \theta_i = \theta_f \left(1 + \frac{\delta}{l}\right)^{1/2} \implies \theta_f = \theta_i \left(1 + \frac{\delta}{l}\right)^{-1/2} \implies \theta_f = \theta_i \left(1 - \frac{\delta}{2l}\right) \]

24. A block of mass 0.38kg is kept at rest on a frictionless surface and attached to a wall with a spring of negligible mass. A bullet weighing 0.02kg moving with a speed of \( \frac{200}{s} \) hits the block at time \( t = 0 \) and gets stuck to it. The displacement of the block(in metre) with respect to the equilibrium position is given by (Spring constant = 640 N/m )

\[\text{(a) } 2 \sin 5t \quad \text{(b) } \cos 10t \quad \text{(c) } 0.4 \cos 25t \quad \text{(d) } 0.25 \sin 40t\]

IITJAM 2016

Ans: d

Sol: Given that \( m = 0.38 \text{ kg}, m' = 0.02 \text{ kg}, v = \frac{200m}{s} \text{ and } k = 640 \text{ N/m} \)
We have $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{m + m'}} = 40 \text{ rad/sec}$(On substitution of given values)

Let $v'$ be the velocity acquired by the block $m$, when the bullet strikes it and comes to rest.

From momentum conservation,

$m' v = (m + m')v' \Rightarrow v' = \frac{m' v}{m + m'}$

On substitution of given values, $v' = 10 \text{ m/s}$ which is the maximum velocity acquired by the block,

$\therefore v' = A\omega = 10 \Rightarrow A = \frac{10}{40} = 0.25m$

Hence the displacement of the block with respect to equilibrium position,

$x = A\sin \omega t = 0.25 \sin 40t$

25. Consider the following differential equation that describes the oscillations of a physical system:

$$\alpha \frac{d^2y}{dt^2} + \beta \frac{dy}{dt} + \gamma y = 0$$

If $\alpha, \beta$ are held fixed and $\gamma$ is increased, then

(a) The frequency of oscillations increases

(b) The oscillations decay faster

(c) The frequency of oscillations decreases

(d) The oscillations decay slower

IIT JAM 2020
Solved problems and Exercises

Ans: a

\[
\frac{d^2 y}{dt^2} + \frac{\beta}{\alpha} \frac{dy}{dt} + \gamma y = 0
\]

\[
2\gamma = \frac{\beta}{\alpha} \Rightarrow \gamma = \frac{\beta}{2\alpha}, \quad \omega_n = \sqrt{\frac{\gamma}{\alpha}}
\]

(a) \[
\omega = \sqrt{\omega_n^2 - \gamma^2} = \sqrt{\frac{\gamma^2 - \beta^2}{4\alpha^2}} = \frac{1}{2\alpha} \sqrt{4\gamma - \beta^2} \Rightarrow \omega = \frac{1}{2\alpha} \sqrt{4\gamma - \beta^2}
\]

So, option (a) is correct and option (c) is wrong.

(b) \[A = A_0 e^{-\gamma t}\]

\[
\gamma = \frac{\beta}{2\alpha} = \text{constant}
\]

So, option (b) and (d) are is wrong.

26.

Consider a particle of mass \(m\) suspended between two identical springs. The mass \(m\) is displaced towards right by a distance \(x\) (much smaller than \(l\)) as shown in figure and then left to oscillate. In leading order, the acceleration of the mass is proportional to

\[A. \ -x\]
\[B. \ -x^2\]
\[C. \ -x^3\]
\[D. \ -e^{-x}\]

HCU 2018

27. For an under damped harmonic oscillator with velocity \(v\) (t\)

(a) Rate of energy dissipation varies linearly with \(v\) (t\)
Solved problems and Exercises

(b) Rate of energy dissipation varies as square of $v(t)$
(c) The reduction in the oscillator frequency, compared to the undamped case, is independent of $v(t)$
(d) For weak damping, the amplitude decays exponentially to zero

IIT JAM 2019
Ans: (b), (c), (d)

Grade your Understanding

1. The frequency of simple harmonic oscillator does not depend on the stretching angle/length if it is small enough [ ]
2. Oscillations can be linear, periodic and non-periodic [ ]
3. If there is a mixture of single and double derivatives in a linear differential equation, then exponential solution is more advantage than sinusoidal solution [ ]
4. Every system in nature has its own single (not more than one) natural frequency [ ]
5. The period of motion of a vertical system will be larger than that of a horizontal system having same mass and spring constant [ ]
6. Motion of system of two pendulums connected to a common string is Simple harmonic [ ]
7. Amplitude of an oscillatory motion can be negative [ ]
8. Quartz clocks work on the principle of undamped harmonic oscillator [ ]
9. More the $Q$-factor, more is the number of oscillations within time constant interval [ ]
10. The forced oscillator is always oscillates with its natural frequency [ ]
11. Locomotive wheels spin at a frequency equal to the frequency of linear oscillations of the steam engine which is mechanically connected to the wheels [ ]
12. Stiffer the spring is, higher the time period [ ]
## Glossary

**Glossary: Undamped, Damped and forced oscillations**

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Amplitude</strong></td>
<td>The maximum distance moved by a point on a vibrating body or wave measured from its equilibrium position</td>
</tr>
<tr>
<td><strong>Complex plane</strong></td>
<td>A modified cartesian plane that represent complex numbers geometrically</td>
</tr>
<tr>
<td><strong>Oscillating Cycle</strong></td>
<td>A complete movement (longitudinal of transverse) over a period of time</td>
</tr>
<tr>
<td><strong>Driving force</strong></td>
<td>An external repeated force that drives an oscillating system to get continuous oscillations</td>
</tr>
<tr>
<td><strong>Damped oscillations</strong></td>
<td>Oscillations with energy dissipation over time</td>
</tr>
<tr>
<td><strong>Equilibrium position</strong></td>
<td>The position where an oscillator would rest without any force applied</td>
</tr>
<tr>
<td><strong>Force constant</strong></td>
<td>It is a measure of stiffness or rigidity of a system</td>
</tr>
<tr>
<td><strong>Harmonic</strong></td>
<td>A wave which is added to the basic fundamental wave in a special way</td>
</tr>
<tr>
<td><strong>Isochronous oscillator</strong></td>
<td>An oscillator with a frequency independent of its amplitude</td>
</tr>
<tr>
<td><strong>Initial conditions</strong></td>
<td>Any of a set of starting point values of an evolving variable at some point in time</td>
</tr>
<tr>
<td><strong>Linear superposition</strong></td>
<td>The net response caused by the overlapping of two or more waves in space</td>
</tr>
<tr>
<td><strong>Order of differential equation</strong></td>
<td>Highest order derivative that a differential equation contains</td>
</tr>
<tr>
<td><strong>Phasors</strong></td>
<td>A vector used to represent a sinusoidal function</td>
</tr>
<tr>
<td><strong>Phase</strong></td>
<td>The position of a point in time on a cycle of a waveform</td>
</tr>
<tr>
<td><strong>Glossary</strong></td>
<td></td>
</tr>
<tr>
<td>-----------------</td>
<td>---------------------------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>Resonance</strong></td>
<td>The phenomenon of increased amplitude when the applied frequency is equal to natural frequency</td>
</tr>
<tr>
<td><strong>Restoring force</strong></td>
<td>An internal force that tries to bring the body to its equilibrium position</td>
</tr>
<tr>
<td><strong>Relaxation time</strong></td>
<td>Time required for an exponentially decreasing variable (like amplitude/energy of damped oscillation) to drop from an initial value to $1/e=0.368$ of that value</td>
</tr>
<tr>
<td><strong>Undamped Oscillations</strong></td>
<td>Oscillations with no loss of energy over time</td>
</tr>
<tr>
<td><strong>Sinusoidal wave</strong></td>
<td>A sine or cosine wave that describes a smooth periodic oscillation</td>
</tr>
<tr>
<td><strong>Time period</strong></td>
<td>The time taken by a complete cycle of the wave to pass a point</td>
</tr>
</tbody>
</table>
UNIT-IV
Chapter-6
Coupled Oscillations
మామిడి తీరస్సు
మార్గం మార్గం

1. జాతి ఉత్పత్తి ఆషం విడి లు కావాలి
2. జాతి ఉత్పత్తి ఆషం విడి లు కావాలి
3. N జాతి ఉత్పత్తి ఆషం విడి లు కావాలి

అభివృద్ధి పద్ధతి

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5. జాతి ఉత్పత్తి ఆషం విడి లు కావాలి
6. జాతి ఉత్పత్తి ఆషం విడి లు కావాలి

CHAPTER 6 పాఠం
Syllabus

Coupled oscillators: Introduction, Two coupled oscillators, Normal coordinates and Normal modes, N-coupled oscillators and wave equation

Learning Objectives
In this chapter students would learn,

1. An introduction to coupled oscillations.
2. Two coupled oscillations, Normal coordinates and normal modes
3. N-Coupled oscillations, wave equation.

Learning Outcomes
By the end of the chapter, student would be able to

1. Identify the need of coupled differential equations in the study of coupled oscillations.
2. Interpret the link between the degrees of freedom and the number of normal modes. Describe the usage of matrix method and decoupling method to solve coupled differential equations.
3. Apply matrix method and decoupling method to various configurations of coupled oscillator systems.
4. Compare and contrast the parameters in spring mass system and simple pendulum system for coupled oscillations.
5. Justify the nature of oscillations and natural modes based on the number of degrees of freedom.
6. Develop mathematical tools suitable to extend the work to various other chemical, geological, electrical and electronic oscillatory systems.
సయ్యాయున్న కాలు సన్న పిక్స్త్రామ సత్యామి యుగ్మ సమయం
సయ్యాయున్న కాలు సన్న పిక్స్త్రామ సత్యామి యుగ్మ సమయం

మామా జూలుస్ ఆంజేలా వన్య పాత్ర అసమానం ||
మామా జూలుస్ ఆంజేలా వన్య పాత్ర అసమానం ||

376
Course Outcomes specific to program and Future directions

By the end of this chapter students from specific programs would be able to identify the need of coupled oscillations in

1. Physics: atomic level interactions to cosmic level astronomical interactions.
2. Chemistry: Oscillatory behaviour of chemical reaction dynamics with multiple reactants.
3. Computer Science: Neural network programming, social network dynamics analysis, pacemaker programming.
4. Geology: various types of seismic waves during earthquakes.
5. Electronics: various electrical LCR circuits and electromagnetic wave generators in communication systems.

Familiar to Unfamiliar

In the previous chapters, you might have learned about simple harmonic oscillations, forced harmonic oscillations and damped harmonic oscillations. All these are basic tools to understand some of the complex phenomena of oscillations that actually occur in nature. In your 11th class, you might have come across one such complex oscillation phenomenon, namely, beats. To generate the beats, two or more oscillations with slightly different frequencies superimpose, to produce a wave with oscillating amplitude. Another such complex oscillating phenomenon is Lissajous figures, where oscillations in two mutually perpendicular spatial directions are superimposed. Fourier series and Fourier transformation, that you may come across in your later studies, deal with superposition of multiple oscillations with varying frequency in time domain.

In your 11th class, you have studied that when multiple pendulums with variable lengths are triggered by oscillating a single pendulum in the series, the one with matching length will respond
6.1 పరిస్థితిలోని సమాధానం

ఈ పరిస్థితిలో సమాధానం అవసరం ఉంది. మొదటి విధానంలో ఈ పరిస్థితిలో సమాధానం అవసరం ఉంది. మొదటి విధానంలో ఈ పరిస్థితిలో సమాధానం అవసరం ఉంది. మొదటి విధానంలో ఈ పరిస్థితిలో సమాధానం అవసరం ఉంది. మొదటి విధానంలో ఈ పరిస్థితిలో సమాధానం అవసరం ఉంది.
more due to resonance. The remaining of them will undergo forced oscillations.

![Diagram of pendulums](image)

**Fig: Forced oscillations and resonance in Pendulums.**

But both these cases are actually examples of coupled oscillations. This is because, the driving force is also getting affected by the reaction effect; i.e., in the above figure, pendulum X executes resonance oscillations with pendulum D and forced oscillations with the rest of the pendulums. But due to the action of the other pendulums, the amplitude of pendulum X also oscillates. This system thus is an example of coupled oscillations. In this chapter, we shall develop matrix method and decoupling method to solve coupled oscillator equations. We also are introduced to the normal coordinates and normal modes of coupled oscillations.

### 6.1 Introduction

**History**

The phenomenon of coupled oscillations in pendulums was first observed by Christian Huygens, while he was sick in bed and watching his pendulum clocks. He observed that one pendulum oscillations were affecting the other pendulum oscillations when both pendulums were tied to the same wooden support. At first it was thought that the oscillations were transmitted through air. Later, it was verified that the oscillations were transmitted through wooden frame.

Further, these coupled oscillators were used to maintain time synchronization in pendulum clocks while travelling on sea.
6.1 Introduction

The secret of the synchronized pendulums – Physics World
Huygens was first to notice that pairs of linked pendulums... 
🔗 https://physicsworld.com/a/the-secret-of-the-synchronized-pendulums/

Christian Huygens:
First...
🔗 https://www.princeton.edu/~hos/Mahoney/articles/huygens/timelong/tim...

The Surprising Secret of Synchronization
Veritasium
🔗 https://www.youtube.com/watch?v=t_VPRCtiUg

N-Sync | MythBusters
Discovery
🔗 https://www.youtube.com/watch?v=e-c6S6SdkPo
6.1 Introduction

The pendulum clock loses its rhythm as a result of the oscillations of the ship, change of longitudes and hence will show up wrong time. Instead, if two pendulums coupled to each other were used, then, one will take care of the other in maintaining the frequency in a synchronous manner.

In three dimensions, the location of a particle in spherical coordinate system may be given by three variables namely \((r, \theta, \phi)\). For a simple pendulum, since length is constant and is allowed to oscillate only on a plane \((\phi = 0)\), the only free variable is the angle \(\theta\). Thus the degrees of freedom for SHM is only ‘one’.

**Fig: Spherical coordinate system (Mathematician’s version).**

**Note:** In general, in physics, another convention is used to define angles in spherical coordinate system. The above convention is usually followed in mathematics and not in Physics.

**Fig: Spherical coordinate system (Physicist’s version).**
3D యొక్క రచిపట్టు మాత్రమే ర, θ, ϕ ఆపంచాలు (r,θ,ϕ) తో సూచించబడింది. వేరు వేరు సంఖ్యల వాటి లేదా వాటి వాటి చేసే ఫంక్షన్ సూచించబడింది. మనం ఎందుకు కట్టించాలని తెలియజేయగలిగే లేదు. ర = రుస్తంబరు, ఫ్యాంటంబరు. రాత్రి పచ్చి నాయి యొక్క సంబంధాలను కలిగి ఉంటాయి. మనం రుస్తంబరు విశేషాత్మక మార్గం సూచించే విధానం. అంటే రుస్తంబరు దిశలో వస్తున్న ప్రకారం వస్తున్న కానం. 

అమేష కాక పచ్చలేదు. పొందిని అంటే అనేక సమస్యలు ఉంటాయి. అంటే అంటే అక్కడ అంతర్భాగాన్ని విలువ పెంచవచ్చు. అంటే కాను కాను అనేక సమస్యలు ఉంటాయి. అంటే అంటే అక్కడ అంటే అక్కడ ఉంటాయి. అంటే అంటే అక్కడ అంటే అక్కడ ఉంటాయి. అంటే అంటే అక్కడ ఉంటాయి. అంటే అంటే అక్కడ ఉంటాయి. అంటే అంటే అక్కడ ఉంటాయి. అంటే అంటే అక్కడ ఉంటాయి. అంటే అంటే అక్కడ ఉంటాయి. అంటే అంటే అక్కడ ఉంటాయి. అంటే అంటే అక్కడ ఉంటాయి. 

అవా విధానంలో అంటే అతి పెద్ద పదార్థాలు ఉండవచ్చు. అంటే అంటే అతి పెద్ద పదార్థాలు ఉండవచ్చు. అంటే అంటే అతి పెద్ద పదార్థాలు ఉండవచ్చు. 

F - Corner
#Diagonalizable_matrices
https://home.iiserb.ac.in/~sebastian/material/ClassMech/Week9_CoupledOscillators_v2.pdf
https://yyknosekai.wordpress.com/2015/11/26/a-system-of-n-coupled-oscillators/

![QR Code 1](https://example.com/qrcode1.png)
![QR Code 2](https://example.com/qrcode2.png)

382
6.1 Introduction

If two pendulums are connected, then the resultant system will have two degrees of freedom. Thus, if multiple systems are connected, each with single degree of freedom, or if single system with multiple degrees of freedom is considered, then one needs to use coupled differential equations to solve the equation of motion of such systems.

Then, corresponding to each degree of freedom, there will be one normal mode of oscillation for the given system. This is also called natural mode of oscillation. If two simple pendulums are connected in series or alternately if two spring mass systems are connected in series, then the resultant system will have two degrees of freedom and two normal modes of oscillations. If the system has N number or infinite number of simple harmonic systems connected together, then it will have N or infinite number of degrees of freedom and respective number of modes of vibrations.

The Taylor series expansion of force for coupled oscillator system is given by

\[
F(x - x_0) = F(x_0) + \sum_i \frac{\partial F}{\partial x_i} \bigg|_{x_i=x_0} (x_i - x_0) \\
+ \frac{1}{2!} \sum_i \frac{\partial^2 F}{\partial x_i^2} \bigg|_{x_i=x_0} (x_i - x_0)^2 + \cdots
\]

Here, again, the SHM approximation implies, \(F(x_0) = 0\), \(\frac{\partial F}{\partial x_i} \bigg|_{x_i=x_0} = -k_i\) and all other higher order terms will vanish.

In the above, the force given is on a single particle in a coupled oscillator system. If differential equations are written for the entire system, then we are looking at a system of coupled differential equations.

The only issue while solving these coupled differential equations is that, the equations are now no more linear in nature, which means that one cannot take linear combinations of solutions to develop new or physically meaningful solutions. To avoid this discrepancy one must take symmetric systems, i.e., the matrix that represents the force constant must be symmetric. Only in that case, one can diagonalize or decouple the k matrix to convert the system.
6.2 అంకేయ చరిత్ర

6.2 అంకేయ చరిత్రం

సాధారణం తిని యంత్రక-యంత్ర విస్తీర్ణ ముఖం చేసేది. రాది మనము యంత్రక యంత్ర చరిత్రం k
క విభాగం. మనం పరిమితం తిని యంత్రక-యంత్ర చరిత్రం మాత్రమే k 12 యంత్రక యంత్ర చరిత్రం ల పరిమితం కేవలం సమానం ఉందని
చెప్పడం విస్తీర్ణ మొత్తం సమానం. మనం మాత్రమే x1 యంత్రక x2 యంత్రక సమాన విస్తీర్ణ కేవలం సమానం
విస్తీర్ణ మొత్తం సమానం శుష్క విస్తీర్ణ మొత్తం సమానం ఉందని
చెప్పడం విస్తీర్ణ మొత్తం సమానం. మనం మాత్రమే x1 యంత్రక x2 యంత్రక సమాన విస్తీర్ణ మొత్తం సమానం ఉందని
చెప్పడం విస్తీర్ణ మొత్తం సమానం. యంత్రక యంత్ర యొక్క విస్తీర్ణ x1 - x2 యంత్రక యంత్ర యొక్క విస్తీర్ణ x2 - x1 యంత్రక యంత్ర యొక్క విస్తీర్ణ
x1 - x2 యంత్రక యంత్ర యొక్క విస్తీర్ణ x2 - x1 యంత్రక యంత్ర యొక్క విస్తీర్ణ
x1 - x2 యంత్రక యంత్ర యొక్క విస్తీర్ణ x2 - x1 యంత్రక యంత్ర యొక్క విస్తీర్ణ
x1 - x2 యంత్రక యంత్ర యొక్క విస్తీర్ణ x2 - x1 యంత్రక యంత్ర యొక్క విస్తీర్ణ
x1 - x2 యంత్రక యంత్ర యొక్క విస్తీర్ణ x2 - x1 యంత్రక యంత్ర యొక్క విస్తీర్ణ

$$m \ddot{x}_1 = -kx_1 - k_{12}(x_1 - x_2)$$

$$m \ddot{x}_2 = -kx_2 - k_{12}(x_2 - x_1)$$

విషయం ప్రతిభావం సమీపం

$$
\begin{pmatrix}
\ddot{x}_1 \\
\ddot{x}_2
\end{pmatrix} = 
\begin{pmatrix}
\frac{(-k - k_{12})}{m} & k_{12}/m \\
k_{12}/m & \frac{(-k - k_{12})}{m}
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2
\end{pmatrix} - - - (1)
$$

మొట్టము విస్తీర్ణ వ్యాఖ్యాత్రం

$$
\begin{vmatrix}
\frac{(-k - k_{12})}{m} - \lambda & k_{12}/m \\
k_{12}/m & (-k - k_{12})/m - \lambda
\end{vmatrix} = 0
$$

$$
\Rightarrow \left( \frac{k + k_{12}}{m} + \lambda \right)^2 - \left( \frac{k_{12}}{m} \right)^2 = 0 \Rightarrow \left( \frac{k + 2k_{12}}{m} + \lambda \right) \left( \frac{k}{m} + \lambda \right) = 0
$$

$$
\lambda_1 = -\frac{k}{m} \text{ and } \lambda_2 = -\frac{(k + 2k_{12})}{m} - - - (2)
$$

అందువలన ఆ రేఖా సంఖ్యలకు ఈ సమరూపా సంఖ్యాపదానికి సంబంధితం ఉంది.

$$
\begin{pmatrix}
\ddot{x}_1 \\
\ddot{x}_2
\end{pmatrix} = 
\begin{pmatrix}
\lambda_1 & 0 \\
0 & \lambda_2
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2
\end{pmatrix}
\Rightarrow \ddot{x}_1 + \frac{k}{m} x_1 = 0 \text{ and } \ddot{x}_2 + \frac{k + 2k_{12}}{m} x_2 = 0
$$

ప్రశ్నాంప్రశ్నాత్రం

384
of equations into independent, linear equations. Thus in the rest of this chapter, we consider only symmetric matrices for force constants and diagonalize them to obtain independent eigenvalues or normal modes of oscillations. This symmetry condition implies that the coupling element between two oscillators gives similar response to the force from either of the oscillators; i.e., \( k_{12} = k_{21} \).

### 6.2 Two coupled oscillators

Consider a double spring mass system with an interconnection. Let the masses and springs be identical, each with mass \( m \) and spring constant \( k \); and further, let \( k_{12} \) be the coupling constant.

![Fig: Two coupled spring mass system.](image)

Let \( x_1 \) and \( x_2 \) be the displacements produced in the two masses respectively towards right. Then the first spring is stretched by \( x_1 \), second spring is compressed by \( x_2 \) and the coupling spring is compressed by \( x_1 \) as well as stretched by \( x_2 \). Net compression in coupling spring is \( x_1 - x_2 \) or net expansion is \( x_2 - x_1 \). Compression in coupling spring causes restoring force on first mass and expansion in it causes restoring force on second mass. Thus

\[
\begin{align*}
    m\ddot{x}_1 &= -kx_1 - k_{12}(x_1 - x_2) \\
    m\ddot{x}_2 &= -kx_2 - k_{12}(x_2 - x_1)
\end{align*}
\]

They can be written in matrix form as

\[
\begin{pmatrix}
    \dot{x}_1 \\
    \dot{x}_2
\end{pmatrix} = \begin{pmatrix}
    -k - k_{12}/m & k_{12}/m \\
    k_{12}/m & -(k - k_{12})/m
\end{pmatrix} \begin{pmatrix}
    x_1 \\
    x_2
\end{pmatrix}
\]

By equating the determinant of the matrix to zero gives the characteristic equation and solution of it gives the eigenvalues.

\[
\chi(t) = \chi(0) \cos \omega_1 t \quad \text{and} \quad \chi_{II}(t) = \chi_{II}(0) \cos \omega_2 t
\]
Two coupled oscillators

\[ \omega_1 = \sqrt{\frac{k}{m}}, \quad \omega_2 = \sqrt{\frac{k + 2k_{12}}{m}} \]  \quad (5)

\[ \lambda = -\frac{k}{m} \quad \text{and} \quad \lambda = -\frac{k + 2k_{12}}{m} \]

The characteristic equations are:

\[ \begin{pmatrix} (-k - k_{12})/m - (-k/m) & k_{12}/m \\ k_{12}/m & (-k - k_{12})/m - (-k/m) \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = 0 \]

\[ \Rightarrow \begin{pmatrix} (-k_{12}/m & k_{12}/m \\ k_{12}/m & -k_{12}/m \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = 0 \]

\[ \Rightarrow k_{12}\alpha_1 - k_{12}\alpha_2 = 0 \Rightarrow \alpha_1 = \alpha_2 \]

\[ \alpha_1^2 + \alpha_2^2 = 1 \quad \text{and} \quad \alpha_1 = \alpha_2 \]

\[ \frac{(\alpha_1)}{(\alpha_2)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \]  \quad (6)

\[ \begin{pmatrix} (-k - k_{12})/m - (-k_{12}/m) & k_{12}/m \\ k_{12}/m & (-k - k_{12})/m - (-k_{12}/m) \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = 0 \]

\[ \Rightarrow \begin{pmatrix} k_{12}/m & k_{12}/m \\ k_{12}/m & k_{12}/m \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = 0 \]

\[ \Rightarrow k_{12}\beta_1 + k_{12}\beta_2 = 0 \Rightarrow \beta_1 = -\beta_2 \]

\[ \beta_1^2 + \beta_2^2 = 1 \quad \text{and} \quad \beta_1 = -\beta_2 \]

\[ \frac{(\beta_1)}{(\beta_2)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \]  \quad (7)

The solutions are:

\[ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} \begin{pmatrix} x_{i1} \\ x_{i2} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x_{i1} \\ x_{i2} \end{pmatrix} \]  \quad (8)
6.2 Two coupled oscillators

\[
\begin{vmatrix}
-k - k_{12}/m - \lambda & k_{12}/m \\
k_{12}/m & -k - k_{12}/m - \lambda
\end{vmatrix} = 0
\]

\[\Rightarrow \left(\frac{k + k_{12}}{m} + \lambda\right)^2 - \left(\frac{k_{12}}{m}\right)^2 = 0 \Rightarrow \left(\frac{k + 2k_{12}}{m} + \lambda\right)\left(\frac{k}{m} + \lambda\right) = 0\]

The eigenvalues are given by

\[\lambda_1 = -\frac{k}{m} \text{ and } \lambda_2 = -\frac{(k + 2k_{12})}{m} \quad -(2)\]

Thus, the decoupled matrix is given by

\[
\left(\begin{array}{c}
\ddot{x}_l \\
\ddot{x}_{ll}
\end{array}\right) = \left(\begin{array}{cc}
\lambda_1 & 0 \\
0 & \lambda_2
\end{array}\right) \left(\begin{array}{c}
x_l \\
x_{ll}
\end{array}\right)
\]

\[= \left(\begin{array}{cc}
-k/m & 0 \\
0 & -k - 2k_{12}/m
\end{array}\right) \left(\begin{array}{c}
x_l \\
x_{ll}
\end{array}\right) \quad -(3)\]

\[\Rightarrow \ddot{x}_l + \frac{k}{m}x_l = 0 \text{ and } \ddot{x}_{ll} + \frac{k + 2k_{12}}{m}x_{ll} = 0\]

The solution of these equations, with zero initial velocity, may be written as

\[x_l(t) = x_l(0) \cos \omega_1 t \quad \text{and} \quad x_{ll}(t) = x_{ll}(0) \cos \omega_2 t \quad -(4)\]

where

\[\omega_1 = \sqrt{\frac{k}{m}}, \quad \omega_2 = \sqrt{\frac{k + 2k_{12}}{m}} \quad -(5)\]

The eigenfunction for \(\lambda = -k/m\) is

\[
\begin{pmatrix}
-k - k_{12}/m - (-k/m) \\
k_{12}/m
\end{pmatrix} - \begin{pmatrix}
-k/m & k_{12}/m \\
k_{12}/m & -k - k_{12}/m - (-k/m)
\end{pmatrix} \begin{pmatrix}
\alpha_1 \\
\alpha_2
\end{pmatrix} = 0
\]

\[\Rightarrow \begin{pmatrix}
k_{12}/m & k_{12}/m \\
k_{12}/m & -k_{12}/m
\end{pmatrix} \begin{pmatrix}
\alpha_1 \\
\alpha_2
\end{pmatrix} = 0 \Rightarrow k_{12}\alpha_1 - k_{12}\alpha_2 = 0 \Rightarrow \alpha_1 = \alpha_2\]

The normalization condition gives \(\alpha_1^2 + \alpha_2^2 = 1\). From these two,

\[
\begin{pmatrix}
\alpha_1 \\
\alpha_2
\end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 \\
1
\end{pmatrix} \quad -(6)\]
6.2 Two coupled oscillators

\[
\begin{pmatrix}
  x_1 \\
  x_{II}
\end{pmatrix} = \begin{pmatrix}
  \alpha_1 & \beta_1 \\
  \alpha_2 & \beta_2
\end{pmatrix}^{-1} \begin{pmatrix}
  x_1 \\
  x_2
\end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix}
  1 & 1 \\
  1 & -1
\end{pmatrix} \begin{pmatrix}
  x_1 \\
  x_2
\end{pmatrix} - - - (9)
\]

\[
x_1(t) = \frac{1}{\sqrt{2}} [x_1(t) + x_{II}(t)] = \frac{1}{\sqrt{2}} [x_1(0) \cos \omega_1 t + x_{II}(0) \cos \omega_2 t]
\]

\[
\Rightarrow x_1(t) = \frac{1}{\sqrt{2}} \left[ \frac{x_1(0) + x_2(0)}{\sqrt{2}} \cos \omega_1 t + \frac{x_1(0) - x_2(0)}{\sqrt{2}} \cos \omega_2 t \right]

- - - (10)
\]

\[
x_2(t) = \frac{1}{\sqrt{2}} [x_1(t) - x_{II}(t)] = \frac{1}{\sqrt{2}} [x_1(0) \cos \omega_1 t - x_{II}(0) \cos \omega_2 t]
\]

\[
\Rightarrow x_2(t) = \frac{1}{\sqrt{2}} \left[ \frac{x_1(0) + x_2(0)}{\sqrt{2}} \cos \omega_1 t - \frac{x_1(0) - x_2(0)}{\sqrt{2}} \cos \omega_2 t \right]

- - - (11)
\]

\[
x_2(0) = 0, \text{ అందువల్ల}
\]

\[
x_1(t) = x_1(0) \cos \left( \frac{\omega_1 + \omega_2}{2} \right) t \cos \left( \frac{\omega_1 - \omega_2}{2} \right) t - - - (12)
\]

\[
x_2(t) = x_1(0) \sin \left( \frac{\omega_1 + \omega_2}{2} \right) t \sin \left( \frac{\omega_1 - \omega_2}{2} \right) t - - - (13)
\]

అందువల్ల చిన్నతో కూడా,

\[
\cos A + \cos B = 2 \cos \frac{A + B}{2} \cos \frac{A - B}{2}
\]

\[
\cos A - \cos B = 2 \sin \frac{A + B}{2} \sin \frac{A - B}{2}
\]

Eq (12), Eq (13) లా ఉండి అందుచితం చేయడం అదేశం వంటిది ఉంటే ప్రకారం నుండి చాలా పరిమితాలు ఉన్నాయి. ఏమిదా చిత్రాల వంటిదికి ఎప్పుడూ ఉంటే ఎప్పుడూ అందానిలో ఉంటే పరిమితాలు ఉన్నాయి.
The eigenfunction for $\lambda = -(k + 2k_{12})/m$ is
\[
\begin{pmatrix}
\frac{(-k - k_{12})}{m} & \frac{k_{12}/m}{m} \\
\frac{k_{12}/m}{m} & \frac{(-k - k_{12})}{m} & \frac{k_{12}/m}{m}
\end{pmatrix}
\begin{pmatrix}
\beta_1 \\
\beta_2
\end{pmatrix}
= 0
\Rightarrow \begin{pmatrix}
k_{12}/m & k_{12}/m \\
k_{12}/m & k_{12}/m
\end{pmatrix}
\begin{pmatrix}
\beta_1 \\
\beta_2
\end{pmatrix}
= 0
\Rightarrow k_{12}\beta_1 + k_{12}\beta_2 = 0 \Rightarrow \beta_1 = -\beta_2
\]
Again, the normalization condition gives $\beta_1^2 + \beta_2^2 = 1$. From these two,
\[
\begin{pmatrix}
\beta_1 \\
\beta_2
\end{pmatrix}
= \frac{1}{\sqrt{2}} \begin{pmatrix}
1 \\
-1
\end{pmatrix}
\quad \ldots \quad (7)
\]
The relation between the old and the new bases is given by
\[
\begin{pmatrix}
x_1 \\
x_2
\end{pmatrix}
= \begin{pmatrix}
\alpha_1 & \beta_1 \\
\alpha_2 & \beta_2
\end{pmatrix}
\begin{pmatrix}
x_I \\
x_{II}
\end{pmatrix}
= \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & 1 \\
1 & -1
\end{pmatrix}
\begin{pmatrix}
x_I \\
x_{II}
\end{pmatrix}
\quad \ldots \quad (8)
\]
\[
\begin{pmatrix}
x_I \\
x_{II}
\end{pmatrix}
= \begin{pmatrix}
\alpha_1 & \beta_1 \\
\alpha_2 & \beta_2
\end{pmatrix}^{-1}
\begin{pmatrix}
x_1 \\
x_2
\end{pmatrix}
= \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & 1 \\
1 & -1
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2
\end{pmatrix}
\quad \ldots \quad (9)
\]
Here Eq. (8), Eq. (9) represent the normal modes of oscillation with frequencies given by Eq. (5). Here the first normal mode represents the case of symmetric mode where both the masses move together towards left or towards right. Then there won’t be any stretch in the coupling spring. The second normal mode represents the anti-symmetric mode where both the masses move opposite to each other. Then the coupling spring will have double the stretch.
\[
x_1(t) = \frac{1}{\sqrt{2}} [x_I(t) + x_{II}(t)] = \frac{1}{\sqrt{2}} [x_I(0) \cos \omega_1 t + x_{II}(0) \cos \omega_2 t]
\Rightarrow x_1(t) = \frac{1}{\sqrt{2}} \left[\frac{x_1(0) + x_2(0)}{\sqrt{2}} \cos \omega_1 t + \frac{x_1(0) - x_2(0)}{\sqrt{2}} \cos \omega_2 t\right]
\quad \ldots \quad (10)
\]
\[
x_2(t) = \frac{1}{\sqrt{2}} [x_I(t) - x_{II}(t)] = \frac{1}{\sqrt{2}} [x_I(0) \cos \omega_1 t - x_{II}(0) \cos \omega_2 t]
\]
The standard procedure to diagonalize a matrix is as follows.

i. Suppose the given matrix equation is $AX = Y$

ii. The transformation matrix is obtained by writing eigenvectors in columns. Let it be $S$.

iii. Then the diagonalized matrix equation is $DX' = Y'$ where $D = S^{-1}AS$.

iv. Here the new basis and old basis are related by $X' = S^{-1}X$ or $X = SX'$.

v. The old and new resultant vectors are related by $Y' = S^{-1}Y$ or $Y = SY'$.

**Activity**

Plot these $x_1(t)$ and $x_2(t)$ for various values of $\omega_1$, $\omega_2$ as a function of time in octave.

**E - Corner**


http://web.csulb.edu/~jchang9/m247/m247_poster_J_Burgess_sp09.pdf

6.2 Two coupled oscillators

\[ x_2(t) = \frac{1}{\sqrt{2}} \left[ \frac{x_1(0) + x_2(0)}{\sqrt{2}} \cos \omega_1 t - \frac{x_1(0) - x_2(0)}{\sqrt{2}} \cos \omega_2 t \right] \]

If \( x_2(0) = 0 \), the above expression simplifies to,

\[ x_1(t) = x_1(0) \cos \left( \frac{\omega_1 + \omega_2}{2} \right) t \cos \left( \frac{\omega_1 - \omega_2}{2} \right) t \quad (12) \]

\[ x_2(t) = x_1(0) \sin \left( \frac{\omega_1 + \omega_2}{2} \right) t \sin \left( \frac{\omega_1 - \omega_2}{2} \right) t \quad (13) \]

Here \( \frac{\omega_1 + \omega_2}{2} \) is a high frequency wave overlapped by a low frequency wave \( \frac{\omega_1 - \omega_2}{2} \). Further, at \( t = 0 \), sine function will have zero value and cosine function will have maximum value.

A plot of them gives the beats pattern, with \( x_1(t) \) complement of \( x_2(t) \).

**Fig: Plot of \( x_1(t) \) and \( x_2(t) \) as a function of time.**

**Decoupling method:**
Consider a double spring mass system with an interconnection. Let the masses and springs be identical with mass \( m \) and spring constant \( k \) and let \( k_{12} \) be the coupling constant.

**Fig: Two coupled spring mass system.**
6.2 Two coupled oscillators

Two coupled oscillators

\[ m \ddot{x}_1 = -k x_1 - k_{12} (x_1 - x_2) \quad - - - (1) \]
\[ m \ddot{x}_2 = -k x_2 - k_{12} (x_2 - x_1) \quad - - - (2) \]

Eql (1), Eql (2)

\[ m(x_1 + x_2) = -k(x_1 + x_2) \quad - - - (3) \]
\[ m(x_1 - x_2) = -(k + 2k_{12})(x_1 - x_2) \quad - - - (4) \]

Let

\[ x_1 + x_2 = X \quad \text{and} \quad x_1 - x_2 = X' \quad - - - (5) \]

Then

\[ \ddot{X} + \frac{k}{m} X = 0 \quad \text{and} \quad \ddot{X}' + \frac{k + 2k_{12}}{m} X' = 0 \quad - - - (6) \]

Let

\[ X = A_1 \cos \omega_1 t \quad \text{and} \quad X' = A_2 \cos \omega_2 t \quad - - - (7) \]

Then

\[ \omega_1 = \sqrt{\frac{k}{m}}, \quad \omega_2 = \sqrt{\frac{k + 2k_{12}}{m}} \quad - - - (8) \]

Eql (7) & Eql (5)

\[ x_1 = \frac{X + X'}{2} = \frac{A_1}{2} \cos \omega_1 t + \frac{A_2}{2} \cos \omega_2 t \]
\[ = \frac{D}{4} \left( \frac{\omega_1 + \omega_2}{2} \right) t \cos \left( \frac{\omega_1 - \omega_2}{2} \right) t \quad - - - (9) \]
Let $x_1$ and $x_2$ be the displacements produced in the two masses respectively towards right. Then the first spring is stretched by $x_1$, second spring is compressed by $x_2$ and the coupling spring is compressed by $x_1$ as well as stretched by $x_2$. Net compression in coupling spring is $x_1 - x_2$ or net expansion is $x_2 - x_1$. Compression in coupling spring causes restoring force on first mass and expansion in it causes restoring force on second mass. Thus

$$m\ddot{x}_1 = -kx_1 - k_{12}(x_1 - x_2) \quad \quad (1)$$
$$m\ddot{x}_2 = -kx_2 - k_{12}(x_2 - x_1) \quad \quad (2)$$

Adding and subtracting these two gives

$$m(x_1 + x_2) = -k(x_1 + x_2) \quad \quad (3)$$
$$m(x_1 - x_2) = -(k + 2k_{12})(x_1 - x_2) \quad \quad (4)$$

Defining

$$x_1 + x_2 = X \quad \text{and} \quad x_1 - x_2 = X'$$

Gives

$$\ddot{X} + \frac{k}{m}X = 0 \quad \text{and} \quad \ddot{X}' + \frac{k + 2k_{12}}{m}X' = 0 \quad \quad (5)$$

The solutions are given by

$$X = A_1 \cos \omega_1 t \quad \text{and} \quad X' = A_2 \cos \omega_2 t \quad \quad (6)$$

Where

$$\omega_1 = \sqrt{\frac{k}{m}}, \quad \omega_2 = \sqrt{\frac{k + 2k_{12}}{m}} \quad \quad (7)$$

Substituting Eq. (7) in Eq. (5) one can get

$$x_1 = \frac{X + X'}{2} = \frac{A_1}{2} \cos \omega_1 t + \frac{A_2}{2} \cos \omega_2 t$$
$$= \frac{D}{4} \cos \left(\frac{\omega_1 + \omega_2}{2}\right) t \cos \left(\frac{\omega_1 - \omega_2}{2}\right) t \quad \quad (8)$$
6.2 Two coupled oscillators

\[ x_2 = \frac{X - X'}{2} = \frac{A_1}{2} \cos \omega_1 t - \frac{A_2}{2} \cos \omega_2 t = \frac{D}{4} \sin \left(\frac{\omega_1 + \omega_2}{2}\right) t \sin \left(\frac{\omega_1 - \omega_2}{2}\right) t - - - (9) \]

E q (8), E q (9) అంటే ఆమె తెలుగు అక్షాలు ఉంటాయి. అంటే ఇక్కడ అంధకారం వలన ట్విగమంచవ ఇక్కడ అంధకారం వలన ట్విగమంచవ ఇక్కడ అంధకారం వలన ట్విగమంచవ ఇక్కడ అంధకారం వలన ట్విగమంచవ ఇక్కడ అంధకారం వలన ట్విగమంచవ.

ఇక్కడ ఒక అక్షాలు ఉంటాయి. అంటే ఇక్కడ అంధకారం వలన ట్విగమంచవ.

ఇక్కడ అంధకారం వలన ట్విగమంచవ. ఇక్కడ అంధకారం వలన ట్విగమంచవ. ఇక్కడ అంధకారం వలన ట్విగమంచవ. ఇక్కడ అంధకారం వలన ట్విగమంచవ. ఇక్కడ అంధకారం వలన ట్విగమంచవ. ఇక్కడ అంధకారం వలన ట్విగమంచవ.

https://scholar.harvard.edu/files/schwartz/files/lecture4-oscillators-to-waves.pdf


https://www2.physics.ox.ac.uk/sites/default/files/2012-09-04/normalmodes_iandii_pdf_96820.pdf


\[ m\ddot{x}_1 = -\frac{mg}{l} x_1 - k_{12} (x_1 - x_2) = -kx_1 - k_{12} (x_1 - x_2) \]

\[ m\ddot{x}_2 = -\frac{mg}{l} x_2 - k_{12} (x_2 - x_1) = -kx_2 - k_{12} (x_2 - x_1) \]
6.2 Two coupled oscillators

\[ x_2 = \frac{X - X'}{2} = \frac{A_1}{2} \cos \omega_1 t - \frac{A_2}{2} \cos \omega_2 t \]
\[ = \frac{D}{4} \sin \left( \frac{\omega_1 + \omega_2}{2} \right) t \sin \left( \frac{\omega_1 - \omega_2}{2} \right) t \]

Here \( \left( \frac{\omega_1 + \omega_2}{2} \right) \) is a high frequency wave overlapped with a low frequency wave \( \left( \frac{\omega_1 - \omega_2}{2} \right) \). Also at \( t = 0 \), sine function will have zero value and cosine function will have maximum value.

A plot of them will give beats pattern with \( x_1(t) \) compliment to \( x_2(t) \).

**Fig: Plot of \( x_1(t) \) and \( x_2(t) \) as a function of time.**

Here Eq. (5) represents the normal modes of oscillations with frequencies given by Eq. (8). Here the first normal mode represents the case of symmetric mode where both the masses move towards left or towards right. Then there won’t be any stretch in the coupling spring. The second normal mode represents the anti-symmetric mode where both the masses move opposite to each other. Then the coupling spring will have double the stretch.

**Fig: Anti-Symmetric and Symmetric modes of oscillations.**
6.3 నిపుణుడు అనువాదం

మేమని, అందువల్ల అనువాదని పంచబం. అందువల్ల అనువాదని పంచబం.

\[
\begin{pmatrix}
\ddot{x}_1 \\
\ddot{x}_2 \\
\vdots \\
\ddot{x}_{n-1} \\
\ddot{x}_n
\end{pmatrix} = -\frac{k}{m}
\begin{pmatrix}
2 & -1 & 0 & \cdots & \cdots & 0 \\
-1 & 2 & -1 & 0 & \cdots & 0 \\
0 & -1 & 2 & -1 & \cdots & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & \cdots & -1 & 2
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
\vdots \\
x_{n-1} \\
x_n
\end{pmatrix}
\]

ఇకడ దినిస్తాం ప్రతి అంశాలని పంచబం. అందువల్ల లేదా అందాని పంచబం. అందువల్ల లేదా అందాని పంచబం.

\[
\begin{pmatrix}
x_1 \\
x_2 \\
\vdots \\
x_{n-1} \\
x_n
\end{pmatrix} = \begin{pmatrix}
1 \\
e^{i\phi} \\
e^{2i\phi} \\
\vdots \\
e^{i(n-1)\phi} \\
e^{in\phi}
\end{pmatrix} e^{i\omega t}
\]

ఇకడ కృతియుడు స్వయంభూతం చేసారు.
In the case of coupled pendulum, the equation of motion is given by

\[ m\ddot{x}_1 = -\frac{mg}{l} x_1 - k_{12} (x_1 - x_2) = -k x_1 - k_{12} (x_1 - x_2) \]
\[ m\ddot{x}_2 = -\frac{mg}{l} x_2 - k_{12} (x_2 - x_1) = -k x_2 - k_{12} (x_2 - x_1) \]

**6.3 N-Coupled oscillator**

Consider a system of \( N \) coupled simple harmonic oscillators. In this system, except for the 1st and \( n^{th} \) oscillator, all other oscillators will have two coupling systems on either side. If we consider a spring mass system, then there will be \( N \) masses and \( N + 1 \) springs in the \( N \)-coupled oscillator system. Let the force constant and the coupling constants among various oscillators be identical and equal to \( k \). The equations of motion for such a system are given by

\[ m\ddot{x}_1 = -k x_1 - k (x_1 - x_2) \]
\[ m\ddot{x}_2 = k (x_1 - x_2) - k (x_2 - x_3) \]
\[ m\ddot{x}_3 = k (x_2 - x_3) - k (x_3 - x_4) \]
\[ \ldots \ldots \]
\[ m\ddot{x}_p = k (x_{p-1} - x_p) - k (x_p - x_{p+1}) \]
\[ \ldots \ldots \]
\[ m\ddot{x}_{n-1} = k (x_{n-2} - x_{n-1}) - k (x_{n-1} - x_n) \]
\[ m\ddot{x}_n = k (x_{n-1} - x_n) - k (x_n) \]
6.3 N-Coupled oscillator

\[-\omega^2 \begin{pmatrix}
  1 \\
  e^{i\phi} \\
  e^{2i\phi} \\
  \vdots \\
  e^{i(n-1)\phi} \\
  e^{in\phi}
\end{pmatrix} e^{i\omega t} = -\frac{k}{m} \begin{pmatrix}
  2 & -1 & 0 & \ldots & 0 \\
  -1 & 2 & -1 & 0 & \ldots & 0 \\
  0 & -1 & 2 & -1 & \ldots & 0 \\
  \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
  0 & \ldots & \ldots & -1 & 2 & -1 \\
  0 & \ldots & \ldots & 0 & -1 & 2
\end{pmatrix} \begin{pmatrix}
  1 \\
  e^{i\phi} \\
  e^{2i\phi} \\
  \vdots \\
  e^{i(n-1)\phi} \\
  e^{in\phi}
\end{pmatrix} e^{i\omega t}\]

\[\omega_n^2 e^{in\phi} e^{i\omega t} = \frac{k}{m} (-e^{-i\phi_n} + 2 - e^{i\phi_n}) e^{in\phi} e^{i\omega t}\]

\[\Rightarrow \omega_n^2 = \frac{k}{m} (2 - e^{-i\phi} - e^{i\phi}) = 2 \frac{k}{m} (1 - \cos \phi) = 4 \frac{k}{m} \sin^2 \frac{\phi}{2}\]

\[\Rightarrow \omega = 2 \sqrt{\frac{k}{m} \left| \sin \frac{\phi}{2} \right|} = 2 \omega_0 \left| \sin \frac{\phi}{2} \right| - - - (1)\]

\[\text{అప్పుడు న}^n +1 \text{ప్రశ్నాయి} \text{ఎం/వాయ్యా} \text{సంపాదించి వచ్చే, సంపాదించిన ప్రశ్నాయి ప్రశ్నాయి యొక్క కంప్యూటర్ వాయ్యా, మాఖిత్తే బుద్ధి ప్రస్తుతించిన బాధ్యత న}^n \phi \text{ ఉంటుంది. ఇంద్రం ఆయామం వెలుగు జోటు పడుతూ ఉంటుంది.}

\[\therefore (N + 1)\phi_n = n\pi\]

\[\text{అప్పుడు} \text{N}^{\text{th}} \text{ప్రశ్నాయి} \text{ఎంప్యుట్ చికిత్సం ఉంటుంది, ఎంప్యుట్ ఎంప్యుట్ చికిత్సం ఉంటుంది,}

\[\phi_n = n\phi = \frac{n\pi}{N + 1} - - - (2)\]

\[\text{అప్పుడు} \text{N}^{\text{th}} \text{వేర్తిపండిత్ ఉంటుంది, ఉంటుంది,}

\[\omega_n = 2\omega_0 \sin \left[ \frac{n\pi}{2(N + 1)} \right] - - - (3)\]

\[\text{అప్పుడు} \text{నేటికి} \text{ఎంప్యుట్ రేమం} \text{ఎంప్యుట్ రేమం} \text{తిస్తంటి ఉంటుంది,}

\[\frac{x_n}{x_{n-1}} = \frac{e^{in\phi}}{e^{i(n-1)\phi}} = e^{i\phi} = e^{i\pi} - - - (4)\]

\[\text{N}^{\text{th}} \text{ప్రశ్నాయి} \text{ఎంప్యుట్ ప్రశ్నాయి ఉంటుంది, ఉంటుంది,}

\[x_n = A e^{in\phi} e^{i\omega t} = A e^{i\pi/(N+1)} e^{i\omega t} - - - (5)\]
This can be written in matrix form as

\[
\begin{pmatrix}
\ddot{x}_1 \\
\ddot{x}_2 \\
\vdots \\
\ddot{x}_{n-1} \\
\ddot{x}_n
\end{pmatrix} = -\frac{k}{m} \begin{pmatrix}
2 & -1 & 0 & \cdots & \cdots & 0 \\
-1 & 2 & -1 & 0 & \cdots & 0 \\
0 & -1 & 2 & -1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & -1 & 2 & -1 \\
0 & \cdots & \cdots & 0 & -1 & 2
\end{pmatrix} \begin{pmatrix}
x_1 \\
x_2 \\
\vdots \\
x_{n-1} \\
x_n
\end{pmatrix}
\]

Since all the force constants and the coupling constants are equal, any force applied to the system will be distributed with uniform phase change across the system. Thus, if the phase difference between first and second oscillator is \(\phi\), then the phase difference between second and third will also be \(\phi\). Then a guess solution of the type below can be written.

\[
\begin{pmatrix}
x_1 \\
x_2 \\
\vdots \\
x_{n-1} \\
x_n
\end{pmatrix} = \begin{pmatrix}
1 \\
e^{i\phi} \\
e^{2i\phi} \\
\vdots \\
e^{i(n-1)\phi} \\
e^{in\phi}
\end{pmatrix} e^{i\omega t}
\]

Substituting this in the above gives,

\[
-\omega^2 \begin{pmatrix}
1 \\
e^{i\phi} \\
e^{2i\phi} \\
\vdots \\
e^{i(n-1)\phi} \\
e^{in\phi}
\end{pmatrix} e^{i\omega t}
= -\frac{k}{m} \begin{pmatrix}
2 & -1 & 0 & \cdots & \cdots & 0 \\
-1 & 2 & -1 & 0 & \cdots & 0 \\
0 & -1 & 2 & -1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & -1 & 2 & -1 \\
0 & \cdots & \cdots & 0 & -1 & 2
\end{pmatrix} \begin{pmatrix}
1 \\
e^{i\phi} \\
e^{2i\phi} \\
\vdots \\
e^{i(n-1)\phi} \\
e^{in\phi}
\end{pmatrix} e^{i\omega t}
\]

A general \(n^{th}\) term is given by

\[
\omega_n^2 e^{in\phi} e^{i\omega t} = \frac{k}{m} (-e^{-i\phi_n} + 2 - e^{+i\phi_n}) e^{in\phi} e^{i\omega t}
\]

\[\Rightarrow \omega_n^2 = \frac{k}{m} (2 - e^{-i\phi} - e^{+i\phi}) = 2 \frac{k}{m} (1 - \cos \phi) = 4 \frac{k}{m} \sin^2 \frac{\phi}{2}
\]
6.4 పరమాణములు

6.4 పరమాణములు

కొంతమందికి మాత్రమే సంఖ్యాతిల్లో కొనసాగించాడు, N రెండు వంటిది p రెండు పోటింది తొడు (కంయ) వ కంయంగా కంయంగా. N రెండు వంటిది వ పరమాణము p th జలాశయ మనం అందువల్ల చిహ్నపడింది,

\[
\frac{d^2 y_p}{dt^2} = \frac{k}{m} (y_{p+1} - 2y_p + y_{p-1}) = \frac{T/\delta x}{m} (y_{p+1} - 2y_p + y_{p-1})
\]

ఈ గ్రహణం ద్వారా ప్రాంతానికంగా పరమాణమును అనుగ్రహించడానికి కింద యొక్క విధానాన్ని ప్రకటిస్తుంది.

\[
\Rightarrow \frac{d^2 y_p}{dt^2} = \frac{T}{m} \left( \frac{y_{p+1} - y_p}{\delta x} - \frac{y_p - y_{p-1}}{\delta x} \right) = \frac{T}{m} \left( \frac{dy}{dx}_{p+1} - \frac{dy}{dx}_p \right)
\]

\[
= \frac{T}{m} \frac{d^2 y}{dx^2} \ dx
\]

\[
\Rightarrow \frac{d^2 y_p}{dt^2} = \frac{T}{m/dx} \frac{d^2 y}{dx^2} = \frac{T}{\rho_l} \frac{d^2 y}{dx^2}
\]

or

\[
\frac{d^2 y_p}{dt^2} = v^2 \frac{d^2 y}{dx^2}
\]

అని అనుభుతం ఉంటుంది అ ష మిస్టర్ స్టాన్ టిస్ట్రియియ (పాగా వియా), \( \rho_l = \frac{m}{dx} \) అర్ధ నుంచి మేడ్ మెంటు ఊరు ఉరిత్వ దీని పరమాణము మేడ్ అంటాం. అందుకే మంచ ఇంటిని వంటిది పరమాణము ద్వారా పరమాణము లాంటి ఎంపాట్ ఉండగానికి దానికి ఉపగపడుతుంది.

6.5 సాధనములు

1. సంఖ్యాతిల్లో జ్యోమిషియి కొంతమందికి చెందిన పరమాణము కొంతమందికి సెంట్రను చాటుకుంటుంది ఎంపాట్ లేదు. ఎంపాట్ కూడా అంటే ఇది కొంతమందికి చెందిన పరమాణము లేదు. ఎంపాట్ కూడా అంటే ఇది కొంతమందికి చెందిన పరమాణము లేదు. ఎంపాట్ కూడా అంటే ఇది కొంతమందికి చెందిన పరమాణము లేదు.

2. భావంలే భావాలు కుంభంలే భావాలు కుంభంలే భావాలు కుంభంలే భావాలు కుంభంలే భావాలు. అంటే భావంలే భావాలు కుంభంలే భావాలు కుంభంలే భావాలు కుంభంలే భావాలు. అంటే భావంలే భావాలు కుంభంలే భావాలు కుంభంలే భావాలు.
6.4 Wave equation

\[ \Rightarrow \omega = 2 \sqrt{\frac{k}{m}} \left| \sin \frac{\phi}{2} \right| = 2\omega_0 \left| \sin \frac{\phi}{2} \right| \quad - - - \quad (1) \]

Since there will be \( N + 1 \) springs/couplers in the system, at the other end of the coupled system, the wave has to vanish. So the total phase change produced must be some integer multiple of \( \pi \).

\[ (N + 1)\phi_n = n\pi \]

Thus the phase of the \( n^{th} \) oscillator is given by

\[ \phi_n = n\phi = \frac{n\pi}{N + 1} \quad - - - \quad (2) \]

With this substitution, the frequency of \( n^{th} \) normal mode frequency is given by,

\[ \omega_n = 2\omega_0 \sin \left[ \frac{n\pi}{2(N + 1)} \right] \quad - - - \quad (3) \]

The ratio of amplitudes of two successive oscillators is given by

\[ \frac{x_n}{x_{n-1}} = \frac{e^{in\phi}}{e^{i(n-1)\phi}} = e^{i\phi} = e^{\pi + i} \quad - - - \quad (4) \]

Thus the displacement of \( n^{th} \) oscillator is given by

\[ x_n = A e^{in\phi} e^{i\omega t} = A e^{in\pi/(N+1)} e^{i\omega t} \quad - - - \quad (5) \]

Here \( \phi, \omega \) are given by Eq. (2) and Eq. (1). It is observed that the resultant frequency of the system is twice the frequency of the individual oscillator modulated by sine of the phase factor which reduces with rise in number \( N \) and remains independent of \( n \). In other words, \( \phi \) remains same for all oscillators.

6.4 Wave equation

In the continuous limit, a system of \( N \) coupled oscillators can be considered as a vibrating string. The equation of motion for a \( p^{th} \) particle in such an \( N \) coupled oscillator system is given by

\[ \frac{d^2y_p}{dt^2} = \frac{k}{m} (y_{p+1} - 2y_p + y_{p-1}) = \frac{T/\delta x}{m} (y_{p+1} - 2y_p + y_{p-1}) \]
6.4 Wave equation

3. The wave equation is a fundamental equation in physics that describes how waves propagate through a medium. It is used in various fields such as acoustics, optics, and electromagnetism.

4. The equation can be derived from the laws of motion and the conservation of energy. It is expressed as

\[ \frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \]

where \( u \) is the displacement, \( t \) is time, \( x \) is the spatial coordinate, and \( c \) is the wave speed.

5. The wave equation has many solutions, including sinusoidal waves, plane waves, and plane waves in three dimensions.

6. The solutions can be used to model a wide range of physical phenomena, such as sound waves, light waves, and electromagnetic waves.

7. The solutions can also be used to predict the behavior of waves in different media, such as water, air, and solids.

8. The wave equation is a key tool in the development of theories in physics, such as quantum mechanics and general relativity.

9. The wave equation is also used in the design and analysis of devices such as antennas, lenses, and microphones.

E - Corner

https://en.wikipedia.org/wiki/El_Ni%C3%B1o
https://en.wikipedia.org/wiki/La_Ni%C3%B1a
https://oceanservice.noaa.gov/facts/ninonina.html
https://www.nature.com/articles/srep16994
https://www.youtube.com/watch?v=xjrrXO0oJVQ
In the above equation, the force constant is given by the tension produced per unit length of the system.

\[ \frac{d^2y_p}{dt^2} = \frac{T}{m} \left( \frac{y_{p+1} - y_p}{\delta_x} - \frac{y_p - y_{p-1}}{\delta_x} \right) = \frac{T}{m} \left( \frac{dy}{dx}_{p+1} - \frac{dy}{dx}_p \right) \]

\[ = \frac{T}{m} \frac{d^2y}{dx^2} dx \]

\[ \Rightarrow \frac{d^2y_p}{dt^2} = \frac{T}{m/\rho_l} \frac{d^2y}{dx^2} = \frac{T}{\rho_l} \frac{d^2y}{dx^2} \]

or

\[ \frac{d^2y_p}{dt^2} = v^2 \frac{d^2y}{dx^2} \]

Here \( v = \sqrt{\frac{T}{\rho_l}} \) is the velocity of the wave, \( \rho_l = \frac{m}{dx} \) is the linear mass density of the oscillator system, \( T \) is the tension of the string. The above equation represents the wave equation of an \( N \) -coupled oscillator system under continuous medium approximation \((N \to \infty)\). Here the system will have \( \infty \) degrees of freedom.

### 6.5 Applications

Study of coupled oscillations finds applications in the following:

1. In Solid state physics, where the nuclei are fixed at lattice points and at most vibrate around their equilibrium position when energy is given. Using coupled oscillations one can explain thermal interactions in crystals.
2. In particle physics, neutral particle oscillations made a major breakthrough deviation from standard model, as they do not obey the standard conservation laws. This lead to a completely new branch of particle physics called Neutrino Physics, B-Physics.
3. In atmospheric science, El-nino and La-nina cause a drastic variation in weather conditions which repeat for every a few years similar to beats pattern.
4. In chemistry, there are several chemical reactions whose dynamics can be studied by using coupled oscillations theory.
5. Normal modes of seismic wave oscillations of earth lead to the development of computational tools that exactly locate earthquake center (epicentre).
6. Population dynamics and neural network analysis requires the usage of coupled oscillation equations for better understanding.
7. In electronics and communication systems LCR circuit is the heart of all oscillations. Combination of several LCR circuits behaviour can be studied by using coupled oscillation theory.
8. In music industry, the grand piano is an instrument that produces music by using coupled oscillations principle.
9. In medical field and computer science field, the programming of pacemaker requires a knowledge about coupled oscillations of it along with human heart.

### Solved Problems & Exercise

#### Solved Problems

1. **Molecular coupled oscillators**:

We know that in a molecule each atom oscillates about its equilibrium position. These oscillations involve nearest neighbour coupling.

Consider a diatomic molecule which is treated as a system of two masses m and M coupled with a spring of spring constant k

![Fig: Diatomic molecule](image)

If \(x_1\) and \(x_2\) are the displacements of masses \(m\) and \(M\) respectively from their equilibrium, then for each mass we have,  
\[
mx_1 = -k(x_1 - x_2) \quad ---(1)
\]
\[
Mx_2 = -k(x_2 - x_1) \quad ----- (2)
\]
\[
(2) \times m - (1) \times M
\]
\[ mM(\ddot{x}_2 - \ddot{x}_1) = -k(x_2 - x_1)(m + M) \]

\[ \Rightarrow \quad \frac{d^2}{dt^2}(x_2 - x_1) = -\frac{k}{\mu}(x_2 - x_1) \quad \text{-----(3)} \]

where \( \mu=\frac{mM}{(m+M)} \) is called reduced mass of the coupled system.

The periodic attributes are given by the same expressions, (3) which are valid for oscillation of single spring with relative displacement \( x_2 - x_1 \). We only need to use equivalent mass \( \mu \) in the expression as \( \mu=\frac{mM}{(m+M)} \)

\[ \therefore \quad \omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{kmM}{(m + M)}} \]

**Problem 1:** If a molecule NaCl has a natural frequency of \( 1.14 \times 10^{13} \text{ Hz} \), then the inter atomic force constant is (\text{Atomic mass of Na is 23 amu and Cl is 35.5 amu and 1 amu}=1.66 \times 10^{-27} \text{ Kg})

\text{a)} 140 N/m \quad \text{b)} 120 N/m \quad \text{c)} 166 N/m \quad \text{d)} 180 N/m

**Ans:** b

**Hint:**

\[ k = \omega^2 \frac{m_1m_2}{(m_1+m_2)} = 4\pi^2 \times (1.14 \times 10^{13})^2 \left[ \frac{23 \times 35.5}{(23+35.5)} \right] = 120 \text{N/m} \]

**Problem 2:** Vibrations of diatomic molecules can be represented as those of harmonic oscillators. Two halogen molecules \( X_2 \) and \( Y_2 \) have fundamental vibrational frequencies \( \nu_x = 16.7 \times 10^{12} \text{ Hz} \) and \( \nu_y = 26.8 \times 10^{12} \text{ Hz} \) respectively. The respective force constants are \( K_x = 325 \text{ N/m} \) and \( K_y = 446 \text{ N/m} \). The atomic masses of F, Cl and Br are 19,35.5 and 79.9 atomic mass unit respectively. The halogen molecules \( X_2 \) and \( Y_2 \) are

\text{a)} \quad X_2 = F_2 \quad \text{and} \quad Y_2 = Cl_2

\text{b)} \quad X_2 = Cl_2 \quad \text{and} \quad Y_2 = F_2
c) \( X_2 = Br_2 \) and \( Y_2 = F_2 \)

d) \( X_2 = F_2 \) and \( Y_2 = Br_2 \)

IITJAM 2015

Ans: b

**Sol:** Given \( \nu_x = 16.7 \times 10^{12} \text{Hz} \) and \( \nu_y = 26.8 \times 10^{12} \text{Hz} \)

\( K_x = 325 \text{ N/m} \) and \( K_y = 446 \text{ N/m} \)

We know that the oscillation frequency of diatomic molecule with reduced mass \( \mu \) is 

\[
\omega = \sqrt{\frac{k}{\mu}} \text{ rad/sec} \Rightarrow \nu = \frac{1}{2\pi} \sqrt{\frac{k}{\mu}}
\]

We have reduced mass of \( X_2 \) is \( \mu_x = \frac{m_x m_x}{m_x + m_x} = \frac{m_x}{2} \)

reduced mass of \( Y_2 \) is \( \mu_y = \frac{m_y}{2} \)

\[
\therefore \nu_x = \frac{1}{2\pi} \sqrt{\frac{k_x}{\mu_x}} = \frac{1}{2 \times 3.14} \sqrt{\frac{325}{(m_x/2)}}
\]

\[
m_x = \frac{325}{(16.7 \times 10^{12})^2 (2 \times 3.14)^2} \times 2 = 59.07 \times 10^{-27} \text{ Kg}
\]

\[
= \frac{59.07 \times 10^{-27}}{1.67 \times 10^{-27}} = 35.5 \text{ amu} \text{ This is the atomic mass of Cl}
\]

Similarly we get \( m_y = 19 \text{ amu} \) which gives the atom F

2. **Coupled Oscillator: String bead system:**
Here two small massive beads, each of mass \( M \) are on a taut massless string as shown. Let \( T \) be the tension in the string.

To find normal modes of the system, we take an analogy from a spring mass system.

\[
Spring\ constant\ k \equiv \frac{Tension\ in\ the\ string\ T}{Length\ of\ string\ l}
\]

Hence first normal mode frequency \( \omega_1 = \sqrt{\frac{k}{m}} = \sqrt{\frac{T}{3lt}} = \sqrt{\frac{T}{3lm}} \)

Second normal mode frequency \( \omega_2 = \sqrt{\frac{k+2k_{12}}{m}} = \sqrt{\frac{T}{3lt} + \frac{2T}{3lt}} = \sqrt{\frac{4T}{3lm}} \)

3. **Coupled Oscillators kept in parallel:**

Consider a mass \( M (=2m) \) connected with two masses by two identical springs of spring constant \( k \) as shown. In this problem, we have 3 degrees of freedom (All three masses constrained/restricted to move in one direction ie here along X-axis). Hence we have three normal mode.

To find normal mode frequencies and coordinates:

Normal mode 1

![Diagram](image)

In this mode, both the small masses are in out of phase as shown. Hence the net force on the mass \( M \) will be zero because stretch in first spring compensate the compression in second spring. Now the two masses behaves as a simple harmonic oscillation with frequency \( \omega_1 = \sqrt{\frac{k}{m}} \)
Normal coordinates: we know in normal mode oscillation every system should oscillate with the same frequency and phase constant. Hence in this mode,

\[ x_1 = 0 \]

\[ x_2 = A \cos(\omega_1 t + \phi_1) \]

\[ x_3 = -A \cos(\omega_1 t + \phi_1) \]

Normal mode 2

Displace simultaneously all the masses as shown in fig. As both the small masses move in the same direction, we can take that as a single mass 2m. Now middle of the springs is at rest during this normal mode oscillation. Now the problem becomes **Two masses coupled with Single spring of spring constant** \( k' = 2k \). Hence

\[ \omega_2 = \sqrt{\frac{kr}{\mu}} = \sqrt{\frac{2k}{m}} \]

**Normal coordinates:**

\[ x_1 = B \cos(\omega_2 t + \phi_2) \]

\[ x_2 = -B \cos(\omega_2 t + \phi_2) \]

\[ x_3 = -B \cos(\omega_2 t + \phi_2) \]

Normal mode 3: All the three masses moves in the same direction. So in this mode oscillation is almost with zero frequency and infinite amplitude. Hence \( \omega_3 = 0 \)

Normal coordinates: \( x_1 = x_2 = x_3 = \text{constant} \)
5. **Coupled Electrical oscillators:**

Consider two identical LC oscillators which are capacitively coupled together as shown in fig. Ignoring the damping effect i.e.; taking resistance as zero, we get the following equations.

Applying Kirchhoff’s voltage law (KVL) to first loop,

\[
\frac{q_1}{c} = L \frac{d}{dt}(i_1 - i_3) \tag{1}
\]

Applying Kirchhoff’s voltage law to third loop,

\[
\frac{q_2}{c} = L \frac{d}{dt}(i_2 + i_3) \tag{2}
\]

Applying Kirchhoff’s voltage law to second loop,

\[
\frac{q_3}{c_1} = L \frac{d}{dt}(i_2 + i_3) + L \frac{d}{dt}(i_3 - i_1) \tag{3}
\]

Substituting 1 and 2 in 3,

\[
\frac{q_3}{c_1} = q_2 - q_1 \tag{4}
\]

Differentiating equation 4 with respect to time,

\[
\frac{d}{dt}\left(\frac{q_3}{c_1}\right) = \frac{d}{dt}\left(\frac{q_2}{c}\right) - \frac{d}{dt}\left(\frac{q_1}{c}\right) = \frac{i_3}{c} = \frac{i_2}{c} - \frac{i_1}{c}
\]

\[
\therefore i_3 = \frac{c_1}{c}(i_2 - i_1)
\]

Adding 1 and 2,

\[
\frac{q_1}{c} + \frac{q_2}{c} = L \frac{d}{dt}(i_1 + i_2)
\]
Let \( q_+ = q_1 + q_2 \) and \( (i_1 + i_2) = \frac{d}{dt}(q_1 + q_2) = \frac{d}{dt}(q_+) \)

\[
\Rightarrow \frac{q_+}{c} = L \left( \frac{d}{dt} \frac{d}{dt}(q_+) \right) = L \frac{d^2 q_+}{dt^2}
\]

\[
\frac{d^2 q_+}{dt^2} + \frac{1}{Lc} q_+ = 0 \quad - - - 5
\]

Similarly, subtracting 1 and 2, we get

\[
\frac{d^2 q_-}{dt^2} + \frac{1}{L(c + 2c_1)} q_- = 0 \quad - - - 6
\]

where \( q_- = q_1 - q_2 \)

Thus equations 5 and 6 are the equations of simple harmonic oscillations having the solutions

\[
q_+ = q_0^+ \sin(\omega_+ t) \quad \text{and} \quad q_- = q_0^- \sin(\omega_- t)
\]

Here \( q_+ \) and \( q_- \) are called normal coordinates of the given electrical system with the normal frequencies

\[
\omega_+ = \frac{1}{\sqrt{Lc}} \quad \text{and} \quad \omega_- = \frac{1}{\sqrt{L(c + 2c_1)}}
\]

https://demoweb.physics.ucla.edu/

A molecule with \( N \) atoms has \( 3N \) degrees of freedom. There are three degrees of freedom for translation and three degrees of freedom for rotation leaving \( 3N - 6 \) degrees of freedom for vibrations. For linear molecules, rotation along the axis of the molecule is neglected. Thus it will have only \( 3N - 5 \) degrees of freedom.

How many vibrational modes are there in the linear \( CO_2 \) molecule?

Ans: \( 3(3) - 5 = 4 \)

How many vibrational modes are there in the non-linear \( SO_2 \) molecule?

Ans: \( 3(3) - 6 = 3 \)

How many vibrational modes are there in the tetrahedral \( CH_4 \) molecule?

Ans: \( 3(5) - 6 = 9 \)
How many vibrational modes are there in the nonlinear C₆₀ molecule?

Ans: 3(60)-6=174.


MCQs

Section A:

1. In a coupled pendulum the phase angle between the two pendulums in the first normal mode is made equal to
   a) 0°     b) 90°     c) 180°     d) 270°  
   AUCET 2020
   Ans: a

2. The following quantity is exchanged between individual oscillators of coupled oscillators
   a) Linear momentum b) Angular momentum c) Energy d) all
   Ans: c

3. The principle of coupled oscillation is used in
   a) Rectifier   b) Amplifier   c) Transformer   d) Inverter
   Ans: c

4. An oxygen molecule vibrates with a frequency of 10¹³ 1/sec. What is the effective force constant for the coupling between the atoms (Mass of oxygen atom 16 amu)
   a) 100 N/m   b) 70 N/m   c) 50 N/m   d) 150 N/m
   Ans: c

5. The equation of travelling wave is given as
   \( y = 10\cos\left(\frac{\pi}{5cm}x - \frac{\pi}{20s}t\right) \). What is the speed of the wave?
   a) 1 cm/s   b) 0.5 cm/s   c) 0.25 cm/s   d) 0.1 cm/s
   HCU 2015
   Ans: c
6. Consider the fig as shown. A and B have equal masses m and all the springs have the same spring constant k. What are the normal frequencies for the system, if the oscillations are assumed to be small?

\[ a) \sqrt{\frac{k}{m}} \sqrt{\frac{2k}{m}} \quad b) \sqrt{\frac{k}{m}} \sqrt{\frac{k}{2m}} \quad c) \sqrt{\frac{k}{m}} \sqrt{\frac{3k}{m}} \quad d) \sqrt{\frac{k}{m}} \sqrt{\frac{k}{m} + 1} \]

HCU 2015

Ans: c

7. Two waves are described by \( y_1 = (4\pi t) \) and \( y_2 = (4\pi t) \). The phase difference between these two waves is

\[ a) \pi \quad b) 0 \quad c) 2\pi \quad d) \frac{\pi}{2} \]

HCU 2012

Ans: d

8. Differential form of wave equation

\[ a) \frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2} \quad b) \frac{\partial^2 y}{\partial t^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial x^2} \quad c) \frac{\partial^2 y}{\partial x^2} = v^2 \frac{\partial^2 y}{\partial t^2} \quad d) \frac{\partial^2 y}{\partial x^2} = \sqrt{\frac{1}{v^2}} \frac{\partial^2 y}{\partial t^2} \]

AUCET 2019

Ans: a

9. Two tuning forks kept side by side as shown. The coupling between them occurs only when

a) Both the forks have nearly same frequency

b) Both tuning forks made with same material but have different masses

c) Both tuning forks having same masses but different length

d) No coupling between them

Ans: a
10. The frequencies of normal modes of longitudinal oscillations in a spring mass system and transverse oscillations in a string mass system respectively given as (Each of the system has N identical masses m and fixed at both the ends. \(l\) is the distance between neighbouring masses on string, \(T\) is the tension in string and \(k\) is spring constant of each identical massless spring)

\[a) \quad \omega_n = 2\sqrt{\frac{k}{m}} \sin\left[\frac{n\pi}{2(N+1)}\right], \quad \omega_n = 2\sqrt{\frac{T}{lm}} \sin\left[\frac{n\pi}{2(N+1)}\right]\]

\[b) \quad \omega_n = 2\sqrt{\frac{k}{m}} \sin\left[\frac{n\pi}{2(N+1)}\right], \quad \omega_n = 2\sqrt{\frac{T}{lm}} \sin\left[\frac{n\pi}{2(N+1)}\right]\]

\[c) \quad \omega_n = \sqrt{\frac{k}{m}} \sin\left[\frac{n\pi}{2(N+1)}\right], \quad \omega_n = \sqrt{\frac{T}{lm}} \sin\left[\frac{n\pi}{2(N+1)}\right]\]

\[d) \quad \omega_n = 2\sqrt{\frac{k}{m}} \cos\left[\frac{n\pi}{2(N+1)}\right], \quad \omega_n = 2\sqrt{\frac{T}{lm}} \cos\left[\frac{n\pi}{2(N+1)}\right]\]

Ans: b

Section B:

11. Two pendulums, each of length \(l\) and \(m\) are coupled with a spring of natural frequency \(\omega_c\). The resultant frequency, of the out of phase vibration is

\[a) \left(\frac{g}{l} - 2\omega_c^2\right)^{\frac{1}{2}} \quad b) \left(\frac{g}{l} - \omega_c^2\right)^{\frac{1}{2}} \quad c) \left(\frac{g}{l} - \omega_c^2\right)^{\frac{1}{2}} \quad d) \left(\frac{g}{l} + 2\omega_c^2\right)^{\frac{1}{2}}\]

HCU 2019

Ans: d

Hint: for spring-mass system \(\omega_c = \left(\frac{k}{m}\right)^{\frac{1}{2}} \quad \omega_{out \ of \ phase} = \left(\frac{g}{l} + \frac{2k}{m}\right)\)

12. Two masses \(6 \times 10^{-3} \text{kg}\) and \(2 \times 10^{-3} \text{kg}\) are connected with a massless spring of force constant \(N/m\). If smaller mass vibrates with amplitudes 0.01m, the larger mass move with the amplitude

\[a) \quad 0.01 \text{ m} \quad b) \frac{0.01}{3}\text{ m} \quad c) \frac{0.01}{2}\text{ m} \quad d) \frac{0.01}{6}\text{ m}\]

Ans: b
Hint: Amplitudes of oscillations of masses are inversely proportional to their masses

\[ \frac{A_1}{A_2} = \frac{m_2}{m_1} \]

**Grade your understanding**

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Natural frequency, fundamental frequency/harmonics, normal mode frequency and resonant frequency: all are same in general</td>
<td>[Y/N]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>All coupled oscillators are forced oscillators</td>
<td>[Y/N]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Wave is always associated with oscillations/vibrations of material particles</td>
<td>[Y/N]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>We can also get coupled oscillations with a single degree of freedom</td>
<td>[Y/N]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>A diatomic molecule has two normal mode frequencies</td>
<td>[Y/N]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Under normal mode oscillations, every system should oscillate with the same frequency and phase constant</td>
<td>[Y/N]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>In a two coupled oscillator, the first normal mode represents a symmetric mode with highest energy and both the masses vibrate 180° out of phase</td>
<td>[Y/N]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Spring constant is equivalent to string tension per unit length</td>
<td>[Y/N]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Oscillations of a block which is connected either in series or parallel to two springs can be treated as coupled oscillator</td>
<td>[Y/N]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Ans:** 1. Yes  2. Yes  3. No (Keep in mind EM wave) 4. Yes (can get coupling between rotation and vertical motion of Wilberforce). 5. No (No. of degrees of freedom of vibrations for nonlinear molecule dof=3N-6, for linear molecule dof=3N-5, hence it has only one normal mode) 6. Yes (all masses move with that normal mode frequency). 7. Yes (As each spring has maximal extension or compression in symmetric mode). 8). Yes  9. No (Here acceleration of block depends only on single displacement)
Glossary

Matching
1. Resonance ( ) A) Simple Harmonic Oscillator
2. Strings ( ) B) Damped Harmonic Oscillator
3. Large amplitude Oscillations ( ) C) Forced harmonic oscillator
4. Exchange of energy ( ) D) Coupled Oscillator
5. Hook’s Law ( ) E) Anharmonic oscillator
6. Non-oscillator ( ) F) Vibrator
7. Logarithmic decrement ( ) G) Rotor

Glossary

<table>
<thead>
<tr>
<th>Antisymmetric mode</th>
<th>Mode of vibration with all masses moving in the same direction at the same time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Characteristic equation</td>
<td>An equation in the variable for given matrix upon which we obtain eigen values of that variable</td>
</tr>
<tr>
<td>Coupling systems</td>
<td>Oscillating systems that allow exchange of energy between them</td>
</tr>
<tr>
<td>Coupling element</td>
<td>Element that provide interactive force between the coupling systems</td>
</tr>
<tr>
<td>Beats</td>
<td>Periodic and repeating fluctuations when two waves of very similar frequencies interfere with one another</td>
</tr>
<tr>
<td>Beeded string</td>
<td>A set of masses connected by a continuous string</td>
</tr>
<tr>
<td>Diagonolization</td>
<td>The process of converting any square matrix into a diagonal matrix that shares the same fundamental properties of the given matrix</td>
</tr>
<tr>
<td>Eigen value</td>
<td>A special set of scalar solutions associated with a linear system of equation (Eigen-German word : own)</td>
</tr>
<tr>
<td>Exponential property</td>
<td>The property that the exponential function itself reappears after every operation of differentiation /integration</td>
</tr>
<tr>
<td>Term</td>
<td>Definition</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Fourier Transformation</td>
<td>A mathematical technique that transforms a function of time to a function of frequency and can be used to find the base frequencies that a wave is made of</td>
</tr>
<tr>
<td>Lissajous figures</td>
<td>Figures that shows the resultant motion of two perpendicular SHMs depending on the frequency ratio and initial phase difference</td>
</tr>
<tr>
<td>Natural mode</td>
<td>The frequency at which a system tends to oscillate in the absence of any external force (Natural frequency)</td>
</tr>
<tr>
<td>Normal mode</td>
<td>A pattern of motion in which all masses move sinusoidally with same frequency and with a fixed phase relation</td>
</tr>
<tr>
<td>N- coupled oscillator</td>
<td>A system of N oscillators in which each oscillator is coupled with all of its neighboring oscillator</td>
</tr>
<tr>
<td>Symmetric matrix</td>
<td>A square matrix that is equal to its transpose</td>
</tr>
<tr>
<td>Symmetric mode</td>
<td>Mode of vibration with all masses moving in the opposite direction at the same time and get stretched or compressed</td>
</tr>
</tbody>
</table>
UNIT-V
Chapter-7
VIBRATING STRINGS
ఘనమైన చిక్కి
చిత్రాల అధ్యాయం

శుభేష్చిద్ది (చిత్రాల కంపెన్సై) ని అందమైన సంపృక్తి కోసం వేడి పేరుగా ఉంటుంది.

1) కాంగ్రేస్ నియమిత కార్యక్రమం (కాంగ్రేస్ నియమిత కార్యక్రమం)
2) కార్యక్రమాల విస్తారం పోరాడించడానికి కార్యక్రమాల ఎత్తు పెంచడానికి అంతర్గత కార్యక్రమాలను తప్పించడానికి.
3) చిత్రాల అధ్యాయం అధ్యాయాలను కార్యక్రమం తో కంపెన్సై చేసాడు.
4) చిత్రాల అధ్యాయం అధ్యాయాలను కార్యక్రమం అధ్యాయంలో వివిధ సంస్థలలో చేసాడు.
5) కాంగ్రేస్ నియమిత కార్యక్రమం నియమిత కార్యక్రమం అధ్యాయం

అధ్యాయం 7

సంస్థ ప్రతిసామంలో వివిధ సమాచారం

1. అంతర్‌గత కార్యక్రమాల సాంస్కృతిక కార్యక్రమాల నియమిత కార్యక్రమాల.
2. అంతర్‌గత కార్యక్రమాల సాంస్కృతిక కార్యక్రమాల వివిధ సంస్థలలో అధ్యాయం.
3. అంతర్‌గత కార్యక్రమాల సాంస్కృతిక కార్యక్రమాల వివిధ సంస్థలలో వివిధ సంస్థలలో అధ్యాయం నియమిత కార్యక్రమాల.
4. అంతర్‌గత కార్యక్రమాల సాంస్కృతిక కార్యక్రమాల వివిధ సంస్థలలో పాలన ప్రామాణ్య సంస్థలలో అధ్యాయం.
5. అంతర్‌గత కార్యక్రమాల సాంస్కృతిక కార్యక్రమాల వివిధ సంస్థలలో వివిధ సంస్థలలో అధ్యాయం నియమిత కార్యక్రమాల.
6. అంతర్‌గత కార్యక్రమాల సాంస్కృతిక కార్యక్రమాల వివిధ సంస్థలలో వివిధ సంస్థలలో పాలన ప్రామాణ్య సంస్థలలో అధ్యాయం.
VIBRATING STRINGS

Chapter 7

Syllabus

Transverse wave propagation along a stretched string, General solution of wave equation and its significance, Modes of vibration of stretched string clamped at ends, overtones and Harmonics, Melde’s strings

Learning Objectives

In this chapter students would learn
1. Transverse wave equation in stretched strings.
2. General solution of wave equation and its significance in stretched strings.
3. Modes of vibration in stretched strings clamped at both the ends.
4. Harmonics and overtones in stretched string clamped at both the ends.
5. Melde’s experiment to measure the vibrational frequency of stretched strings.

Learning Outcomes

By the end of the chapter, student would be able to
1. Identify the cause of vibrations in stretched strings.
2. Describe the nature of transverse waves in stretched strings.
3. Predict proper material characteristics to generate transverse waves of desired frequency in stretched strings.
4. Classify various harmonics and overtones in stretched string vibrations.
5. Select suitable boundary conditions for the analysis of vibrations in stretched strings.
6. Develop prototype models of vibrational string systems for various applications.
సిలబస్

పతనే క్రమంగా మొదలు మంది మందమ మనిషి మగోడను తయారు చేయాలని మనం ఉపయోగిస్తుంది.

a) మానవములు స్వాతంత్ర్యం కొరకు కూడా స్త్రీ స్త్రి టిప్పుడికీ అనుసరించి ఉండాలి.

b) మతములు తయారు చేసిక ఉంటుంది అనుకీలి ఉండాలి.

c) సంస్కృతి విశ్లేషించాలి అనుకీలి ఉండాలి.

d) సంస్కృతి గుండా తయారు చేసిక ఉండాలి.

e) విద్యాది నిర్ధారించాలి అనుకీలి ఉండాలి.

ప్రామాణిక విషయాలను అధ్యయించడానికి మాటలు ఉండేవి.

ఇతర విషయాలలో ఆదరించడానికి మాటలు ఉండేవి.

ప్రామాణిక విషయాలను అధ్యయించడానికి మాటలు ఉండేవి.

ప్రామాణిక విషయాలను ఆదరించడానికి మాటలు ఉండేవి.

ప్రామాణిక విషయాలను అధ్యయించడానికి మాటలు ఉండేవి.

ప్రామాణిక విషయాలను ఆదరించడానికి మాటలు ఉండేవి.
Course Outcomes specific to program and Future directions
By the end of this chapter students from specific programs would be able to identify the need of vibrational string analysis

a) Physics: in a new branch of physics namely string theory.
b) Chemistry: in mechanical strength analysis of suture yarns and textile yarns.
c) Computers: in robotic surgery suture quality analysis.
d) Geology: in deep sea mineral exploration pipe lining analysis.
e) Electronics: in analyzing vibration sensing electronic yarns in industrial gloves.
f) Renewable energy: in vortex wind mill development.
g) Statistics: in data analysis and interpretation from various vibrational sensors.

Familiar to Unfamiliar
In the previous classes (in 8\textsuperscript{th} class) you might have learnt about various musical instruments that use vibration of strings to generate music; for example, Veena, Violin, Guitar etc. You might have also learned that shorter the vocal card, larger is the frequency/pitch of sound generated (more pronounced in women and kids). You also might be familiar with how to make a toy telephone using paper cups and stretched strings. You might have been taught to spot the difference between noise and music. In your 11\textsuperscript{th} class, you might have been introduced to the concept of longitudinal and transverse waves, the speed of transverse waves in stretched strings, normal modes and harmonics in strings clamped at both the ends.

In the previous chapters we have studied about the oscillations in pendulums and spring - mass systems, along with coupled oscillations and normal modes in them. In this chapter, we shall learn to distinguish vibrations from oscillations and study the equation of motion of waves on strings, solution of wave equation. We shall also study waves on a string clamped at both the ends, using
7.1 తమనుసారం

ఉంచబడిన విషయానికి ప్రత్యేక పాఠాలు ఉంటాయి, తప్పణి నిలువు కలిగి ఉంటాయి. ఆయా ఆయా గ్రహించడానికి లేదా పరిమితి విషయం పోలిస్తూ, వ్యాఖ్యలు ఈమృతి లేదా విషయానికి ప్రత్యేక పాఠాలు ఉంటాయి.

కంపెన్స్ మధ్య ఏటం ఇండా ఉంంది, ఇది వక్షన్బంతంగా ఉంటాయి. బాగా ఇది లేదా విషయం పోలిస్తూ ఉంటాయి. సమను ఈమృతి కలిగి ఉంటాయి. ఉండగా లేదా విషయం పోలిస్తూ ఉంటాయి.

https://www.wikihow.com/Calculate-Tension-in-Physics

వాటిలో సెట్టింగ్ చేసే పరిమితి ద్వారా సుమారు 180° ఉంంది. ఇది వాటిలో ఇన్ని పరిమితి ఉంంది. సమను ఉండి కొనసాగి ఉంంది. అందువలసి వాడుక అందయ్యా కోసం చేసారొచ్చా. చాలా వాడుక ఉన్నాయి. వాటిలో ఉండగా కొనసాగి ఉంంది. వాటిలో ఇందులో ఉండగా కొనసాగి ఉంంది. తప్పణి నిలువు కలిగి ఉంటాయి.
general wave equation, its solution with boundary conditions. Further, harmonics and overtones are introduced, together with usage of Melde’s apparatus to study transverse waves in strings.

7.1 Introduction

The difference between oscillations and vibrations is that oscillations have a point of symmetry, about which the object undergoes to and fro motion with equal amplitude on either side. Vibrations do not have such a point of symmetry, though they are periodic. Heart beat is a simple example of oscillations which has no symmetry but periodicity. This does not mean vibrations will never have symmetry. The vibrating objects may have either symmetric or asymmetric or both symmetric and asymmetric modes of movement. Another difference that distinguishes oscillations and waves is that for a given configuration of the system, oscillating systems can have only one natural frequency whereas vibrating systems can have all natural number multiples of some fundamental frequency. They are also called overtones or 2nd and other higher order harmonics. In the case of vibrations, all the higher order harmonics or overtones are called the modes of vibrations. Thus, in the case of oscillations, various modes of oscillations have quite independent frequencies. But in the case of vibrations, all modes are multiples of some fundamental frequency.

Fig: Modes in coupled pendulum oscillations and double clamped vibrating string.
7.1 Introduction

Fig: wave reflection on rigid and flexible boundary.

https://phet.colorado.edu/en/simulation/wave-on-a-string

https://in.mathworks.com/matlabcentral/fileexchange/35746-vibrating-string-simulator
The restoring force that is responsible for vibrations in strings is the tension in the string. The tension force is generated opposite to the applied force. But it always acts along the length of the string, in a direction that establishes equilibrium. A link to some problems wherein one can resolve applied force along the tension generating direction to calculate the resultant tension is given below.

When a periodic force is applied on a stretched string, perpendicular to its length, transverse waves will be generated in it. Let the equilibrium configuration of the string be a straight line. Then the act of generating a transverse wave moves the particles of the string away from equilibrium. Here the red arrow heads represent the direction of applied force. When one end of the string is tied to a rigid support and a wave is generated in it, a reflected wave is generated at the rigid support. The wave will have same amplitude but is 180° opposite in phase. This can be explained by Newton’s third law. If we hit the wall, wall will hit us back with equal and opposite force. Thus a wave of equal amplitude opposite phase is generated upon reflection. Suppose the string is connected to a flexible support. This can be achieved by tying the string to a ring and insert it onto a rigid metal bar. Then the ring will be free to move on the bar in accordance with the movement of the incoming wave. In that case, the reflected wave will have the same amplitude as well as same phase. Thus, the reflected wave traces the same path as the incoming wave.
7.2 ప్రమాణస్థాయి అనుష్ఠానానికి యోగ్యమైన మరణాలు

ప్రమాణం లో తరంగాన గాని, తరంగాన మాదిరి పారం నుండి గల రతంగ పారం నుండి 7.2 ప్రమాణం లో ఈ దాటకతో ఇంకా పారంస్తూ ఉంది. తరంగాన నుండి ఆక్రమణ నుండి తరంగాన గాని, తరంగాన మాదిరి పారం నుండి గల రతంగ పారం నుండి యోగ్యమైన మరణాలు ప్రమాణానికి యోగ్యమైన మరణాలు.

ఏమి ఏమి దీన్ని నుండి ఎందూ యోగ్యమైన మరణాలు ప్రమాణానికి యోగ్యమైన మరణాలు.

భాషలు చెప్పడంపెట్టలో, $F_x = T.\cos(\theta + d\theta) - T.\cos \theta$

$F_y = T.\sin(\theta + d\theta) - T.\sin \theta$

$\theta$ అనే పొందగా, $\cos \theta = 1$ మరణానికి $\sin \theta = \theta \approx \tan \theta$ అవసరం. అంటే,

$F_x \approx T.1 - T.1 = 0$

$F_y \approx T.\tan(\theta + d\theta) - T.\tan \theta = T \left(\frac{d\gamma}{dx}\right) - T \left(\frac{d\gamma}{dx}\right)$

$F_y = m.a_y = \mu . dx \frac{d^2y}{dt^2}$

$\frac{d^2y}{dt^2} = F_y \frac{\mu}{dx} = \frac{T}{\mu} \frac{(\frac{d\gamma}{dx}) - (\frac{d\gamma}{dx})}{dx} = \frac{T}{\mu} \frac{d^2y}{dx^2}$

$\frac{d^2y}{dt^2} = \frac{T}{\mu} \frac{d^2y}{dx^2}$

అతనే చికిత్సల సమస్యలకు విషయం ప్రత్యేకం. ఒక చికిత్సల సమస్యలకు విషయం ప్రత్యేకం, అంటే,

$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}$

మరణానికి ఈ స్థాయి, $v = \sqrt{T/\mu}$. ఏదుగుచేస్తే నిపుణుల చికిత్సల సమస్యలకు విషయం ప్రత్యేకం,
Consider a stretched string with tension $T$ and linear density $\mu$. Let the string be stretched along positive $x$-axis and plucked along $y$-axis to produce transverse waves. This results in a displacement by an amount $dx$ and $dy$ along $x$ and $y$ directions respectively. Let $\theta$ and $\theta + d\theta$ be the angles made by the tension with the horizontal axis at the coordinates $(x, y)$ and $(x + dx, y + dy)$ respectively.

The components of force along $x$ and $y$ directions, in terms of tension are given by

$$F_x = T \cdot \cos(\theta + d\theta) - T \cdot \cos \theta$$

$$F_y = T \cdot \sin(\theta + d\theta) - T \cdot \sin \theta$$

For small values of $\theta$, $\cos \theta \approx 1$ and $\sin \theta \approx \theta \approx \tan \theta$. Then the above equations become,

$$F_x \approx T \cdot 1 - T \cdot 1 = 0$$

$$F_y \approx T \cdot \tan(\theta + d\theta) - T \cdot \tan \theta = T \left( \frac{\partial y}{\partial x} \right)_2 - T \left( \frac{\partial y}{\partial x} \right)_1$$

But $F_y$ also is equal to
7.3 చదువు మీద సమీఖ్యాంశాల సమయం వంటించి ప్రదర్శనం

$$\frac{L}{T} = \sqrt{\frac{MLT^{-2}}{ML^{-1}}} = \sqrt{L^2T^{-2}} = \frac{L}{T}$$

7.3 చదువు మీద సమీఖ్యాంశాల సమయం వంటించి ప్రదర్శనం

ఇకడ పరంస్థితి మెండు సంక్షిప్తంగా ప్రదర్శించడం ప్రారంభించటం అవసరం, ప్రతి వంటించి సమయం మీద సమయం వంటించి ప్రదర్శనం అయితే తరసితుంది. ఇతర పరంస్థితులు ప్రదర్శించలోకి ప్రారంభించారు. పావిత సమయం ఒక ప్రదర్శనం కంటే ప్రదర్శనం అయితే అంతర్జాతీయ సంఖ్యాతి అంగం ప్రారంభించారు. 

అనుకుంటుందులు, వేరే వంటించి ప్రదర్శనం

$$y(x, t) = X(x). T(t)$$

ఇతర పరంస్థితి మీద సమయం వంటించి ప్రదర్శనం

$$X \frac{\partial^2 T}{\partial t^2} = v^2 T \frac{\partial^2 X}{\partial x^2}$$

ఇకడ పరంస్థితి మీద పరంస్థితి మీద సంక్షిప్తంగా ప్రదర్శించడం ప్రారంభించటం అవసరం, ప్రతి వంటించి సమయం వంటించి ప్రదర్శనం అయితే తరసితుంది. ఇతర పరంస్థితులు ప్రదర్శించలోకి ప్రారంభించారు. పావిత సమయం ఒక ప్రదర్శనం కంటే ప్రదర్శనం అయితే అంతర్జాతీయ సంఖ్యాతి అంగం ప్రారంభించారు.

$$\frac{1}{T} \frac{\partial^2 T}{\partial t^2} = \frac{v^2}{X} \frac{\partial^2 X}{\partial x^2} = -\omega^2$$

ఇతర పరంస్థితి,

$$\frac{\partial^2 T}{\partial t^2} + \omega^2 T = 0 \quad \text{and} \quad \frac{\partial^2 X}{\partial x^2} + \left(\frac{\omega^2}{v^2}\right) X = 0 \Rightarrow \frac{\partial^2 X}{\partial x^2} + k^2 X = 0.$$

ఇతర పరంస్థితి,

$$X(x) = A \sin kx + B \cos kx \quad \text{and} \quad T(t) = C \sin \omega t + D \cos \omega t$$

ఇది ప్రతి వంటించి ప్రదర్శనం

$$y(x, t) = (A \sin kx + B \cos kx)(C \sin \omega t + D \cos \omega t)$$

ఇకడ పరంస్థితి మీద సమయం వంటించి ప్రదర్శనం అయితే తరసితుంది. 

ఇతర పరంస్థితి మీద పరంస్థితి మీద సంక్షిప్తంగా ప్రదర్శించడం ప్రారంభించటం అవసరం, ప్రతి వంటించి సమయం వంటించి ప్రదర్శనం అయితే తరసితుంది. ఇతర పరంస్థితి మీద పరంస్థితి మీద సంక్షిప్తంగా ప్రదర్శించడం ప్రారంభించటం అవసరం, ప్రతి వంటించి సమయం వంటించి ప్రదర్శనం అయితే తరసితుంది.
7.3 Solution of Wave Equation

\[ F_y = m \cdot a_y = \mu \cdot dx \frac{\partial^2 y}{\partial t^2} \]

Here \( \mu = \frac{dm}{dx} \), the linear density (mass per unit length) of the string.

From the two equations above, we obtain

\[ \frac{\partial^2 y}{\partial t^2} = \frac{F_y}{\mu \cdot dx} = \frac{T \left( \frac{\partial y}{\partial x} \right)_2 - \left( \frac{\partial y}{\partial x} \right)_1}{\mu \cdot dx} = \frac{T \cdot \partial^2 y}{\mu \cdot dx^2} \]

or

\[ \frac{\partial^2 y}{\partial t^2} = \frac{T \cdot \partial^2 y}{\mu \cdot dx^2} . \]

This is the equation for transverse wave on a stretched string.

Comparing it with the general wave equation results in the following:

\[ \frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2} , \]

The wave velocity can be expressed as

\[ v = \sqrt{\frac{T}{\mu}} , \]

which can be verified using dimensional analysis:

\[ \frac{L}{T} = \sqrt{\frac{MLT^{-2}}{ML^{-1}}} = \sqrt{L^2 T^{-2}} = \frac{L}{T} \]

7.3 Solution of Wave Equation

The equation of transverse wave on a stretched string is a second order differential equation in both space and time. Hence it will have oscillating solutions in both space and time. Let the space
7.4 స్పష్టమైన విశోధనాలు నిర్మాణ శాసన విశేషాలను

7.4 స్పష్టమైన విశోధనాలు నిర్మాణ శాసన విశేషాలను

పాఠము కంపెంటి అంధకారం కంపెంటి పాఠాలను అంధకారం కంపెంటి పాఠాలను.

\[ \frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}. \]

విధానం కూడా విషాదానికి తప్పించిన ప్రతి ప్రతి తప్పించిన ప్రతి.

యు (x, t) = X(x). T(t) = (A \sin kx + B \cos kx)(C \sin \omega t + D \cos \omega t).

ఇందు లేదా లేదా తోడ్డించిన విషయాలు, కంపెంటి పాఠాల తప్పించిన ప్రతి తప్పించిన ప్రతి.

\[ X(x) = 0 \text{ at } x = 0 \text{ and } X(x) = 0 \text{ at } x = L. \]

మరియు సాధారణ కంపెంటి పాఠాలను నిర్మాణ శాసన విశేషాలను,

\[ A \sin k.0 + B \cos k.0 = 0 \Rightarrow A.0 + B.1 = 0 \Rightarrow B = 0. \]

మరియు సాధారణ కంపెంటి పాఠాలను నిర్మాణ శాసన విశేషాలను,

\[ A \sin kL = 0 \Rightarrow kL = \pm n\pi \Rightarrow X(x) = A \sin \left( \frac{n\pi x}{L} \right) \]

ఇందు లేదా లేదా తోడ్డించిన విషయాలు, కంపెంటి పాఠాల తప్పించిన ప్రతి తప్పించిన ప్రతి.

\[ y_n(x, t) = A \sin \frac{n\pi x}{L} . D \cos \omega t \]

ఇందు

\[ y_n(x, t) = y_0 \sin \frac{n\pi x}{L} \cos \omega t = y_0 \sin kx \cos \omega t, \]

అందువలన \( y_0 = A.D \)

మరియు సాధారణ కంపెంటి పాఠాలను కూడా తోడ్డించిన ప్రతి తప్పించిన ప్రతి,

\[ y_n(x, t) = \sum_{n=1}^{\infty} A \sin \frac{n\pi x}{L} (C \sin \omega t + D \cos \omega t) \]
part of the solution and time part of the solution be independent of each other. Then one can write

\[ y(x, t) = X(x)T(t) \]

Substituting this into wave equation, we obtain

\[ X \frac{\partial^2 T}{\partial t^2} = v^2 T \frac{\partial^2 X}{\partial x^2} \]

Since space part and time part are independent of each other, both must be equal to a constant. A simple dimensional analysis yields the result that it must have dimensions of inverse time square. Let it be denoted by \( \omega^2 \).

\[
\frac{1}{T} \frac{\partial^2 T}{\partial t^2} = \frac{v^2}{X} \frac{\partial^2 X}{\partial x^2} = -\omega^2
\]

Then one can write

\[
\frac{\partial^2 T}{\partial t^2} + \omega^2 T = 0 \quad \text{and} \quad \frac{\partial^2 X}{\partial x^2} + \left(\frac{\omega}{v}\right)^2 X = 0 \Rightarrow \frac{\partial^2 X}{\partial x^2} + k^2 X = 0.
\]

The solutions are given by

\[ X(x) = A \sin kx + B \cos kx \quad \text{and} \quad T(t) = C \sin \omega t + D \cos \omega t \]

Thus

\[ y(x, t) = (A \sin kx + B \cos kx)(C \sin \omega t + D \cos \omega t) \]

One need to apply proper boundary conditions to arrive at a specific solution. Here one can see the solution is a combination of a symmetric part and an asymmetric part both in space and time. Thus boundary conditions decide whether a given system has symmetric solutions or asymmetric solutions or combination of both.

7.4 Modes of vibration of stretched string clamped at ends

The equation of motion of transverse wave on a string is given by

\[
\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}.
\]
7.4 Modes of vibration of stretched string clamped at ends

$$f_n = \frac{v}{\lambda} = v \cdot \frac{k}{2\pi} = v \cdot \frac{n\pi}{L \cdot 2\pi} = \frac{n}{2L} \cdot \frac{v}{\mu} = \frac{n}{2L} \sqrt{\frac{T}{\mu}}$$

$$f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}}$$

https://iwant2study.org/lookangejss/04waves_12generalwaves/ejss_model PIPESTRINGWEE02/pipestringwhee02_Simulation.xhtml


https://www.acs.psu.edu/drussell/demos/string/fixed.html
7.4 Modes of vibration of stretched string clamped at ends

The general solution of this wave equation is given by
\[ y(x, t) = X(x) \cdot T(t) = (A \sin kx + B \cos kx)(C \sin \omega t + D \cos \omega t). \]

Since the string considered is clamped at both the ends separated by a distance, (say \( L \)), it follows that \( X(x) = 0 \) at \( x = 0 \) and \( X(x) = 0 \) at \( x = L \).

Substituting the first condition gives
\[ A \sin k. 0 + B \cos k. 0 = 0 \Rightarrow A. 0 + B. 1 = 0 \Rightarrow B = 0. \]

Substituting the second condition gives
\[ A \sin kL = 0 \Rightarrow kL = \pm n\pi \Rightarrow X(x) = A \sin \left( \frac{n\pi x}{L} \right) \]

Since there are no boundary conditions on the time dependent part of the solution, \( T(t) \) remains the same. The only condition that one can impose is for the displacement to have time reversal symmetry which is expressed as \( T(t) = T(-t) \). This requirement implies that the solution of \( T(t) \) will be a function of cosine terms only. This would make \( C = 0 \). Thus, with all the above information incorporated, the solution is given by
\[ y_n(x, t) = A \sin \frac{n\pi x}{L} \cdot D \cos \omega t \]
or
\[ y_n(x, t) = y_0 \sin \frac{n\pi x}{L} \cos \omega t = y_0 \sin kx \cos \omega t, \]

where \( y_0 = A \cdot D \)

A more general solution can be written as
\[ y_n(x, t) = \sum_{n=1}^{\infty} A \sin \frac{n\pi x}{L} \left( C \sin \omega t + D \cos \omega t \right) \]

The frequency of oscillation is given by
\[ f_n = \frac{v}{\lambda} = v, \quad k = \frac{n\pi}{2L} = \frac{n\pi}{2\pi} = \frac{n}{2L} \quad \Rightarrow \quad \frac{n}{2L} \sqrt{\frac{T}{\mu}} \]
\[ f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}} \]
చాలా సరిపోయినప్పటికీ కంచి గలంగిన అంశాలు మీద పిలిచిన ఉండాలి అంశాలు కంచి గలం ప్రతిభ కంపెన్సీలు

\[ f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}} \]

ఇది మోడల్ మేలు, ప్రతి ఉత్పత్తి ప్రామాణిక మీద పిలిచిన ఉండాలి మీద ప్రతిభ కంపెన్సీలు

1) కానికి మేలు ప్రామాణిక: ప్రతి ఉత్పత్తి ప్రామాణిక మీద పిలిచిన ఉండాలి కంపెన్సీ లక్షణాలు

2) తను మేలు ప్రామాణిక: ప్రతి ఉత్పత్తి ప్రామాణిక మీద పిలిచిన ఉండాలి కంపెన్సీ లక్షణాలు

3) తను మేలు ప్రామాణిక: ప్రతి ఉత్పత్తి ప్రామాణిక మీద పిలిచిన ఉండాలి కంపెన్సీ లక్షణాలు

4) తను మేలు ప్రామాణిక: ప్రతి ఉత్పత్తి ప్రామాణిక మీద పిలిచిన ఉండాలి కంపెన్సీ లక్షణాలు

The frequency of vibrations in stretched string clamped at both the ends is given by

\[ f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}} \]

From this equation, three laws of transverse vibrations in stretched string can be deduced.

a) Law of Length: The fundamental frequency of vibration of stretched string is inversely proportional to the length of the string, provided the tension in the string and the linear density of the string remain constant.

b) Law of Tension: The fundamental frequency of vibration of the stretched string is directly proportional to the square root of the tension produced in the string, provided the length and linear density of the string remain constant.

c) Law of Linear density: The fundamental frequency of vibration of stretched string is inversely proportional to the square root of the linear density of the string provided the length of the string and the tension produced in the string remain constant.

For a given string with a fixed tension applied, the term \( \sqrt{T/\mu} \) remains constant. Thus, based on the value of \( n \) there are infinitely many possible frequencies.

The lowest value of \( n \) for which a nontrivial solution is obtained is when \( n = 1 \), which is known as the fundamental frequency. This is also called the first harmonic. Higher values of \( n \)
7.5 మామిడి విస్తారములు

7.5 మామిడి విస్తారములు

ధారా విస్తారములు మేలు విస్తారములకు సమాధానం చేస్తుంది.

\[ f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}} \]

అమలుగా ఈదుద్దించబడింది, \( n \) ఆంశికంగా విస్తారముల సంఖ్య. నిమ్మితమ, మూడు సంఖ్యను అంటే, \( n = 1, 2, 3 \).

\[ f_1 = \frac{1}{2L} \sqrt{\frac{T}{\mu}} \]

మూడు సంఖ్యలు ప్రామాణికంగా పంచబడింది. \( f_1 \) సంఖ్య సమాన ఉంటుంది.

\[ f_2 = \frac{2}{2L} \sqrt{\frac{T}{\mu}} \]

మూడు సంఖ్యలు సమానంగా ఉంటుంది. \( f_2 \) సంఖ్యాలు సమానంగా ఉన్నాయి.

\[ f_3 = \frac{3}{2L} \sqrt{\frac{T}{\mu}} \]

మూడు సంఖ్యలు సమానంగా ఉంటుంది. \( f_3 \) సంఖ్యలు సమానంగా ఉన్నాయి.

మూడు సంఖ్యలు సమానంగా ఉన్నాయి. భాగం 1 యొక్క మండల యొక్క శాంతి ఉంటుంది. భాగం 2 యొక్క మండల యొక్క శాంతి ఉంంది. భాగం 3 యొక్క మండల యొక్క శాంతి ఉంంది.
give rise to higher harmonics/overtones. For \( n = 2 \), we obtain the second harmonic or the first overtone. The value \( n = 3 \) corresponds to the third harmonic or second overtone and so on.

\[
f_1 = \frac{1}{2L} \sqrt{\frac{T}{\mu}} \quad \text{First harmonic or Fundamental Frequency}
\]

\[
f_2 = \frac{2}{2L} \sqrt{\frac{T}{\mu}} \quad \text{Second harmonic or First Overtone}
\]

\[
f_3 = \frac{3}{2L} \sqrt{\frac{T}{\mu}} \quad \text{Third harmonic or Second Overtone}
\]

For the first harmonic, the wavelength is given by

\[
\lambda_1 = \frac{v}{f_1} = \frac{\sqrt{T/\mu}}{\frac{1}{2L} \sqrt{T/\mu}} = 2L
\]

For other harmonics, the wavelength is given by

\[
\lambda_2 = \frac{v}{f_2} = \frac{2L}{2} = L; \quad \lambda_3 = \frac{v}{f_3} = \frac{2L}{3}; \ldots
\]

Thus, one can fit half a wave of wavelength \( \lambda_1 \) within a length \( L \); two half waves of wavelength \( \lambda_2 \) and three half waves of wavelength \( \lambda_3 \) within the length \( L \). The wave patterns for various harmonics and overtones are shown below.

Here these harmonics and overtones are also called natural vibrational modes of the stretched string and the first harmonic is also called the fundamental mode of vibration.

When a guitar string is plucked at \( L/7 \) distance, then one cannot produce 7th harmonic, as that portion must remain as node forever to produce 7th harmonic. Similarly, if the guitar string is plucked at \( L/3 \) distance, then one cannot produce 3rd and 6th harmonics. Thus musicians control the number of harmonics to be produced/avoided by selecting the plucking point suitably.

https://www.acs.psu.edu/drussell/demos/string/fixed.html
Did You Know?

Harmonic series in mathematics is named after harmonics in music. In music 2, 3, 4 … harmonics are blocked by plucking the string at 1/2, 1/3, 1/4 etc lengths. Similarly harmonic series is produced by adding all numbers 1, 1/2, 1/3, 1/4, etc in the series. Simple harmonic motion (SHM) is considered harmonic because it can be described by same harmonic functions (sine and cosine) that describe the harmonics in music. More specifically the fundamental mode.

Did You Know?

Undertones or sub-harmonics are the resultant harmonics of division of fundamental frequency instead of multiplication. One need to use double length of the string or crossed strings or by using third bridge in guitars. In air instruments, overblowing creates sub-harmonics. They play major role in the perception of Ultrasonic sounds by human ear through skull bone. (Details are in next chapter)
7.6 Melde’s Experiment

1) Melde’s experimental setup consists of an electrically driven tuning fork.

2) One of the prongs of the tuning fork is connected to a string.

3) The other end of the string is passed through a smooth pulley and attached to a pan carrying weights. By changing the weights, required tension can be generated in the string.

4) The tuning fork, when vibrated, generates standing waves in the string. The orientation of the tuning fork gives rise to two configurations.

5) Longitudinal mode: Here the tuning fork prong’s vibrations are along the length of the string. In this case, one full oscillation of the tuning fork generates only half a wave in the string. Thus $f_{\text{string}} = 2f_{\text{tuning fork}}$.

6) Transverse mode: Here the tuning fork’s vibrations are perpendicular to the length of the string. In this case, one full oscillation of the tuning fork generates one full wave in the string. Thus $f_{\text{string}} = f_{\text{tuning fork}}$.

7) The length of the string is adjusted so that a perfect standing wave is formed on the string. Then the frequency of oscillations of the string is given by $f_{\text{string}} = \frac{n}{2L} \sqrt{\frac{T}{\mu}}$. Here $n$ is the number of loops (half waves) in the standing wave.

8) Using this experiment, one can estimate either the frequency of the tuning fork or the tension produced in the string or the linear density of the string provided the remaining parameters are known.

Fig: Melde’s experimental setup
(a) Longitudinal mode
(b) Transverse mode.
7.5 మంది సమయం పంచానం

7.5 మంది సమయం పంచానం

1) మంది మూలను విశేషాలు ప్రదర్శించి తీసుకుని కంటి లేకపోయిన ప్రత్యేకం ముందు లేదు. 

2) వారికి కంటి ముందు విధానం కనుగొని విధానానికి లేదు.

3) ప్రత్యేకం తీసుకుని ఎవరికి చెప్పకాని కట్టడానికి గురించి పరిమితం చేయడానికి తగినం చేయడానికి. అందుకే నిహారం చేయబడి వీటి కూడా సంఖ్యలు తోందినాం.

4) త్రిశాఖ పత్రిక పంచానం ప్రవచనాలు, ప్రత్యేకం నిఖరంగా పంచానం చేయడానికి. వారికి కంటి ముందు విధానానికి లేదు.

5) సంఖ్యలను కూడా: ఎందుకంటి విధానం కంటి పత్రిక తీసుకుండా ఉండాం. ఉండాలి, తేడా విధానం కంటి పత్రిక ఉండాం. ఒక విధానం చేయడానికి తగినం చేయడానికి.

6) ఒక విధానం చేయడానికి: ఎందుకంటి విధానం కంటి విధానం ఉండాం. ఉండాలి, వీటి తీసుకుండా ఉండాం. ఒక విధానం చేయడానికి తగినం చేయడానికి.

7) ఐన కొుదు సంఖ్యలను నిఖరంగా విధానం ఉండాం. కొుదు దాని ప్రత్యేకం దానితో విధానం ఉండాం.

8) ఐ మూలం ప్రత్యేకంగా చెప్పడానికి, కంటి పత్రిక పత్రిక వివరణాత్మకంగా ఇంగ్లీషులో పరిమితం చేయడానికి. ఉండాలి, తేడా విధానం ఉండాం. 

7.6 ప్రత్యేకం ప్రత్యేకం విద్యా పంచానం

1) ప్రత్యేకం విద్యా ప్రదర్శించడానికి (10-35 మి) మూలం విద్యా ప్రదర్శించడానికి మరియు మూలం విద్యా ప్రదర్శించడానికి మరియు మూలం విద్యా ప్రదర్శించడానికి.

2) తేడా విధానం, కంటి, మూలం విద్యా ప్రదర్శించడానికి మరియు మూలం విద్యా ప్రదర్శించడానికి మరియు మూలం విద్యా ప్రదర్శించడానికి.

3) తేడా విధానం ప్రదర్శించడానికి, కంటి, మూలం విద్యా ప్రదర్శించడానికి మరియు మూలం విద్యా ప్రదర్శించడానికి మరియు మూలం విద్యా ప్రదర్శించడానికి.

4) తేడా విధానం ప్రదర్శించడానికి, కంటి, మూలం విద్యా ప్రదర్శించడానికి మరియు మూలం విద్యా ప్రదర్శించడానికి మరియు మూలం విద్యా ప్రదర్శించడానికి.
7.6 Applications of vibrating strings

1) String theory analyses the vibrations of string like fields in the Planck length scale \((10^{-35}\text{m})\) where short range nuclear interactions play a major role.

2) All stringed musical instruments like Violin, Guitar, Veena etc. function based on the principle of resonant vibrations and harmonics in stretched strings.

3) Analysis of vibrating strings is essential in textile industry for the strength analysis of yarns. They are also useful in assessing the quality of sutures as well as the quality of suture yarns in robotic surgery.

4) Electronic yarns are developed for vibration sensing to analyse hand transmitted vibrations in industrial areas.

5) Vibrational analysis of pipe line tube strings in underground gas storage systems, deep sea floor mining etc. is useful in quality checking of the systems.

6) Industrial conveyor belts, cable car wires, crane wires, elevator cables etc. need vibrational analysis for their strength and stability analysis, during dynamic working conditions.

7) Various sensors developed for such industrial applications require programming and statistical analysis of the data collected from those sensors for sensible and informed decision making.

8) Vortex wind mills use vibrating metal strings to generate power from wind, instead of turbines.
7.6 Applications of vibrating strings

5) The use of vibrating strings in various musical instruments, such as the violin, oboe, and flute, has been extensively studied.

6) String vibrations are also used in various mechanical systems for motion detection and control.

7) The application of vibrating strings in the field of acoustics is crucial for understanding sound propagation and the behavior of membranes.

8) Research on the vibrational properties of strings has led to advancements in materials science and technology.

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**F - Corner**


https://d1wqtxs1xzle7.cloudfront.net/35303058/Manuscript36-Bark-Hamlyn2012-Final-with-cover-page-v2.pdf


https://www.mdpi.com/1424-8220/21/8/2780/pdf

http://www.autexrj.com/cms/zalaczone_pliki/6d.pdf


https://www.jstor.org/stable/20022151

http://home.mit.bme.hu/~bank/publist/jasa05.pdf
Solved Problems and Exercises

Solved Problems

1. The equation of a wave along a stretched string is \( y = A \sin \sin (kx - \omega t) \). Find the particle velocity amplitude and maximum value of the slope of the displacement curve.

Sol: Given the wave equation of a travelling wave along the positive x direction
\[ y = A \sin \sin (kx - \omega t) \]
The particle velocity is
\[ u = \frac{\partial y}{\partial t} = A\omega \cos(kx - \omega t) \]
\[ \therefore \] Particle velocity amplitude = \( A\omega \)

The wave velocity is
\[ v = f \lambda \]
We know that the slope of the displacement curve is
\[ \frac{\text{Particle velocity}}{\text{wave velocity}} = \frac{A\omega \cos(kx - \omega t)}{f\lambda} \]
\[ \therefore \text{maximum value of the slope of the displacement curve} \]
\[ = \frac{A\omega}{f\lambda} \]

2. The vibrations of a stretched string of the length 60cm and fixed at both the ends are given by the equation \( y = 7 \sin \sin \left(\frac{\pi x}{15}\right) \cos(50\pi t) \) with x and y in cm and t in sec. Find the locations of nodes and antinodes.

Sol: Given length of wire \( L = 50 \text{ cm} \)
Also the given equation is a stationary wave equation \( y = 7 \sin \sin \left(\frac{\pi x}{15}\right) \cos(50\pi t) \)
We know that nodes are the points of minimum displacement and hence for nodes
\[ y = 0 \text{ for all times} \]
\[ ie \text{ Space dependent term should be zero for nodes} \]
\[ \sin \sin \left(\frac{\pi x}{15}\right) = 0 \]
3. A string is stretched between two rigid supports having 100 cm separation. Find the longest possible wavelength of the stationary wave in the string.

Sol: We know that the mode of vibration with longest possible wavelength of the stationary wave in the string is fundamental mode.
In fundamental mode, 2 nodes are formed at rigid supports and 1 antinode at the middle.
Also we know that distance between two successive nodes is \( \frac{\lambda}{2} \).
\[ L = \frac{\lambda}{2} \Rightarrow \lambda = 2L = 200\text{ cm} \]
which is the longest possible wavelength.

4. In a guitar, a string of length 25 inches and of mass 1gm is stretched uniformly under a tension of 1 N. After setting the string into vibrations, it is slightly touched at a point of distance 5 inches from a fixed end. Find the possible frequencies of the string.

Sol: Given the string is touched at 5 inches. Hence node should be there. i.e., only those vibrations will be present for which the touched point will be a node.
Also given the length of string \( L = 25\text{ inches} \).
\( \therefore \) The minimum number of loops in which the string vibrates when it is touched at 5 inches is
\[ N = \frac{25}{5} = 5 \]

The possible number of loops in which the string vibrates is 5, 10, 15, 20, …

Hence the frequencies will be \( f_5, f_{10}, f_{15}, f_{20} \), …

Let us first find fundamental frequency \( f_1 = \frac{1}{2L} \sqrt[3]{\frac{T}{\mu}} \) where

\( T = 1N \) and \( L = 25 \text{ inches} = 25 \times 2.54 = 63.5 \text{ cm} \) and

linear density \( \mu = \frac{m}{L} = \frac{0.001}{6 \times 35 \times 10^{-2}} = 0.0016 \text{ kg/m} \)

\[ f_1 = \frac{1}{2 \times 6 \times 35 \times 10^{-2}} \sqrt[3]{\frac{1}{0.0016}} = 0.79 \times 25 \approx 20 \text{ Hz} \]

\[ f_5 = 5f_1 = 100 \text{ Hz}, f_{10} = 10f_1 = 100 \text{ Hz}, f_{15} = 15f_1 = 300 \text{ Hz}, f_{20} = 400 \text{ Hz} \frac{}{} \]

5. A string of length 36 cm was in unison with a fork of frequency 256 Hz. It was in unison with another fork when the vibrating length was 48 cm, the tension being unaltered. Find the frequency of second fork.

Sol: Given \( f_1 = 256 \text{ Hz} \) and its corresponding resonating length of string is \( L_1 = 36 \text{ cm} \)

\( f_2 = ? \text{ Hz} \) and its corresponding resonating length of string is \( L_2 = 48 \text{ cm} \)

\[ f_n = \frac{n}{2L} \sqrt[3]{\frac{T}{m}} \]

Here \( n, T \) and \( m \) are constant then \( f \propto \frac{1}{L} \)

\[ f_2 = \frac{f_1 L_2}{L_1} = 256 \times \frac{48}{36} = 341 \text{ Hz} \]

6. The fundamental frequency of sonometer wire is 600 Hz.

When length is shorted by 25%, the frequency of first overtone will be

Sol: Given \( f_{\text{fundamental}} = 600 \text{ Hz} \), \( L_1 = L \) and \( L_2 = \frac{(100-25)}{100}L = 0.75L \)
Solved Problems and Exercises

\[ \therefore f_{\text{fundamental}} = \frac{f_1 L_2}{L_1} = 600 \times 0.75 = 450 \text{ Hz} \]

The frequency of first overtone for the shortened wire will be

\[ f_{\text{first overtone}} = 2 f_{\text{fundamental}} = 900 \text{ Hz} \]

7. Transverse wave on a string is given by \( y = 0.05 \sin \sin (3x - t) \) in SI system. Find the frequency and wavelength

Sol: We have general solution \( y = A \sin \sin (kx - \omega t) \)
On comparison, we have \( \omega = 1 \text{ rad/sec} \). Hence frequency

\[ f = \frac{1}{2\pi} \text{ Hz} = 0.31 \text{ Hz} \]

and wave vector \( k = 3 \text{ m}^{-1} \). Hence wave length \( \lambda = \frac{2\pi}{k} = 2.09 \text{ m} \)

8. A string of length 1m and linear density 0.1 g/m is kept under a tension of 1 N. Find the frequency of first and second overtone, when the string is plucked at its mid point

Sol: When the string is plucked at its mid point, only odd harmonics ie n, 3n, 5n 7n....alone are present.
Hence first overtone or second harmonics is not possible

Second overtone = \( 3n = \frac{3}{2L} \sqrt{\frac{T}{m}} = \frac{3}{2} \sqrt{\frac{1}{0.0001}} = 150 \text{ Hz} \)

9. The speed of a transverse wave on a stretched string of 1m length is 500m/s. Find the frequency in fundamental mode.

Sol: We have \( f_n = \frac{n}{2L} \sqrt{\frac{T}{m}} \)

\[ f_n = \frac{n}{2L} \nu \]

Given \( n = 1 \), \( L = 1 \text{ m} \) and \( \nu = 500 \text{ m/s} \)
\( f_1 = \frac{1}{2} \times 500 = 250 \text{ Hz} \)

10. A monochord is having a taut wire of total length L and has a frequency of 100Hz . Its vibrating length can be adjusted with a movable bridge. If the length is halved and the tension is changed so that the frequency becomes 50Hz. How many times the tension is increased/decreased?
Sol: Given $L_1 = L$ and $L_2 = \frac{L}{2}$; $f_1 = 100 Hz$ and $f_2 = 50Hz = \frac{1}{2}f_1$

We have the relation among $L, f$ and $T$ is $f \propto \sqrt{\frac{T}{L}}$

\[ \frac{f_1}{f_2} = \frac{\sqrt{T_1L_2}}{\sqrt{T_1L_1}} \]

\[ 2 = \sqrt{\frac{T_1}{T_2}} \cdot \frac{1}{2} \] and hence we get $T_1 = 8T_2$ ie tension should be increased by 8 times

11. Find the speed of transverse wave in a wire of 1mm$^2$ cross section and under the tension provided using 0.1 kg mass. (Specific gravity of the wire is 9.8 gm/cm$^3$)

Sol: Given $\mu = 9.8 \times \frac{10^{-3}}{(10^{-2})^3} \times (10^{-3})^2 \frac{kg}{m} = 9.8 \times 10^{-3} \frac{kg}{m}$

Tension $T = Mg = 0.1 \times 9.8$

\[ \therefore \text{Speed of transverse wave} \ n = \frac{T}{\mu} = \sqrt{\frac{0.1 \times 9.8}{9.8 \times 10^{-3}}} = 10 \text{ m/s} \]

**MCQs**

1. A stationary wave is produced in a string of length 1.25 m. If three nodes and two antinodes are produced in the string then the wavelength of the wave is
   a) 2.5 m   b) 3.75 m   c) 5m   d) 1.25m
   HCU 2020

Ans: d

2. The Young’s modulus of a rod of mass $m$, cross sectional area $a$ and length $l_0$, fixed at one end is $Y$. If a force is applied to stretch the rod, the period of small oscillation will be

   a) $2\pi \sqrt{\frac{ml_0}{AY}}$   b) $\pi \sqrt{\frac{ml_0}{AY}}$   c) $2\pi \sqrt{\frac{m}{AY}}$   d) $2\pi \sqrt{\frac{ml_0Y}{A}}$

   HCU 2019

Ans: a
3. Consider a sonometer wire made by soldering two wires of radii \( r \) and \( 2r \) and kept under tension \( T \). Stationary vibrations are set up in the wire such that the soldered joint is midway between the two bridges and is a node. The ratio of the number of loops formed in the wires is

\[ \frac{1}{2} \]

HCU 2018

Ans: ?

4. A wave is propagating in a string of tension \( T \) and mass per unit length \( \mu \). The travelling wave can be described as \( y(x, t) = A \sin(kx - \omega t) \). What is the kinetic energy in one wavelength \( \lambda \) of the travelling wave?

\[ \frac{2TA^2\pi^2}{\lambda} \quad \frac{T \Lambda^2 \pi^2}{\lambda} \quad \frac{T \Lambda^2 \pi^2}{2\lambda} \quad \frac{T \Lambda^2 \pi^2}{4\lambda} \]

HCU 2015

Ans: d

Hint: Kinetic energy \( K.E = \frac{1}{2} \)

5. A violin string is held under Tension \( T \). What will be the fractional change in the frequency of its fundamental mode of vibration if the tension is increased by the amount \( \delta T \)?

\[ \frac{\delta T}{T^2} \quad \frac{\delta T \sqrt{T}}{\sqrt{T}} \quad \frac{\delta T}{T} \quad \frac{\delta T^2}{2T} \]

HCU 2015

Ans:

6. The standing wave on a string of length \( L \) that is fixed at both ends have a speed \( v \). The three lowest frequencies of vibration are

\[ \frac{v}{L}, \frac{2v}{L}, \frac{3v}{L} \quad \frac{v}{2L}, \frac{v}{L}, \frac{3v}{2L} \quad \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2} \quad \frac{L}{v}, \frac{2L}{v}, \frac{3L}{v} \]

HCU 2014

Ans: b
7. Which of the following equations does not represent a travelling wave

a) \( y = f(x - vt) \)

b) \( y = f(x^2 - v^2 t^2) \)

c) \( y = y_{max} k(x + vt) \)

d) \( y = y_{max} e^{i(x-vt)} \)

HCU 2012

Ans: b

Hint: Function should contain \( x \) and \( t \) only in the combination of either \( x + vt \) or \( x - vt \)

8. Beats are the results of

a) Diffraction

b) Destructive Interference

c) Constructive interference

d) Super position of two waves with almost same frequency

AUCET 2020

Ans: d

9. The velocity of a transverse wave along a stretched string is

a) \( \sqrt{\frac{F}{m}} \)

b) \( \sqrt{\frac{m}{\ell}} \)

c) \( \sqrt{\frac{1}{mT}} \)

d) \( \sqrt{\frac{T}{m}} \)

AUCET 2020

Ans: d

10. If the energy flows across every plane in the direction of propagation of the wave then it is

a) stationary wave  

b) progressive wave  

c) electromagnetic wave  

d) both a and c

AUCET 2020

Ans: b

11. A Steel wire 550 cm long has mass of 5 gm. It is stretched with a tension of 400 N. Find the frequency of the wire in fundamental mode of vibration
a) 100 Hz  b) 125 Hz  c) 200 Hz  d) 216 Hz

Ans: 50Hz

12. The speed of a transverse wave on a stretched string is 500 m/s, when it is stretched under a tension of 19.6 N. If the tension is altered to a value of 78.4 N. what will be the speed of the wave

a) 100 m/s  b) 500 m/s  c) 800 m/s  d) 1000 m/s

Ans: d

13. A wave is travelling on a string is given by $y(x, t) = 10 \sin \sin (0.1x - 22t)$. The wavelength ($\lambda$) and frequency ($n$) of the wave are given by

a) 62.9 cm, 3.5 Hz  b) 31.4 cm, 7 Hz  c) 0.002 cm, 0.3 Hz  d) 6.28 cm, 35 Hz

Ans: a

14. Which of the following tuning fork is used by the doctors to access a patient’s hearing

a) C-438  b) C-128  c) C-416  d) C-512

Ans: d

15. In Melde’s experiment, eight loops are formed with a tension of 0.75 N. If the tension is increased to four times then the number of loops produced will be

a) 2  b) 4  c) 8  d) 16

Ans: b

16. In Melde’s experiment, the tuning fork is arranged such that it’s prongs are parallel to the string. During 200 vibrations of the tuning fork, the string completes vibrations
Solved Problems and Exercises

17. The apparatus of Melde's experiment can be used to test the relationship between

A. Tension   B. Mass per unit length   C. Frequency   D. Wavelength

Choose appropriate answer

a) A, B only     b) A, D only     c) A, C only     d) None of the above

Ans: d

18. A knife edge divides a sonometer wire in two parts which differ in length by 2 mm. The whole length of the wire is 1 meter. The two parts of the string when sounded together produce one beat per second. Then the frequency of the smaller and longer pans in Hz, are

a) 250.5 and 249.5     b) 249.5 and 250.5     c) 124.5 and 125.5     d) 125.5 and 124.5

Ans: d

19. A wave represented by \( y = 20 \sin \sin (2t + \phi) \) is

A. Simple harmonic wave   B. Progressive wave
C. Standing wave   D. Stationary wave

Choose appropriate answer

b) A, B only     b) A, D only     c) C, D only     d) A only

Ans: d

20. A string vibrates according to the equation where \( x, y \) are in cms and \( t \) in sec then the distance between two successive nodes is

a) 15 cm     b) 3 cm     c) 5 cm     d) 6 cm

21. A wave is described by the equation where all distances are in cms and time in second then
Solved Problems and Exercises

a) The amplitude is 4 cm  b) The wavelength is 
c) The period is sec  d) The wave is travelling in –ve x
direction

https://www.aliensbrain.com/quiz/19885/melde-s-experiment-
class-xi (Problems)

Grade your understanding

1. Modes of coupled oscillations have independent [ ]
frequencies where as modes of vibrations are [ ]
multiples of some fundamental frequencies [ ]
2. When we apply a force on string, a tension force is [ ]
generated perpendicular to the length of the string [ ]
3. Strings have both longitudinal and transverse [ ]
vibrations [ ]
4. Heart beat variations represent a wave [ ]
5. Wave equation of a string have oscillating solutions [ ]
   only in space but not in time [ ]
6. Tension in the string depends on the material of the [ ]
   string [ ]
7. The displacement of elements of the string has time [ ]
   reversal symmetry [ ]
8. Harmonics and overtones depends on the resonant [ ]
   length of a string when keeping applied frequency, [ ]
   tension and material of string constant [ ]
9. Tight string gives high pitch and that pitch value [ ]
   depends on mode of vibration [ ]
10. Transverse waves in the string are unpolarized [ ]
11. In transverse mode tuning fork prong length is [ ]
    transverse to string length [ ]
12. nth harmonic has (n + 1) nodes and n antinodes [ ]
13. A string being under tension between two fixed ends [ ]
    has continuous frequency spectrum [ ]
14. A simple harmonic wave is always either a [ ]
    progressive wave or stationary wave [ ]
## Glossary: Vibrating Strings

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antinode</td>
<td>A point where the amplitude of the standing wave is at maximum</td>
</tr>
<tr>
<td>Constrain</td>
<td>Compel or force (something) to follow a particular course of action</td>
</tr>
<tr>
<td>Crest</td>
<td>Highest surface part (highest amplitude) of a wave</td>
</tr>
<tr>
<td>Eigen frequency</td>
<td>One of the natural resonant frequencies of a system</td>
</tr>
<tr>
<td>Elasticity</td>
<td>The ability of an object or material to resume its normal shape after being stretched or compressed; stretchiness</td>
</tr>
<tr>
<td>Harmonic</td>
<td>A wave with a frequency that is a positive integer multiple of the frequency of the original wave</td>
</tr>
<tr>
<td>Monochord</td>
<td>A monochord, also known as sonometer, is an ancient musical and scientific laboratory instrument, involving one (mono-) string (chord)</td>
</tr>
<tr>
<td>Node</td>
<td>A point along a standing wave where the wave has minimum amplitude</td>
</tr>
<tr>
<td>Prong</td>
<td>A projecting part on various other devices</td>
</tr>
<tr>
<td>Quantum leap</td>
<td>A sudden, very noticeable and significant advance</td>
</tr>
<tr>
<td>Slackening</td>
<td>Reduce or decrease in speed or intensity</td>
</tr>
<tr>
<td>Sonometer</td>
<td>An instrument used to measure the sensitivity of hearing</td>
</tr>
<tr>
<td>Term</td>
<td>Definition</td>
</tr>
<tr>
<td>-------------</td>
<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Spindle</td>
<td>A rod or pin serving as an axis that revolves or on which something revolves.</td>
</tr>
<tr>
<td>Standing wave</td>
<td>Combination of two waves moving in opposite directions</td>
</tr>
<tr>
<td>Tension</td>
<td>The state of being stretched tight</td>
</tr>
<tr>
<td>Trough</td>
<td>Lowest surface part (lowest amplitude) of a wave</td>
</tr>
<tr>
<td>Ventral</td>
<td>Being or located near or on the anterior or lower surface of an animal opposite the back/plant structure.</td>
</tr>
</tbody>
</table>
UNIT-V
Chapter-8
ULTRASONICS
చిత్రాలు

1) అభిసంధి ధృత్రానికి, కింద అధ్యాయం నిలిచి నాణయం అందించాలి.

2) లోకం సమాధానం అభిసంధి కర్తా తయారు చేసి ఆశ్చర్యం చేశాలి.

3) అభిసంధి కర్తా వాక్యం, ఆంగ్లం నుంచి తెలుగు పదార్థాలను ప్రతిష్ఠించాలి.

4) అభిసంధి కర్తా జవాంశానికి చెప్పాలి. అహ్మద్ ప్రాంతం స్తంభం నెహ్వాల ప్రాంతం ప్రతిష్ఠించాలి.

5) లోకం సమాధానం అభిసంధి కర్తా జవాంశానికి నాణయం అందించాలి. సంస్కృతం నుంచి తెలుగు పదార్థాలను ప్రతిష్ఠించాలి.

6) అభిసంధి కర్తా లోకం సమాధానం తెలుగు పదార్థాలను ప్రతిష్ఠించాలి. ఆంగ్లం నుంచి తెలుగు పదార్థాలను ప్రతిష్ఠించాలి.

అభయిత్తులు వాదాలు

1) అభిసంధి నంది అధ్యాయం నిలిచి లోకం సమాధానం అభిసంధి కర్తా జవాంశానికి నాణయం అందించాలి.

2) లోకం సమాధానం అభిసంధి కర్తా వాక్యం, ఆంగ్లం నుంచి తెలుగు పదార్థాలను ప్రతిష్ఠించాలి.

3) అభిసంధి కర్తా జవాంశానికి చెప్పాలి. అహ్మద్ ప్రాంతం స్తంభం నెహ్వాల ప్రాంతం ప్రతిష్ఠించాలి.

4) అభిసంధి కర్తా జవాంశానికి నాణయం అందించాలి. సంస్కృతం నుంచి తెలుగు పదార్థాలను ప్రతిష్ఠించాలి.

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**Syllabus**

Ultrasonics, General properties of ultrasonic waves, Production of ultrasonics by piezoelectric and magnetostrictive methods, Detection of ultrasonics, Applications of ultrasonic waves, SONAR

**Learning Objectives**

In this chapter students would learn,

1. Properties of ultrasonic waves
2. Methods of generating ultrasonic waves
3. Methods of detecting ultrasonic waves
4. Applications of ultrasonics in various fields

**Learning Outcomes**

By the end of the chapter, student would be equipped with adequate knowledge to

1. Identify specific properties of ultrasonics to be used for a chosen application. In addition, to identify various fields where ultrasonics are being used.
2. Describe the behaviour of ultrasonics relevant to the nature of medium.
3. Predict proper material characteristics to generate and detect ultrasounds of required frequency.
4. Classify various types of applications of ultrasounds based on application and properties.
5. Select suitable methods of production & detection of ultrasonics for specific applications.
6. Design prototype systems suitable for specific application.
1. ప్రతి కార్యాలను ప్రామాణిక అనుబంధాలతో భాషించడం ముఖ్యమైన అవకాశం ఉంది.

2. ఎందుకంటే ప్రతి కార్యాన్ని ఎందుకంటే ప్రతి అంశాన్ని అధీనంగా పరిశీలిస్తాం?

3. మనం ప్రతి కార్యాన్ని ప్రతి అంశాన్ని జోగా నీటి వలన స్థాయి నిర్ణయిస్తాం?

4. మనం ప్రతి కార్యాలను ప్రతి అంశాన్ని సాధనా సమాధానాన్ని అధికంగా పరిశీలిస్తాం?

5. మనం ప్రతి కార్యాన్ని ప్రతి అంశాన్ని అధికంగా పరిశీలిస్తాం?

6. మనం ప్రతి కార్యాన్ని ప్రతి అంశాన్ని అధికంగా పరిశీలిస్తాం?

7. మనం ప్రతి కార్యాన్ని ప్రతి అంశాన్ని అధికంగా పరిశీలిస్తాం?

నృత్యదర్శన రిపోర్ట చేస్తే

కేంద్రానికి (నా మంచాట) అతితర విషయాలతో పాటు మనం, నిర్ణయానికి అన్ని ప్రత్యేకతలు, నిర్ణయించడం కావలసిన, సమాధానానికి, ప్రత్యేక ప్రకారం కావలసినప్పటి పద్ధతులు కనబడాలి. అదే నిర్ణయించడం, అంత పద్ధతి ప్రతి అంశాన్ని చెందిన పద్ధతి విశేషాత్మకంగా ఉంటుంది. అదే నిర్ణయించడం అనుకూలం తన చారుకొండల ఎందుకంటే విషయాలు, అంత పద్ధతి ప్రతి అంశాన్ని ప్రతి అంశాన్ని చెందిన పద్ధతి విశేషాత్మకంగా ఉంటుంది.

చాలాంటి పరంపరలు

20,000Hz ఉన్న ద్రాగుడికంత పరిశీలించడం లేదు చేత అధికంగా విషయాలు. ఇది మంచాన్ని అందించడం.

మనం ప్రతి కార్యాంశాన్ని పరిశీలించడం లేదు చేతి. 1013Hz ఉన్న ద్రాగుడికంత పరిశీలించడం లేదు చేతి. అందువల్ల 5≡ మనం ప్రతి కార్యాంశాన్ని పరిశీలించడం లేదు చేతి. అందువల్ల కార్యాలను communication కారణంగా మనం కొరకు అధీనంగా ఉన్న పద్ధతి. vacuum లేదా మనం పరిశీలించడం.
Course Outcomes specific to program and Future directions

By the end of this chapter students from specific programs would be in a position to identify the

2. Chemistry: Use of ultrasonics in various ultrasound assisted chemical reactions (sonochemistry) and especially in the emulsions industry.
3. Computers: Use of ultrasonic sensors in various fields and industries which are needed to be programmed.
4. Geology: Use of ultrasonics for detecting a variety of geological systems like underground water bodies, mineral deposits and their use in geo-physical imaging.
5. Electronics: Use of ultrasonic sensors in various fields and need for seamless integration of ultrasonic generators and detectors into various electronic equipment.
6. REM: Use of ultrasonics in biomass processing and acquire an understanding of the role of ultrasounds in nature and species, while designing eco-friendly wind and ocean energy systems.
7. Statistics: Use of ultrasounds in various fields and need for integrating statistical tools in analysing data, collected by various ultrasonic detectors in the respective fields.

Familiar to Unfamiliar

In your High School you might have learned about what are ultrasonics and applications of ultrasonics in cleaning, detecting flaws, medical imaging (Ultrasonography), breaking kidney stones (Lithotripsy) and measuring depth of sea (SONAR). You might have also learnt, in 11th class, that Doppler shift produced by ultrasounds is useful in monitoring the heartbeat. Here you will learn various properties of ultrasounds that decide their behaviour in various media, the Production and detection techniques and some further applications in various fields like industry, medicine, research etc.

Prepare a note on chronological developments in the field of ultrasonics from the books given in E-corner
8.1 Introduction

Ultrasonics are sound waves with frequencies greater than human audible range i.e., 20,000Hz. It should not be confused with supersonics, where objects travel with velocity more than the speed of sound. There are also hypersonic waves that travel with Mach number greater than 5 (i.e., velocities 5 times the speed of sound). There are also hypersounds that have frequency greater than $10^{13}$Hz.

Since ultrasonic waves are mechanical waves, they need a material medium to travel. Some of the animals like dogs and cats use these frequencies for communication. Other animal species like bats, dolphins and whales use them for navigation and echolocation of prey.

History

The usage of ultrasounds by human beings dates back to 1830 where Savart used toothed wheel to produce ultrasounds. Here the speed of the wheel decides the frequency of the sound. His apparatus could produce ultrasounds up to 24KHz. Savart’s wheel is considered as the first artificial ultrasound generator.

Fig: Savart’s wheel or disk. (Source: Wikipedia commons)

Further The Galton whistle or Dog whistle was developed in 1876 by Francis Galton. It consists of a resonating wind pipe similar to flute whose length can be adjusted. The relation between the length and resonance frequency is given by

$$\frac{\lambda}{4} = (e + l) \Rightarrow f = \frac{v}{\lambda} = \frac{v}{4(e + l)}$$

This was used to train dogs and was also used to test the reduction in audible range of frequencies, with age, in humans.
8.1 Introduction

Galton Whistle – The first dog whistle
The last reference in e-corner pp:20 says that it is it is an improved version of Galton Whistle in 1900 by Edlemann. – Justify with further references.

Did You Know?
Human ear drum can’t vibrate for frequencies greater than 20KHz. This is because, if a peak in the sound wave pushes the ear drum in, before it comes back to original state, second peak also falls in. Still human brain can perceive ultrasounds. Details are in next pages…

Think
Are there any other methods to generate sound waves other than beating objects and directing wind through pipes?

<table>
<thead>
<tr>
<th>Sl No</th>
<th>Application</th>
<th>Frequency range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Medical imaging</td>
<td>1-18MHz</td>
</tr>
<tr>
<td>2.</td>
<td>Acoustic sensors</td>
<td>10Hz-100GHz</td>
</tr>
<tr>
<td>3.</td>
<td>Guided waves</td>
<td>50KHz-10MHz.</td>
</tr>
<tr>
<td>4.</td>
<td>Acoustic microscopy</td>
<td>10MHz-400MHz</td>
</tr>
<tr>
<td>5.</td>
<td>Cavitation</td>
<td>30KHz-1MHz</td>
</tr>
<tr>
<td>6.</td>
<td>NDE (Non-destructive Evaluation)</td>
<td>100KHz-50MHz</td>
</tr>
<tr>
<td>7.</td>
<td>SAW (Surface Acoustic Waves)</td>
<td>100MHz-10GHz</td>
</tr>
</tbody>
</table>
Later in 1990 Koeing developed tuning forks which can produce ultrasounds up to 90KHz. Richardson in 1912 patented SONAR which is used for under water range finding.

**Did You Know?**

As per the Sanskrit, Chinese and Tibetan scriptures, in ancient India, Jivaka, a physician in the royal court, Bimbisara of Magadha (558BC-491BC), used a gem placed between wooden pieces to see inside the human body. That resembles the modern ultrasound scanning system.

During the First World War ultrasounds were widely used for echo locating icebergs and submarines in deep seas. Meanwhile various ultrasonic production and detection techniques were developed which made it possible to use them in industries and medical fields for various applications.

**Frequency Range of ultrasonic waves**

The range of frequencies of ultrasounds for various applications is as shown below.

**Fig. 3: Ultrasonics frequency range for various applications**
8.1 Introduction

The introduction provides an overview of the topic or research. It sets the stage for the rest of the paper, outlining the purpose, scope, and significance of the study. The introduction typically includes a brief history of the topic, an explanation of the problem or question being addressed, and a statement of the research objectives.

In the context of the document, the introduction likely discusses the background information, the importance of the study, and the research questions or hypotheses. It may also introduce key terms, concepts, or theories relevant to the topic.

The introduction is essential for establishing the context and rationale for the research, guiding the reader's understanding of the subsequent sections. It sets the tone and expectations for the reader, ensuring that they are prepared for the detailed exploration of the study's findings and implications.
Types of Ultrasonic waves

There are basically two types of ultrasonic waves. Longitudinal and Transverse. Longitudinal waves have oscillations of material particles of the medium along the direction of propagation and transverse waves have oscillations perpendicular to the direction of propagation of the wave. Transverse waves are again classified into two types - Shear Vertical (SV) and Shear Horizontal (SH). SV waves have oscillations in vertical direction and SH waves have oscillations in horizontal direction.

![Fig: P, SV, SH waves](image)

There are other important wave forms which are different combinations of longitudinal and transverse modes. Eg: Rayleigh, Love, Lamb, Stoneley, Sezawa etc.

**Rayleigh waves:** Rayleigh waves are the surface acoustic waves which may be longitudinal or transverse. They were named after their discoverer, Lord Rayleigh. The combination of both types of Rayleigh waves results in an ellipsoidal wave. These are found in seismic waves generated by earth quake. These are used in developing sensors for touchscreens, cell phone radiation filters, narrow band frequency filters in communication systems.

**Lamb waves:** These waves were discovered by Horace Lamb. These are the guided Rayleigh waves along a surface. These are combination of longitudinal and transverse waves with resultant circularly polarized waves. These are of two types namely symmetric and anti-symmetric lamb waves. These are used for bulk surface analysis of materials in one shot. These waves are used for flaw detection in thin films.
8.1 Introduction

Identify the difference between SH wave and Love wave. Identify the difference between SV, Rayleigh and Sezawa waves.

**E-Corner**

https://www.geometrics.com/support/different-types-of-seismic-waves/
https://doi.org/10.1109/ULTSYM.1986.198801
https://www.ndt.net/article/v11n05/nesvi/nesvi.htm
https://doi.org/10.1016/j.snb.2018.05.158
https://doi.org/10.1063/1.89772
https://doi.org/10.1121/1.4927633
https://doi.org/10.1063/1.4990443
https://www.mdpi.com/1424-8220/21/10/3421/htm
Love waves: These were discovered by AEH Love. These are horizontally polarized surface waves produced by superposition of many shear waves (S-waves). These are also called Q-waves where Quer in German means lateral. These are observed only when there is a low velocity layer over a high velocity layer. These are also used for surface studies in NDE techniques.

Sezawa waves: These are also similar to Love waves but with their vibration components in vertical direction. These were discovered by Sezawa, a Japanese seismologist. These waves are useful for characterizing surface acoustic wave sensors.

Stoneley waves: These waves propagate along the boundary separating two different media. Hence they are also called as boundary waves or interface waves. These waves are observed at solid-solid interfaces and liquid-solid interfaces. These waves are used in ultrasonic well logging tools which have a frequency of operation around 20KHz and 500KHz. These are also used for non-destructive evaluation (NDE) studies of metal-metal interfaces.

Sholte waves: These are also interface waves. These waves are observed at the interface of fluid and elastic solid medium. These are observed at water sand interfaces at shallow seabed during their
acoustic analysis. These are also used for NDE of lab generated tissue membranes.

అవసరాల కొనసాగ చేసేలే:

1. అవసరాల కొనసాగ 20,000Hz కం ఎవ ల కం ఉంం. 
2. అవసరాల కొనసాగ మాత్రమే ఉదాహరణకి జరిగిన ఉదాహరణలు ఉంంటుంది. 
3. అవసరాల కొనసాగ మాత్రమే ఉదాహరణలు ఉంంటుంది. 
4. అవసరాల కొనసాగ మాత్రమే ఉదాహరణలు ఉంంటుంది. 
5. అవసరాల కొనసాగ మాత్రమే ఉదాహరణలు ఉంంటుంది. 
6. అవసరాల కొనసాగ మాత్రమే ఉదాహరణలు ఉంంటుంది. 
7. అవసరాల కొనసాగ మాత్రమే ఉదాహరణలు ఉంంటుంది. 
8. అవసరాల కొనసాగ మాత్రమే ఉదాహరణలు ఉంంటుంది. 
9. అవసరాల కొనసాగ మాత్రమే ఉదాహరణలు ఉంంటుంది. 
10. అవసరాల కొనసాగ మాత్రమే ఉదాహరణలు ఉంంటుంది.
8.2 Properties of Ultrasonic waves

**Whistler mode acoustic waves:** These are the low frequency (1KHz-30KHz) radio waves generated during lightning. The lightning also generates acoustic waves in the surrounding air or ionosphere. These can be studied to understand the turbulences produced by lightning. These studies are especially useful in the analysis of solar wind.

8.2 Properties of Ultrasonic waves

The properties of ultrasonic waves are listed in the following:

1. Ultrasonic waves have frequencies greater than 20,000Hz.
2. They are mechanical waves and hence need a material medium to travel.
3. Ultrasonic waves travel with the same velocity as other audible sound waves.
4. The velocity of ultrasonic waves depends on the elasticity of the medium and temperature. It is given by $v = \sqrt{\frac{Y}{\rho}} \text{ (for solids)} \sqrt{\frac{K}{\rho}} \text{ (For fluids)}$ Where $Y$ is young’s modulus, $K$ is the bulk modulus of the material and $\rho$ is the density of the material. With the rise in temperature, both bulk modulus and density change and ultrasound wave velocity increases.
5. They can travel longer distances without much loss in intensity, as they have high frequency and short wavelength.
6. They can penetrate through opaque, conducting, non-conducting and dense media without deviation and hence are advantageous over optical, electronic or any other probes.
8.2 Properties of Ultrasonic waves

8.2.1 Ultrasonic Properties

8.2.1.1 Ultrasonic Properties:

Ultrasonic waves are waves that travel through a medium at a frequency higher than the audible range. Ultrasonic waves are used in various applications such as non-destructive testing, medical imaging, and cleaning. The properties of ultrasonic waves include:

- **Wavelength (λ)**: The distance between two consecutive wave peaks or troughs.
- **Frequency (f)**: The number of waves that pass a fixed point per unit time. It is measured in Hertz (Hz).
- **Velocity (v)**: The speed at which the wave travels through a medium. It is given by the equation:
  \[ v = f \cdot \lambda \]

- **Amplitude (A)**: The maximum displacement of a point in the medium from its equilibrium position.
- **Intensity (I)**: The power per unit area of the wave. It is given by the equation:
  \[ I = P / A \]

where \( P \) is the power of the wave and \( A \) is the cross-sectional area of the wave.

Ultrasonic waves can be generated using various methods, such as piezoelectric transducers, which convert electrical energy into mechanical energy.

The equation for the intensity of an ultrasonic wave is:

\[ I = I_0 e^{-2ax} \]

where \( I_0 \) is the intensity at the source, \( x \) is the distance from the source, and \( a \) is the attenuation coefficient.

The equation for the pressure of an ultrasonic wave is:

\[ u = u_0 e^{-ax} \]

where \( u_0 \) is the amplitude of the wave at the source, \( a \) is the attenuation coefficient, and \( x \) is the distance from the source.

The logarithmic relationship between intensity and power is:

\[ \log(I/I_0) = -20 \log(u/u_0) \]

where \( I_0 \) is the intensity at the source, \( I \) is the intensity at a distance \( x \), \( u_0 \) is the amplitude of the wave at the source, and \( u \) is the amplitude of the wave at a distance \( x \).

The equation for the pressure of an ultrasonic wave in decibels (dB) is:

\[ I = I_0 e^{-2.3 \alpha_B x} \]

where \( I_0 \) is the intensity at the source, \( x \) is the distance from the source, and \( \alpha_B \) is the attenuation coefficient in decibels per unit length.

The equation for the power density in decibels (dB) is:

\[ \alpha_{dB} = 10 \log \left( \frac{I_0}{I} \right) = 20 \log \left( \frac{u_0}{u} \right) = 20 \frac{\ln \left( \frac{u_0}{u} \right)}{2.303} \]

where \( u_0 \) is the amplitude of the wave at the source, \( u \) is the amplitude of the wave at a distance \( x \), and \( \alpha_{dB} \) is the attenuation coefficient in decibels per unit length.

The equation for the intensity in decibels (dB) is:

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where \( u_0 \) is the amplitude of the wave at the source, \( u \) is the amplitude of the wave at a distance \( x \), and \( \alpha_{dB} \) is the attenuation coefficient in decibels per unit length.
8.2 Properties of Ultrasonic waves

7. They can be reflected, refracted, focused exactly like ordinary sound waves.

8. They also exhibit “diffraction limit” during focusing, like sound waves. We know that when sound waves are focused into a region smaller than their wavelength, they get deviated.

9. They produce heat when incident on materials.

10. They produce “Cavitation effect”, which is the phenomenon of creation of vacuum bubbles in liquids when exposed to ultrasonic waves under low pressure.

11. They produce sonoluminescence, i.e., when a standing ultrasonic wave is established in liquid medium and if cavitation produces bubbles, the bubble expands at the low pressure region and collapses at the high pressure regions of the standing wave. During sudden collapse, the bubble exhibits luminescence.

A few of the quantitative properties of ultrasonic waves which are essential in deciding their behaviour in various media, are as follows:

Attenuation of ultrasonic waves

Ultrasonic waves have high energy, high frequency and low wavelength. Hence, they can travel longer distances compared to ordinary sound waves without much loss in energy. The amplitude of ultrasonic waves also falls exponentially in a medium in which it travels. The amplitude of ultrasonic wave as a function of distance is given by

\[ u = u_0 e^{-\alpha x}. \]

Here \( \alpha \) is the attenuation coefficient and \( x \) is the distance travelled by ultrasonic waves in the given medium. Thus the variation of intensity as a function of distance in terms of attenuation coefficient is given as

\[ I = I_0 e^{-2\alpha x}. \]

The value of the attenuation coefficient will be 1 Neper/m, when the amplitude of the waves falls by \( e^{-1} \) times its original value.
### 8.2 Properties of Ultrasonic waves

<table>
<thead>
<tr>
<th>Material</th>
<th>( \alpha (dB/m) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air, at 20 °C</td>
<td>1.64</td>
</tr>
<tr>
<td>Blood</td>
<td>0.2</td>
</tr>
<tr>
<td>Bone, cortical</td>
<td>6.9</td>
</tr>
<tr>
<td>Bone, trabecular</td>
<td>9.94</td>
</tr>
<tr>
<td>Brain</td>
<td>0.6</td>
</tr>
<tr>
<td>Dentin</td>
<td>80</td>
</tr>
<tr>
<td>Enamel</td>
<td>120</td>
</tr>
<tr>
<td>Fat</td>
<td>0.48</td>
</tr>
<tr>
<td>Liver</td>
<td>0.5</td>
</tr>
<tr>
<td>Marrow</td>
<td>0.5</td>
</tr>
<tr>
<td>Muscle</td>
<td>1.09</td>
</tr>
<tr>
<td>Tendon</td>
<td>4.7</td>
</tr>
<tr>
<td>Soft tissue (avg.)</td>
<td>0.54</td>
</tr>
<tr>
<td>Water</td>
<td>0.0022</td>
</tr>
</tbody>
</table>

**Fig: List of materials with attenuation coefficients.**

**Think**

The attenuation coefficient consists of a ratio of two intensities at two locations in space. Then what is the standard initial intensity to define \( \alpha \)?

**Did You Know?**

It is observed that \( \alpha \) is inversely proportional to square of frequency of ultrasonic wave. The standard frequency at which \( \alpha \) is defined is 1MHz.

**E - Corner**

https://www.nde-ed.org/Physics/Waves/attenuation.xhtml

https://en.wikipedia.org/wiki/Attenuation

8.2 Properties of Ultrasonic waves

\[ u = u_0 e^{-\alpha_{NP}x} \Rightarrow \ln \left( \frac{u}{u_0} \right) = -\alpha_{NP}x \Rightarrow \alpha_{NP} \]
\[ = -\frac{1}{x} \ln \left( \frac{u}{u_0} \right) \Rightarrow \alpha_{NP} = \frac{1}{x} \ln \left( \frac{u_0}{u} \right). \]

The attenuation coefficient in decibels is defined for intensities as follows. The attenuation coefficient will be 1 Bel/m if the intensity falls by \(10^{-1} = e^{-2.3 \times 0.3}\) times its original value.

The attenuation coefficient in decibels is defined for intensities as follows. The attenuation coefficient will be 1 Bel/m if the intensity falls by \(10^{-1} = e^{-2.3 \times 0.3}\) times its original value.

\[ I = I_0 e^{-2.3 \times 0.3 \alpha_{B}x} \Rightarrow \ln \left( \frac{I}{I_0} \right) = -2.303 \alpha_B x \Rightarrow \log \left( \frac{I}{I_0} \right) = -\alpha_B x \]
\[ \Rightarrow \alpha_B = \frac{1}{x} \log \left( \frac{I_0}{I} \right) \text{bel} \Rightarrow \alpha_{dB} = \frac{1}{10} \log \left( \frac{I_0}{I} \right) \frac{1}{10} \text{Bel} \]
\[ \Rightarrow \alpha_{dB} = \frac{1}{x} 10 \log \left( \frac{I_0}{I} \right) dB \]

In the above, the relation between Neper and decibel is given by

\[ \alpha_{dB} = 10 \log \left( \frac{I_0}{I} \right) = 10 \log \left( \frac{u_0}{u} \right)^2 = 20 \log \left( \frac{u_0}{u} \right) = \frac{20}{2.303} \ln \left( \frac{u_0}{u} \right) \]
\[ = 8.68 \times \alpha_{NP} \]

Here

\[ 1dB = 10 \log \left( \frac{I_0}{I} \right) = 20 \log \left( \frac{u_0}{u} \right) = 8.68 \times 1Np \text{ where } 1Np \]
\[ = \ln \left( \frac{u_0}{u} \right) \]

The units of attenuation coefficient is \(dB/m\) or \(Np/m\).

**Did You Know?**

The zero decibel where \(I_0 = I\), is defined at the threshold of human hearing intensity which causes a pressure of \(20\mu\text{Pascal}\) on human ear drum.
8.2 Properties of Ultrasonic waves

Electrical system | Mechanical system
---|---
Charge Q | Displacement x
Current I | Velocity v
Applied voltage V | Applied force F
Resistance R | Mechanical resistance \( R_m \)
Inductance L | Mass \( m \)
Capacitance C | Compliance \( C=1/k \)
Impedance \( Z=R+j(\omega L-1/\omega C) \) | Mechanical Impedance \( Z_m=R_m+j(\omega m-k/\omega) \)

\[ Z = \frac{F}{c} \]

where \( F \) is force in \( \text{Kg/m}^2\text{sec} \), \( c \) is velocity in \( \text{m/sec} \). The impedance \( Z \) is given by:

\[ Z = \frac{p}{c} = \rho c \]
Acoustic impedance

The mechanical equivalents of electrical terms are given as follows. Charge is analogous to displacement while current is analogous to velocity. The applied voltage is analogous to applied force and thus electrical resistance, which is equal to voltage/current is analogous to mechanical resistance, which is equal to force/velocity. Thus resistance is the amount of driving force required per unit velocity. In electricity, it is the amount of driving potential required per unit current flow. Inverse of that is the conductance which represents the amount of velocity produced per unit driving force. In electricity, it would be the amount of current (electron flow) produced per unit driving potential.

The other reactive terms equivalent to inductance and capacitance are mass and spring compliance \((C = 1/k)\), respectively. The sum of resistance and reactance is the impedance, which is given as

\[
Z = R_m + j(\omega m - k/\omega)
\]

A table of comparison of parameters in electrical and mechanical oscillator systems is given below.

<table>
<thead>
<tr>
<th>Electrical system</th>
<th>Mechanical system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charge Q</td>
<td>Displacement x</td>
</tr>
<tr>
<td>Current I</td>
<td>Velocity v</td>
</tr>
<tr>
<td>Applied voltage V</td>
<td>Applied force F</td>
</tr>
<tr>
<td>Resistance R</td>
<td>Mechanical resistance (R_m)</td>
</tr>
<tr>
<td>Inductance L</td>
<td>Mass m</td>
</tr>
<tr>
<td>Capacitance C</td>
<td>Compliance (C=1/k)</td>
</tr>
<tr>
<td>Impedance</td>
<td>Mechanical Impedance (Z_m)</td>
</tr>
</tbody>
</table>

The acoustic impedance has the units of \(Kg/sec\). This can also be defined as

\[
Z = \frac{F}{c}
\]

The acoustic impedance per unit area is called the specific acoustic impedance. It has the units of \(Kg/m^2/sec\). Another
8.2 Properties of Ultrasonic waves

Table: Specific Acoustic impedance of various materials.

<table>
<thead>
<tr>
<th>Medium</th>
<th>Density (kg/m³)</th>
<th>Speed of Ultrasound (m/s)</th>
<th>Specific Acoustic Impedance (kg/(m² · s))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1.3</td>
<td>330</td>
<td>429 = 429 Rayl</td>
</tr>
<tr>
<td>Water</td>
<td>1000</td>
<td>1500</td>
<td>1.5 × 10⁶ = 1.5MRayl</td>
</tr>
<tr>
<td>Blood</td>
<td>1060</td>
<td>1570</td>
<td>1.66 × 10⁶ = 1.66MRayl</td>
</tr>
<tr>
<td>Fat</td>
<td>925</td>
<td>1450</td>
<td>1.34 × 10⁶ = 1.34MRayl</td>
</tr>
<tr>
<td>Muscle (average)</td>
<td>1075</td>
<td>1590</td>
<td>1.70 × 10⁶ = 1.7MRayl</td>
</tr>
<tr>
<td>Bone (varies)</td>
<td>1400–1900</td>
<td>4080</td>
<td>5.7 × 10⁶ to 7.8 × 10⁶</td>
</tr>
<tr>
<td>Barium Titanate</td>
<td>5600</td>
<td>5500</td>
<td>30.8 × 10⁶ = 30.8MRayl</td>
</tr>
</tbody>
</table>

Table: Specific acoustic impedance of water Vs. temperature.

<table>
<thead>
<tr>
<th>Temp, T (°C)</th>
<th>Speed, c (m/s)</th>
<th>Density of air, ρ (kg/m³)</th>
<th>Specific acoustic impedance z₀ (Pa·s/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>351.88</td>
<td>1.1455</td>
<td>403.2</td>
</tr>
<tr>
<td>30</td>
<td>349.02</td>
<td>1.1644</td>
<td>406.5</td>
</tr>
<tr>
<td>20</td>
<td>343.21</td>
<td>1.2041</td>
<td>413.3</td>
</tr>
<tr>
<td>10</td>
<td>337.31</td>
<td>1.2466</td>
<td>420.5</td>
</tr>
<tr>
<td>0</td>
<td>331.30</td>
<td>1.2922</td>
<td>428.0</td>
</tr>
<tr>
<td>−10</td>
<td>325.18</td>
<td>1.3413</td>
<td>436.1</td>
</tr>
<tr>
<td>−20</td>
<td>318.94</td>
<td>1.3943</td>
<td>444.6</td>
</tr>
</tbody>
</table>

https://courses.lumenlearning.com/physics/chapter/17-7-ultrasound/
standard unit is Rayl and Mega Rayl (MRayl) which is named after Lord Rayleigh. This can be expressed as

\[ z = \frac{p}{c} = \rho c \]

Here \( p \) is the pressure, \( \rho \) is the density of the medium and \( c \) is the velocity of the wave. This acoustic impedance is the guiding parameter for the reflection and transmission of ultrasonic waves at any of the solid-liquid-gas-vacuum interfaces.

The variation of specific acoustic impedance for air at various temperatures is given below. This table shows that with reduction in temperature, the velocity reduces and the density increases resulting in a net increase in the specific acoustic impedance. This implies that in case of air, the specific acoustic impedance of the ultrasonic wave is much less sensitive to a reduction in velocity in comparison to an increase in the density.

**Reflection and Transmission of Ultrasonic waves across interfaces**

Ultrasonic waves behave similar to ordinary sound waves during reflection. They also transmit through material media without much loss in their amplitude. They obey the laws of reflection and refraction as the light rays, which implies that ultrasonic waves also obey Snell’s law.

\[ \frac{\sin i}{\sin r} = \frac{c_1}{c_2} = \frac{z_1 \rho_2}{z_2 \rho_1} \]

**Fig. Reflection and Refraction of ultrasonic waves.**
8.2 Properties of Ultrasonic waves

ఇంట మయమలో అతిపెద్ద అంతరంల పవర నంపం మపం పరంపర ఎవనషంయ కలిగి ఉంం. ఎందుకంటే ఇంట మయమలో అంతరంల పవర నంపం మపం పరంపర ఎవనషంయ కలిగి ఉంం. ఇది వాటిపై అంతరంల పవర నంపం మపం పరంపర ఎవనషంయ కలిగి ఉంం.

ఇంట మయమలో అంతరంల పవర నంపం మపం పరంపర ఎవనషంయ కలిగి ఉంం. ఎందుకంటే ఇంట మయమలో అంతరంల పవర నంపం మపం పరంపర ఎవనషంయ కలిగి ఉంం. ఇది వాటిపై అంతరంల పవర నంపం మపం పరంపర ఎవనషంయ కలిగి ఉంం.

Try to draw Refraction and reflection diagrams for any other missing combinations like solid to liquid interfaces etc.


అమితాబహ్ యు అమరికం పదార్థ సేసి పాత్రం పంపం పండ్లు

మొదట అమితాబహ్ యు అమరికం పదార్థ సేసి పాత్రం పంపం పండ్లు

$$z_1 = \frac{p_1}{u_1} \quad \text{and} \quad z_2 = \frac{p_2}{u_2}$$

మాత్రమే p_1, p_2 నికిరియు ఉ_1 మాత్రమే u_1 నికిరియు ఉ_2 నికిరియు ప_1 మాత్రమే 2p_1

ఎంటిరియం పదార్థ పండ్లు p_1 మాత్రమే p_2 నికిరియు ప_1 మాత్రమే 20 ప్రతిరోధం
Liquids and gases allow only longitudinal modes whereas solids have 3 modes. Longitudinal (L), Transverse with oscillations in the plane (Sagittal/Shear Vertical SV −↓−→) and Transverse with oscillations perpendicular to the plane (Sagittal/Shear Horizontal SH −⊙−→). Out of these, any single L wave or SV wave generates both L and SV waves in solids. However, a SH wave generates only a SH wave. Vacuum doesn’t allow any type of waves or vibrations. Some typical reflections and transmissions at a few interfaces are illustrated in the following figure.

**Fig: Typical reflections and transmissions of Ultrasonic waves at solid, liquid, gas and vacuum interfaces.**

These are highly useful while designing sensors for various applications.

**Reflection and Transmission coefficients of Ultrasonic waves across interfaces**
Consider a boundary at \( x = 0 \) separating two acoustic media with specific impedance.
8.2 Properties of Ultrasonic waves

\[ p_i = P_i e^{j(\omega t - K_1 x)} \quad p_r = P_r e^{j(\omega t - K_2 x)} \quad p_t = P_t e^{j(\omega t - K_2 x)} \]
\[ u_i = U_i e^{j(\omega t - K_1 x)} \quad u_r = U_r e^{j(\omega t - K_1 x)} \quad u_t = U_t e^{j(\omega t - K_2 x)} \]

When the boundaries are rigid, we have:

\[ u_1 |_{x=0} = u_2 |_{x=0} \quad \text{and} \quad p_1 |_{x=0} = p_2 |_{x=0} \]

When the boundaries are fixed, we get:

\[ u_i |_{x=0} - u_r |_{x=0} = u_t |_{x=0} \Rightarrow \frac{p_i}{z_1} - \frac{p_r}{z_2} = \frac{p_t}{z_2} \Rightarrow \frac{P_i e^{j\omega t}}{z_1} - \frac{P_r e^{j\omega t}}{z_2} = \frac{P_t e^{j\omega t}}{z_2} \]

\[ = \frac{P_t e^{j\omega t}}{z_2} \Rightarrow P_i - P_r = \frac{z_1}{z_2} P_t = P_0 - - - - (1) \]

When the boundaries are free, we get:

\[ p_i |_{x=0} + p_r |_{x=0} = p_t |_{x=0} \Rightarrow (P_i + P_r) e^{j\omega t} = P_r e^{j\omega t} \]

\[ \Rightarrow P_i + P_r = P_t = P_0 - - - - (2) \]

The reflection coefficient is:

\[ R = \frac{\text{Amplitude of Reflected wave}}{\text{Amplitude of Incident wave}} = \frac{P_r}{P_i} - - - - (3) \]
\[ T = \frac{\text{Amplitude of Transmitted wave}}{\text{Amplitude of Incident wave}} = \frac{P_t}{P_i} - - - - (4) \]

\[ (4) \text{ occurs if (3) is valid}, \ (2) \text{ occurs if not} \]

\[ T - R = \frac{P_t - P_r}{P_i} = \frac{P_i}{P_i} = 1 - - - - (5) \]

\[ 2P_i = \left(1 + \frac{z_1}{z_2}\right) P_t \Rightarrow T = \frac{P_t}{P_i} = \frac{2}{\left(1 + \frac{z_1}{z_2}\right)} = \frac{2z_2}{z_1 + z_2} - - - - (6) \]
8.2 Properties of Ultrasonic waves

\[ z_1 = \frac{p_1}{u_1} \text{ and } z_2 = \frac{p_2}{u_2} \]

Here \( p_1, p_2 \) are the pressures and \( u_1 \) and \( u_2 \) are the velocities of ultrasonic waves in medium 1 and 2 respectively. Let \( \rho_1 \) and \( \rho_2 \) be the densities of medium 1 and 2 respectively.

The incident wave and the reflected wave are in the first medium and the transmitted wave is in the second medium, i.e., a portion of the incident wave is reflected back into medium 1 and the remaining portion is transmitted into medium 2.

So the pressure and velocity components in medium 1 and medium 2 are given by

\[ p_i = p_i e^{i(\omega t - K_1 x)} \quad p_r = p_r e^{i(\omega t - K_1 x)} \quad p_t = p_t e^{i(\omega t - K_2 x)} \]
\[ u_i = u_i e^{i(\omega t - K_1 x)} \quad u_r = u_r e^{i(\omega t - K_1 x)} \quad u_t = u_t e^{i(\omega t - K_2 x)} \]

The boundary conditions are

\[ u_1 \big|_{x=0} = u_2 \big|_{x=0} \text{ and } p_1 \big|_{x=0} = p_2 \big|_{x=0} \]

Using the first boundary condition gives

\[ u_i \big|_{x=0} - u_r \big|_{x=0} = u_t \big|_{x=0} \Rightarrow \frac{p_i}{z_1} - \frac{p_r}{z_2} = \frac{p_t}{z_1} \Rightarrow \frac{p_i e^{i\omega t}}{z_1} - \frac{p_r e^{i\omega t}}{z_2} = \frac{p_t e^{i\omega t}}{z_1} \]

\[ = \frac{p_t e^{i\omega t}}{z_2} \]

\[ \Rightarrow P_i - P_r = \frac{z_1}{z_2} P_t \quad (1) \]
8.2 Properties of Ultrasonic waves

(5) Suppose \(z_1\) is given by

\[
R = T - 1 = \frac{2z_2}{z_1 + z_2} - 1 = \frac{z_2 - z_1}{z_1 + z_2} \quad \Rightarrow (7)
\]

Case-1: When \(z_2\) is much less than \(z_1\), say, \(z_2 \gg z_1\) \(\Rightarrow z_2 \to \infty\) \(\Rightarrow R = T - 1 = 1 \Rightarrow P_r = P_i\)

Case-2: When \(z_2\) is much greater than \(z_1\), say, \(z_2 \gg z_1\) \(\Rightarrow z_2 \to \infty\) \(\Rightarrow R = T - 1 = 1 \Rightarrow P_r = P_i\)

\[
T = \frac{2}{\left(\frac{z_1}{z_2} + 1\right)} = \frac{2}{\left(\frac{1}{\infty} + 1\right)} = 2 \Rightarrow P_t = 2P_i,
\]

\[
R = T - 1 = 1 \Rightarrow P_r = P_i
\]

Case-3: When \(z_1 = z_2\), say, \(R = 0\) \(\Rightarrow T = 1\).

Polar direction \(\theta\) of sound wave can be determined by:

\[
\theta = 180^\circ - \theta \quad \text{if} \quad z_2 \gg z_1 \quad \Rightarrow \quad \theta = 180^\circ - \theta \quad \text{if} \quad z_2 \ll z_1.
\]

The intensity of sound \(I\) can be determined by:

\[
\frac{I_t}{I_i} = \frac{z_1}{z_2} |T|^2 \quad \text{and} \quad \frac{I_r}{I_i} = |R|^2
\]
Using the second boundary condition, we obtain
\[ p_i \big|_{x=0} + p_r \big|_{x=0} = p_t \big|_{x=0} \Rightarrow (p_i + p_r)e^{j\omega t} = p_t e^{j\omega t} \]
\[ \Rightarrow p_i + p_r = p_t \quad \quad \quad (2) \]

The reflection coefficient & transmission coefficient are defined by
\[ R = \frac{\text{Amplitude of Reflected wave}}{\text{Amplitude of Incident wave}} = \frac{p_r}{p_i} \quad \quad \quad (3) \]
\[ T = \frac{\text{Amplitude of Transmitted wave}}{\text{Amplitude of Incident wave}} = \frac{p_t}{p_i} \quad \quad \quad (4) \]

Subtracting (3) from (4) and using (2) gives
\[ T - R = \frac{p_t - p_r}{p_i} = \frac{p_i}{p_i} = 1 \quad \quad \quad (5) \]

Adding (1) and (2) gives,
\[ 2p_i = \left(1 + \frac{z_1}{z_2}\right)p_t \Rightarrow T = \frac{p_t}{p_i} = \frac{2}{\left(1 + \frac{z_1}{z_2}\right)} = \frac{2z_2}{z_1 + z_2} \quad \quad \quad (6) \]

Using (5) and (6) we get
\[ R = T - 1 = \frac{2z_2}{z_1 + z_2} - 1 = \frac{z_2 - z_1}{z_1 + z_2} \quad \quad \quad (7) \]

Case-1: If the second medium is rigid, i.e., \( z_2 \gg z_1 \) or \( z_2 \to \infty \) then,
\[ T = \frac{2}{\left(\frac{z_1}{z_2} + 1\right)} = \frac{2}{\left(\frac{1}{\infty} + 1\right)} = 2 \Rightarrow p_t = 2p_i, \]
\[ R = T - 1 = 1 \Rightarrow p_r = p_i \]

Then the amplitude and phase of the reflected ray are equal to that of the incident ray. Further, the amplitude of the transmitted ray is double that of the incident ray and both have the same phase.

Fig: Ultrasonic reflection and transmission at rarer-denser medium boundary
8.2 Properties of Ultrasonic waves

Eq. (6) and Eq. (7) state that the incident intensity, reflected intensity, and transmitted intensity can be related by the following equations:

\[ I_{t} + I_{r} = I_{i} \]

where \( I_{t} \) is the transmitted intensity, \( I_{r} \) is the reflected intensity, and \( I_{i} \) is the incident intensity.

The ultrasonic waves are reflected and transmitted at an interface between two different media. The intensities of the reflected and transmitted waves are given by the following equations:

\[ R = \frac{z_2 \cos \theta_2 - z_1 \cos \theta_1}{z_1 \cos \theta_1 + z_2 \cos \theta_2} \]

\[ T = \frac{2z_2 \cos \theta_2}{z_1 \cos \theta_1 + z_2 \cos \theta_2} \]

where \( z_1 \) and \( z_2 \) are the wave impedances of the media on either side of the interface, and \( \theta_1 \) and \( \theta_2 \) are the angles of incidence and reflection, respectively.

The interface between the two media can be smooth or rough, depending on the nature of the materials involved.


http://www.fast.up-psud.fr/~martin/acoustique/support/r%C3%A9fraction.pdf


8.2 Properties of Ultrasonic waves

Case-2: If the first medium is rigid, i.e., \( z_1 \gg z_2 \) or \( z_1 \to \infty \) then,

\[
R = \frac{z_1 \left( \frac{z_2}{z_1} - 1 \right)}{z_1 \left( 1 + \frac{z_2}{z_1} \right)} = \frac{\frac{z_2}{\infty} - 1}{1 + \frac{z_2}{\infty}} = -1 \quad \Rightarrow \quad P_r = -P_i; \quad T = R + 1
\]

\[
= 0 \quad \Rightarrow \quad P_t = 0
\]

Then the reflected ray is opposite in phase to the incident ray and has the same amplitude, while there will be no transmission (or near zero transmission).

![Fig: Ultrasonic reflection at denser-rarer medium boundary](image)

Case-3: If both media are same, i.e., \( z_1 = z_2 \), then \( R=0 \) and \( T=1 \).

This implies that the entire wave will be transmitted without reflection.

**Note:**

It is observed that the ultrasonic waves behave quite differently across boundaries. When reflected on denser medium, the reflected wave is in phase with the incident wave. When reflected on rarer medium, the reflected wave is 180° out of phase with the incident wave. Also while reflecting on denser medium (Case-1, \( z_2 \gg z_1 \)) Incident wave of 1 unit amplitude is producing a reflected wave of equal amplitude and a transmitted wave of double the amplitude. This confusion will be cleared out if the intensities of incident, reflected and transmitted waves are calculated instead of amplitudes. They are given as follows.

The transmitted wave amplitude is given by
పతలానికి 

పతనర్డి నిర్మాణం

పతనర్డి నిర్మాణం సాధనం కొనసాగరు. ఇది జరిగిన ఇతర నిర్మాణానికి ప్రతిపాదితి చేయడం వలన నిర్మాణం కోసం ఉపయోగించబడింది. అలాగా పతనర్డి నిర్మాణం కొనసాగించబడింది.

మానవుల రాయిలు

మానవుల రాయిలు 1928 సంవత్సరం జరిగాయి. అనేక USA యుద్ధ పొలారులు సాధ్రాస్సుల ప్రాజెక్టులకు ఉపయోగయోగ్యమైన సందర్భాల పొందింది (1904-1920). 1921 సంవత్సరం నుండి యుద్ధ పొలారులు ఉపయోగయోగ్యమైన సందర్భాల పొందింది.

పదార్థం:

మానవుల రాయిలు 1928 సంవత్సరం జరిగాయి. అనేక USA యుద్ధ పొలారులు సాధ్రాస్సుల ప్రాజెక్టులకు ఉపయోగయోగ్యమైన సందర్భాల పొందింది (1904-1920). 1921 సంవత్సరం నుండి యుద్ధ పొలారులు ఉపయోగయోగ్యమైన సందర్భాల పొందింది.

పదార్థం:


1. పదార్థం L_1 మరియు L_2 సంయోగం ప్రతి సమయంలో మొత్తం మార్పు వేయడం, అందుకే f_e = 1/(2π√(L_1 C_1))

2. పదార్థం L_2, CE సంయోగం ప్రతి సమయంలో మొత్తం మార్పు వేయడం, అందుకే f_e = 1/(2π√(L_2 CE))

3. పదార్థం సమానం, L_1 మరియు L_2 సంయోగం ప్రతి సమయంలో మొత్తం మార్పు వేయడం, అందుకే f_e = 1/(2π√[(L_1 C_1)(L_2 CE)])

4. పదార్థం L_1 మరియు L_2 సంయోగం ప్రతి సమయంలో మొత్తం మార్పు వేయడం, అందుకే f_e = 1/(2π√((1/ρ)(Y/ρ)))

5. పదార్థం L_1 మరియు L_2 సంయోగం ప్రతి సమయంలో మొత్తం మార్పు వేయడం, అందుకే f_e = 1/(2π√((1/ρ)(Y/ρ)))

486
Substituting Eq. (6) and (7) in the above, one obtains \( \frac{I_t}{I_i} = \frac{z_1}{z_2} |T|^2 \) and \( \frac{I_r}{I_i} = |R|^2 \). This ensures conservation of energy.

When the ultrasonic wave is incident on the interface at oblique angle, then Fresnel equations give the reflection and transmission coefficients. Let \( \theta_1 \) and \( \theta_2 \) be the angle of incidence and angle of refraction respectively. Then the reflection and transmission coefficients are given by

\[
R = \frac{z_2 \cos \theta_2 - z_1 \cos \theta_1}{z_1 \cos \theta_1 + z_2 \cos \theta_2}
\]
\[
T = \frac{2z_2 \cos \theta_2}{z_1 \cos \theta_1 + z_2 \cos \theta_2}
\]

Observe that the indices 1 and 2 are interchanged in the case of ultrasonic waves as compared to the Fresnel equations for light rays. This makes ultrasonic waves different from light rays during reflection and refraction across boundaries.

### 8.3 Production of Ultrasonics

Ultrasonic waves are mechanical waves. Hence several mechanical energy transducers have been proposed and utilized to convert various other forms of energy into ultrasonic (mechanical) wave energy. Some of them are discussed in detail in the following.

### 8.4 Magnetostriction method

This was developed by George Washington Pierce in 1928. He was considered as the father of wireless telegraphy in the USA (1904-1920). His methods of oscillators were further strengthened by the development of quartz crystal stabilized oscillator by Cady in 1921.
8.4 Magnetostriction method

Principle: The principle involved in magnetostriction method, as the name suggests, is the magnetostriction effect. When a ferromagnetic material is placed in an external magnetic field, its length increases along the direction of magnetization. The change in length is proportional to the applied field strength at low field strengths. Thus, if a sinusoidally varying magnetic field is applied to a ferromagnetic bar, its length rises and falls twice for each cycle of the applied field. This implies that the frequency of oscillations of the ferromagnetic bar will be twice the frequency of the applied field. This phenomenon is called magnetostriction effect.

![Magnetostriction Principle](image)

**Fig: Magnetostriction Principle.**

Working:
1. The experimental setup consists of a ferromagnetic bar clamped at the middle and wound with inductors $L_1$ and $L_2$ on either ends.
2. The coil $L_2$ is connected to the input of a CE amplifier and coil $L_1$ is in parallel with a capacitor $C_1$, which is connected at the output of the CE amplifier.
3. When the circuit is turned on, the tank circuit formed by $L_1$ and $C_1$ produces electric field oscillations of frequency given by $f_e = \frac{1}{2\pi\sqrt{L_1C_1}}$.
4. The oscillating electric field produces an oscillating magnetic field in the ferromagnetic bar with frequency
8.4 Magnetostriction method

Fig: Magnetostriction apparatus used by George W. Pierce in 1928

Proceedings of the American Academy of Arts and Sciences, vol. 63, no. 1, April 1928; [https://doi.org/10.2307/20026189](https://doi.org/10.2307/20026189); reprinted with permission of the American Academy of Arts and Sciences.
2\(f_e\). But the natural frequency of the bar is given by
\(f_b = \frac{1}{2\pi} \sqrt{\frac{Y}{\rho}}\) where \(L\) is the length of the bar, \(Y\) is Young’s modulus of the bar and \(\rho\) is the density of the bar.

5. The oscillating magnetic field produces oscillating electric field of frequency \(f_e\) in the coil \(L_2\). Since the fields in \(L_1\) and \(L_2\) are in phase, constructive interference occurs between the input and output signals. Thus the circuit acts as a positive feedback oscillator and the oscillations remain forever without loss in amplitude.

6. The value of \(f_e\) can be varied by tuning the capacitor \(C_1\). Resonance occurs when \(2f_e = f_b\). This results in a high amplitude oscillation of the bar and ultrasonic waves of double the frequency of the tank circuit are emitted into the medium containing the ferromagnetic bar. They are of longitudinal waves.

7. But the transistor cannot pass the signal if the input signal is below 0.7V. Hence a biasing DC voltage is applied at the input section of the transistor. Then the entire signal shifts into positive quadrant and is allowed by the transistor.
సాధారణ విశేషాల

అమ్మకి సందర్భం నిలిచిన 1921 ఆరోగ్య ప్రాముఖ్యతను సంఖ్యారంభం అమలు చేసినది. ఒకసారీ ఉపయోగానికి మాత్రమే తన మూలాలను మార్గంగా మూడు సంఖ్యాథయాస నడిపాలను క్రమంగా తెలిచుకోవచ్చిన వివరాలు అభివృద్ధి చేయబడలేదు.

పరిశీలన:

అనేక సమస్యల మూలంగా రాములు యొక్క ముఖ్య అభివృద్ధి ఉన్నది. ఇందులో రాములు యొక్క ముఖ్య అభివృద్ధి ప్రాప్తి. అందుకు రాములు యొక్క ముఖ్య అభివృద్ధి ప్రాప్తి గా మాత్రమే సాధారణ విశేషాలను ప్రామాణ్యంగా మూడు సంఖ్యాథయాస నడిపాలను క్రమంగా తెలిచుకోవచ్చిన వివరాలు అభివృద్ధి చేయబడలేదు.

1. $\mu L = \frac{1}{\sqrt{LC}}$
2. $L_1 + L_2 = 180^\circ$
3. $L_1 + L_2 = 180^\circ$
4. $L_1 + L_2 = 180^\circ$
8. Sometimes a polarizing DC field stronger than the AC field is also applied to the bar directly. One can also achieve this by applying external magnetic field directly.

9. When such is the case, one end of the bar remains as north pole forever and the other end as south pole. The AC field, at most, changes the degree of magnetization. Thus the length of the bar changes for the entire cycle of the AC field but not the polarity. Thus the resonance condition is given by \( f_e = f_b \).

Advantages:

1. High amplitudes can be achieved for low frequency applications.

2. It is the cheapest method to generate ultrasonic waves.

Disadvantages:

1. Their fundamental frequency may start at 30KHz so the first sub-harmonic (15KHz) falls in audible range. This results in a high noise compared to piezoelectric generators whose fundamental frequency is in the range of 40KHz.

2. They can generate ultrasonics with a maximum frequency of 3MHz or 3000KHz.

3. The frequency of oscillations is more affected by temperature and hysteresis effects.

8.5 Piezoelectric method

This was developed by Walter Guyton Cady in 1921. He was considered as the father of modern piezoelectricity. He has inspired this from Langevin and Boyle’s piezoelectric submarine detector.

Principle:
Quartz crystal consists of a hexagonal prism capped on top and bottom with hexagonal pyramid structure. The vertical axis of the crystal is denoted as the z-axis. The axis parallel to the faces is the y-axis and the axis connecting corners of faces is the x-axis. The
8.5 Piezoelectric method

494

Fig: Quartz crystal oscillator developed by Walter Guyton Kedy
structure is as shown below. Here the x-axis is known as the electrical axis and the y-axis is known as the mechanical axis.

If electric field is applied along the electrical axis, it results in a displacement along mechanical axis. Similarly if pressure is applied along the mechanical axis, it results in a potential difference along the electrical axis.

![Quartz crystal planes](image)

There are two possible cuts for the crystal, the X-cut and the Y-cut.

In X-cut, the crystal along X-axis is chopped off and the plane of the remaining crystal piece is parallel to Y-axis. Since the mechanical axis is perpendicular to the plane of the crystal, the oscillations produced in X-cut crystal are transverse in nature. Since one complete oscillation in transverse mode generates a complete wave in the surrounding medium, the frequency of oscillations will be equal to the frequency of the crystal.

In Y-cut, the crystal along Y-axis is chopped off and the plane of the remaining crystal piece is parallel to X-axis. Since
8.5 Piezoelectric method

(a) Paul Langevin (1872–1946)

(c) Cable, heavily insulated Iron pipe for support

Copper casing

Thin mica

Steel plate

Quartz

Steel plate

Insulating and waterproof mixture

(b) Robert W. Boyle (1883–1955)

Fig: Pioneers of Submarine detection & Ultrasonics.

(b) Walter G. Cady

GEORGES W. PIERCE,
As a Junior in the University.
8.5 Piezoelectric method

the mechanical axis is perpendicular to plane of the crystal, the oscillations produced in Y-cut crystal are longitudinal in nature. Since one complete oscillation in longitudinal mode can generate only half a wave in the surrounding medium, the frequency of oscillations generated will be half of the frequency of the crystal.

Working:
1. The circuit diagram consists of a transistor that is connected as an oscillator.
2. The oscillations are produced in the tank circuit connected at the collector of the transistor. The frequency of oscillations is given by \( f_e = \frac{1}{2\pi\sqrt{LC}} \).
3. The inductor in the LC circuit is a part of a transformer, a portion of which is fed to a piezoelectric crystal and another portion is fed to the input circuit of the transistor.
4. Since transformer produces a 180° phase shift between input and output, there will be a 180° phase shift between \( L_1 \) and \( L_2 \). There will be a further 180° phase shift between \( L_2 \) and \( L_1 \). Thus there will be 360° or 0° phase shift between transistor output and input signals.
5. Thus there will be a positive feedback to input from output of transistor. Thus the transistor works as an oscillator.
6. The electric field induced in the coil \( L_2 \) of the transformer is fed to a Quartz crystal. This converts the electrical oscillations into mechanical oscillations. In other words, it produces ultrasonic waves with frequency given by \( f_c = \frac{n}{2l} \sqrt{\frac{Y}{\rho}} \). Here \( n \) is the number of the vibrational mode and \( l \) is the length of the crystal.

Advantages:
1. The fundamental frequency of these crystals are usually above 40KHz. Thus first sub-harmonic also falls in ultrasonic range and the operation is noise free.
8.5 Piezoelectric method

Piezoelectric method

Piezoelectric method was discovered by Pierre Curie in 1878. The relationship between the mechanical strain and the electric field can be expressed as:

\[ f = \frac{v}{(l+e)} \]

Where:
- \( f \) is the frequency of vibration
- \( v \) is the velocity of sound in the medium
- \( l \) is the length of the piezoelectric element
- \( e \) is the thickness of the piezoelectric element

MEMS (Micro Electro-Mechanical Systems) utilizes piezoelectric materials to convert mechanical vibrations into electrical signals, or vice versa. These devices are widely used in various applications such as sensors, actuators, and electronics.

E - Corner

https://www.osti.gov/pages/servlets/purl/1515412
https://patents.google.com/patent/US1472583
https://doi.org/10.1258/ult.2011.011027
https://en.wikipedia.org/wiki/Ultrasonic_transducer
https://www.britannica.com/science/ultrasonics
2. The frequency is tolerant to temperature fluctuations. (i.e., not affected much by temperature changes.)
3. Ultrasonic waves of frequencies up to 500MHz can be generated using this method.

Fig: Piezoelectric method of generating ultrasonic waves.

Disadvantages:
1. Long run operations may not be possible due to mechanical fatigue generated in the crystal.
2. The life time of the oscillator is also low for the same reason.

Other methods
1. The Galton whistle: This is also called as Dog whistle. It was developed in 1876 by Francis Galton. It consists of a resonating wind pipe similar to flute whose length can be adjusted. The relation between the length and resonance frequency is given by
   \[ \frac{\lambda}{4} = (l + e) \Rightarrow f = \frac{v}{(l+e)}. \]

2. Photo-acoustic effect: In this process, light absorption causes thermal expansion or pressure change in the material. If light intensity is varied periodically or if pulsed light with required frequency is used, one can generate ultrasonic waves of desired
Did You Know?
If piezo electric crystal is pressed just once, charges will be accumulated along electrical axis (potential diff.). If a light bulb is connected to them, they at most produce a spark in the bulb. For continuous flow of electrons (current), one need to apply a periodic force. So that the charge accumulation changes sign alternatively along with applied force. That may be considered as alternating current. Then it can light a bulb.

8.5 Piezoelectric method
frequency using this method. This is widely used in biomedical ultrasound imaging.

3. **Laser induced shock waves** in liquids and gases may have high amplitude and at the same time frequencies of the order of ultrasounds. The limit on the maximum amplitude and frequency of ultrasounds depend on the compressibility limit of the fluid.

4. **Diaphragms made with MEMS** (Micro Electro Mechanical Systems), when excited with high frequency electric fields, produce ultrasonic sounds.

5. **Capacitor microphones** have thin membranes. When they are excited with ultrasonic frequency electrical signals, the membrane produces ultrasonic waves.

8.6 Detection of ultrasonic waves

1. **Kundt’s tube method:** In this method ultrasonic waves are passed onto a smooth powder (Lycopodium powder) placed in a glass tube with one end closed. Ultrasonic waves produce standing wave pattern. Thus at nodes more powder is accumulated while at antinodes, the powder accumulation is low. The separation between any two nodes or antinodes gives half of the wavelength of the ultrasonic waves.

2. **Sensitive flame method:** In this method ultrasonic waves are passed on to narrow sensitive flame. Flame will be stationary at nodes and flickering at antinodes.

3. **Thermal detection method:** A thin platinum wire is placed in the medium where ultrasonic waves are to be detected. The resistance of platinum wire changes due to temperature change produced at compressions and rarefactions. These resistance changes can be identified using sensitive resistance bridge.

4. **Acoustic grating method:** When ultrasonic waves are passed through liquid media, to form stationary waves, the regions at nodes act as transparent and the regions at antinodes act as opaque media. In effect, the liquid medium acts as a transmission grating. By passing light of known wavelength through this liquid grating, the width of opaque and
8.6 Detection of ultrasonic waves

6. To detect ultrasonic waves in a medium, one must first generate ultrasonic waves and then detect their presence. In most cases, the ultrasonic waves are generated using a piezoelectric transducer, and their detection is done using a similar transducer. This is a common method used in various applications, such as in medical imaging and non-destructive testing.

7. MEMS (Micro-Electro-Mechanical Systems) are often used to generate and detect ultrasonic waves in small devices. MEMS technology allows for the integration of electronic and mechanical components on a single chip, making it ideal for applications requiring small, portable devices that can generate and detect ultrasonic waves.

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The text contains technical details about detecting ultrasonic waves, including methods such as using piezoelectric transducers and MEMS technology. It also mentions applications like medical imaging and non-destructive testing.
transparent regions can be calculated. This in turn gives information about the wavelength of ultrasonic waves.

5. Piezo-electric detector: This uses the phenomenon of inverse piezo-electric effect, in which the electric field is produced along electric axis of the piezo-electric crystal when mechanical oscillations are applied along mechanical axis. Here ultrasonic waves, which need to be detected, produce mechanical oscillations in the medium.

6. Capacitor microphones use thin diaphragms that vibrate in the presence of ultrasonic waves. When these diaphragms are connected with magnets in coils, they produce electrical signals. The frequencies of the generated electrical signals give an estimate of ultrasonic waves.

7. MEMS (Micro Electro Mechanical Systems) use thin films that oscillate with the incident ultrasonic waves. These oscillating films are part of a capacitor plate. So the capacitance varies in accordance with the ultrasonic waves. A measurement of capacitance variations give an estimation of ultrasonic wave frequency.

### 8.7 Applications of Ultrasonics

Ultrasonic waves find their applications in various fields from daily life, to industry; research to medical fields; navigation and ranging to signalling and sensing. In the following, a few of the applications of Ultrasonics in some of the above mentioned fields are discussed.

**Ultrasonics in Nature**

Some species in nature use ultrasonic waves for communication and some species use them for navigation and echolocation of their prey.

1. Cats, dogs, rats and frogs use ultrasonic waves for communication and sometimes for mating. Cats and dogs have this ultrasonic wave sensing ability to identify the presence of their prey.
8.7 Applications of Ultrasonics

Did You Know?

The two projections over the head of an owl are not their ears. They are just feathers. Their ears have wide openings on either side of their head and their eyeballs can be seen through their ears.

https://www.nature.com/articles/193594b0
https://www.lsu.edu/deafness/HearingRange.html
2. Bats, Dolphins, Porpoise and whales use them for navigation and for echolocation of their prey.

3. Dolphins are more attracted towards pregnant women, as their ultrasonic sensor system detects two heart beats within a single human body. There are dolphin assisted child birthing centres as well. But they are not medically suggestible.

4. An owl has more sensitive sound sensing system. It is able to recognize a time delay of even 30 micro seconds between its two ears. By rotating its head 360° it can balance the sound heard by two ears such that the target is exactly in front of its head. They have a heightened hearing sensitivity in the range 5KHz to 8.5KHz. Some of the owls like Tawny owl can hear ultrasounds. Their major advantage is that their wings do not produce ultrasonic noise while flying, and hence insensitive to their prey.

5. Moths, a prey for bats, are able to sense ultrasonic waves produced by their predator (bats) and change their path. Tiger moths are able to generate clicks like sounds that could even distract the echolocation activity of bats.

6. Human brain can perceive ultrasonic frequencies, even though the eardrum does not vibrate at those frequencies. If they are injected through skull bone, they may generate sub-harmonics which may produce vibrations in ear drum. By this process, humans can perceive ultrasonic waves up to 33KHz, in the form of mild humming sounds.

In Geology

In geology ultrasonic waves are used for sensing purposes.

1. Ultrasonic seismic physical model imaging technology is a method in which seismic waves are replaced by ultrasonic waves to study the stability of geological structures.

2. Ultrasonic sensors are used in hydrogeological surveys for ground water level sensing applications. Usually
8.7 Applications of Ultrasonics

Applications of Ultrasonics

1. Applications in Medicine and Health: Ultrasonics are widely used in medical imaging, such as ultrasound imaging, which helps in diagnosing various diseases and conditions. They are also used in lithotripsy, a treatment for breaking up urinary tract stones.

2. Applications in Materials Science: Ultrasonics are used to measure the velocity and attenuation of sound waves in materials, which can help in understanding their properties and in the development of new materials.

3. Applications in Geology: Ultrasonics are used to study geological samples, such as rocks and minerals, to understand their composition and structure.

4. Applications in Environmental Science: Ultrasonics are used in the study of marine life, such as monitoring fish populations and studying marine ecosystems.

5. Applications in Aerospace: Ultrasonics are used in the non-destructive testing of aerospace materials to ensure their integrity and safety.

6. Applications in Manufacturing: Ultrasonics are used in the testing of manufactured components to ensure their quality and performance.

7. Applications in Non-Destructive Testing: Ultrasonics are used in various industries for non-destructive testing of materials, ensuring their quality and safety.

E - Corner
https://qjeqh.lyellcollection.org/content/51/2/179
https://www.osapublishing.org/oe/fulltext.cfm?uri=oe-26-8-11025&id=385607
resistivity based and polarizability based tests are more abundant.

3. Ultrasonic sensors are used in oil well logging to monitor the quality of pipe line fitting and to monitor the corrosion effects on pipe lines produced by salt water. Log file carries a record of events that occur in any operation.

4. Ultrasonic waves are widely used in Paleoceanography, where geological history of oceans is studied. Especially, the study of sedimentation at sea bed employing ultrasonic waves enable non-destructive evaluation of different layers of sediments.

5. The age of carbonate rocks can be estimated by performing non-destructive evaluation of the ultrasonic wave velocities. It is observed that the ultrasonic wave velocity reduces in aged, old rocks.

6. Snow density measurement is used for meteorological and hydrological information systems. Here snow depth is measured using ultrasonic sensor. Snow water equivalent is measured by using snow pillows. The ratio of both gives the snow density. It is interesting to note that if the snow density is more than 40%, then melting occurs.

7. Ultrasonic waves are used for study of hardness, grain structure and elastic modulus of rock samples and minerals.
8.7 Applications of Ultrasonics

![Image: Oil well log tool and log file; Snow density measuring device](image)

Fig: Oil well log tool and log file; Snow density measuring device

Make a study project on various industrial applications of ultrasonics and conduct survey on it in your area.
Ultrasonics in Industry

Ultrasonics waves find their importance in various heating, mixing, cutting activities in industry. Some of the important applications of ultrasonic waves in industry are mentioned below.

1. Welding/Soldering: Ultrasonics are used to weld sensitive and micro size electronic equipment like LEDs, light bulbs, integrated circuits, sensitive plastic materials like gas lighters and various medical fixtures made of plastic, surgical masks etc. These are useful in medical industry as they don’t leave any traces of toxic elements of wastage. In aeronautical engineering, aluminium alloys are majorly used. Aluminium creates oxide layers when it is gas welded. To avoid that, gas welding machine is accompanied with ultrasonic cleaner. Otherwise, aluminium parts are welded using direct ultrasonic waves.

2. Drilling: a drill that oscillates with ultrasonic frequency can bore any hard material. As it provides low axial load (downward force), one can drill small holes even in brittle material. Since this is not based on rotatory operation, torque force is not necessary. This is highly difficult to produce while performing microsurgeries. Thus it is highly useful for root canal treatment, bone micro drilling etc. They are useful in gem cutting and drilling industry where precision drilling with low material loss is the primary requirement.

3. Cutting and sealing: Ultrasonic transducers are used for cutting without fraying in textile industry, leather industry and in other industries where sealing is also needed along with cutting. Dissimilar fabric types can be sealed together with ease using this technology. They are also used for gem cutting and polishing.

4. Emulsification: Ultrasonic technology made it possible to develop high quality emulsions, which made possible the preparation of water proof, dust proof and fire proof paints with ease.

Think
Why gas lighter welding is possible only by ultrasonics?
8.7 Applications of Ultrasonics

3. Ultrasonic cleaning: The ultrasonic waves created by the transducer cause micro-bubbles to form in the water, which then collapse, generating high-speed jets of water. These jets dislodge and clean the particles, making it an efficient method for cleaning delicate objects.

4. Ultrasonic welding: Ultrasonic waves can be used to create a strong bond between two materials, such as plastic or metal, by heating the materials at the interface. This method is widely used in the manufacturing of medical devices and electronics.

5. Ultrasonic treatments: Ultrasonic waves can be used to stimulate tissue growth and repair, reducing pain and inflammation. This method is often used in the treatment of joint conditions such as osteoarthritis.

6. Ultrasonic imaging: Ultrasonic waves can be used to create images of internal structures, allowing medical professionals to diagnose and treat conditions such as tumors and injuries.

7. Ultrasonic therapy: Ultrasonic waves can be used to relieve pain and improve tissue healing, often used in the treatment of chronic conditions such as reflux oesophagitis and rhinitis.

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5. Mixing: Ultrasonics can be used to mix powders, liquids and gels in a more homogenised manner. They are also used to drive ultra-smooth powders like milk powder etc. through funnels and conveyor belts with relative ease. Thus they ensure prevention of clogging, agglomeration and adhesion.

6. Cooling: Ultrasonic waves when passed through fluid media, produce cavitation effect, meaning that they create air bubbles which explode and generate shock waves. These shock waves expand the fluid suddenly. This sudden expansion generates cooling in the fluid just like cooling produced outside a refrigerator compressor. This feature finds its applications in food industry.

7. Food industry: In food industry, ultrasounds are used for disinfection and sterilization of food items.

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https://sonotronic.de/technologies/ultrasonic
https://www.hielscher.com/
https://www.hindawi.com/journals/ijce/2011/670108
https://www.preprints.org/manuscript/201802.0168/v1/download
https://www.mdpi.com/2076-0825/7/2/18
8.7 Applications of Ultrasonics

hardt of Ultrasonics

Applications

Applications of Ultrasonics

1. Applications in the field of medical, biotechnology, food processing, automotive, and aerospace industries can benefit from the use of ultrasonics. They are used for cleaning, sterilization, and decontamination processes.

2. In the field of pharmaceuticals, ultrasonics are used for the production of microemulsions, liposomes, and nanoparticles.

3. In the field of agriculture, ultrasonics are used for the production of activated carbon, which can be used as a food additive.

4. In the field of materials science, ultrasonics are used for the production of nanomaterials, such as carbon nanotubes and graphene.

5. In the field of environmental science, ultrasonics are used for the production of sonochemically active organic materials.

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https://www.idosi.org/wasj/wasj10(8)/13.pdf
Chemistry applications

In the field of chemical and pharmaceutical industries, ultrasonic waves have an important role in deciding the chemical reaction speeds and processes. They are also used for cleaning and mixing purposes. Some of the specific applications are mentioned below.

1. Ultrasonic waves find their applications in various processes of pharmaceutical industry like aeration, crystallization, degasification, evaporation, extraction, homogenization, oxidation, synthesis etc. This branch of chemistry is called sonochemistry.

2. There are some special category of reactions which are assisted by ultrasonic waves. They are sonocatalytic reactions and sonophotocatalytic reactions. In these processes, ultrasonic waves and/or ultraviolet waves are used to treat waste water. They are especially used to treat organic soluble contaminants that are present in waste water.

3. Ultrasound assisted compression is used in the preparation of tablets. If stress is more, the chemical structure of drugs may get effected. Hence a low stress applying ultrasonic technique is preferred. Ultrasonic sealing and cutting is also used in blister packing of tablets.

4. Ultrasonic cleaners are highly useful in pharma industry, as there will be huge numbers and volumes of glassware that are needed to be cleaned. They may contain toxic elements as well. Usage of ultrasound cleaners will reduce process time, improves cleaning quality, improves the life time of equipment, and avoids exposure to toxic chemicals during cleaning.

5. Ultrasonics are used for level sensing in industries where ordinary conductivity based level sensors and optical reflection based laser sensors fail like in molten steel tanks, reactive chemical chambers and powder samples.
8.7 Applications of Ultrasonics

The applications of ultrasonics can be broadly classified into various categories. These include medical, industrial, and research applications. In recent years, ultrasonics has gained prominence due to its non-invasive nature and ability to penetrate through different materials.

1. Medical Applications: Ultrasonics find extensive use in medical diagnostics and treatment. They are used to detect and monitor various medical conditions such as pregnancy, heart diseases, and tumors.

2. Industrial Applications: Ultrasonics are used in various industries for quality control, process monitoring, and material testing. They help in detecting defects in materials and components before they reach the final stage.

3. Research Applications: Ultrasonics are also used in research to study various physical and chemical phenomena. They are used to study the behavior of materials under different conditions and to develop new technologies.

In conclusion, ultrasonics have become an integral part of modern technology due to their versatility and effectiveness. They continue to expand their applications and are set to play a significant role in the future.
Research
In the field of research, ultrasonic waves find their applications in materials manipulation and materials characterization needs.

1. Acoustic levitation: This is the phenomenon where pressure generated by ultrasonic sounds is used to levitate objects in air against gravity. This is useful in high purity microchip fabrication. This method is advantageous to levitate non-magnetic materials also.

2. Acoustic tweezers: acoustic tweezers use ultrasonic waves to control the position and motion of micro objects like blood cells, viral vectors, etc. When objects with different physical parameters like density, surface tension are kept in ultrasonic standing wave field, some will be accumulated at nodes and some will be accumulated at antinodes just like butter and buttermilk get separated during churning. The physical phenomenon behind this is the acoustic radiation force. This method is widely used in biotechnology for biological cell separation, cell trapping and cell manipulation applications.

3. Ultrasonic motors are used in lens cameras to control the autofocus operation. This motor is used for high precision control on rotational motion. These motors are used for controlling machines in high magnetic field medical imaging devices as these do not use any magnetic materials. These are highly useful in military and aerospace engineering for high precision control of antennas and guided missiles. These are also used to control the tip of electron microscopes where it has to move at nanometer precision.

Ultrasonic interferometer:
In research field, ultrasonic interferometer has made a major breakthrough in studying the elastic properties of liquids. Several binary, ternary liquid mixtures are also studied for measuring their mechanical properties like compressibility, miscibility, molecular interactions, relaxation time constants, acoustic impedance etc.

- The experimental setup consists of a double walled metal cylindrical cup. The liquid sample to be tested is placed in that cup.
8.7 Applications of Ultrasonics

- The wavelength of ultrasound is determined by the speed of sound, which is given by $\lambda = \frac{c}{f}$, where $\lambda$ is the wavelength, $c$ is the speed of sound, and $f$ is the frequency.

- The range of ultrasound is limited by the absorption in tissue, which reduces the travel distance. The absorption is dependent on the tissue properties such as density and sound speed.

- Design possible experiments using ultrasonic interferometer. Ref. sub links in second reference above.

**T - Corner**

http://www.mittalenterprises.com/products/index/ultrasonic-interferometer-for-liquids_3238.html
https://vlab.amrita.edu/?sub=1&brch=201&sim=803&cnt=1

**Activity**

Design possible experiments using ultrasonic interferometer. Ref. sub links in second reference above.
The bottom surface of the cup is attached to a piezoelectric oscillator. This produces ultrasonic oscillations in the liquid sample.

The top surface of the cup is covered with a movable metal reflector plate. This plate can be moved in vertical direction using a micrometer screw arrangement.

The entire arrangement is kept in hot water bath. Hot water of required temperature is circulated in it.

When the ultrasonic waves from the generator get reflected from the reflector, they form a standing wave when the separation between them is an integral multiple of $\lambda/2$.

Thereafter, standing wave pattern is formed for every $\lambda/2$ displacement of the micrometer screw. Then resonance occurs and the amplitude of oscillations increases. Thus, the oscillator circuit draws more current at resonance which can be measured.

A graph of current ($I$) supplied to the oscillator circuit as a function of micrometer displacement ($d$) will exhibit peaks as shown in the figure. The separation between the peaks is equal to $\lambda/2$.

Then the velocity of ultrasonic wave in the given liquid sample is found from the relation,

$$v = \lambda f = 2df.$$ 

The bulk modulus of the liquid sample is given by $\beta = \frac{1}{\rho v^2}$.
8.7 Applications of Ultrasonics

1. In the medical field, ultrasonics are used for various diagnostic purposes.
2. The main advantage of ultrasonic imaging is that it does not involve ionizing radiation, making it safer for patients.
3. Therapeutic applications include shock wave lithotripsy, where high-intensity focused ultrasound (HIFU) is used.
4. HIFU is also used in aspiration biopsy, a minimally invasive procedure for removing tissue samples.
5. Other applications include HIFU-based therapy for treating prostate cancer.
6. HIFU is also used in non-invasive procedures such as skin tightening.
7. In dentistry, ultrasonic instruments are used for cleaning teeth and removing plaque.
8. In the field of veterinary medicine, ultrasonics are used for imaging and diagnostics.
9. In obstetrics, ultrasonics are used for monitoring fetal development and detecting potential issues.
10. In industry, ultrasonics are used for non-destructive testing of materials.
11. In research, ultrasonics are used for various experiments and investigations.
12. In general, ultrasonics have a wide range of applications due to their non-invasive nature.

https://en.wikipedia.org/wiki/Medical_ultrasound
Medical Field Applications

1. Ultrasonics in the medical field are used to sterilize surgical equipment.
2. They are used as diagnostic tools, to scan the fetus, heart (Echocardiogram), and other abdominal parts. This branch of medical diagnostics is called Ultrasonography.
3. Ultrasound waves are also used for pain relief treatments in physiotherapy.
4. Some cancer related drugs enclosed in nanocapsules can be triggered on site using ultrasound.
5. Some of the non-cancerous tumours can be removed by focusing low intensity ultrasound waves from multiple directions onto a target site, in such a way that the intensity of beams is harmless to the rest of the body but at the target site, the net intensity is sufficient to perform the required operation. This is called HIFU (High Intensity Focused Ultrasound) non-invasive surgery.
6. In minimal invasive surgeries using ultrasonic waves, the surgical probe such as drill bit or blade is controlled by piezoelectric ultrasonic vibrator. These are used for bone and kidney stone surgeries.
7. The pressure and temperature generated by ultrasound probes can also be used to block blood vessels to reduce bleeding during surgeries.
8. Ultrasounds are also useful for transdermal drug delivery systems where ultrasounds enhance the absorption capacity of tissues underneath the skin for the required drug delivery.
9. Ultrasound guided needles are used for biopsy sample collection. Here ultrasonic waves give a visual of the needle as well as tissue under observation.
10. Low Intensity Pulsed UltraSound (LIPUS) is used externally for quick healing of the bone fractures.
11. In non-drug acne therapy, ultrasonic waves are used to heat the sebaceous (oil) glands underneath the skin externally. Heating reduces the size of the gland, kills bacteria and thus heals the acne.
12. They are used to measure blood flow rate in blood vessels.
8.7 Applications of Ultrasonics

A-తెలుగు: అంచలు: ఎదురు అంచనా మొదలుగా నిర్ధారించిన ఆరేయం ఈ మెరుగు శిఖరం y-మేళుగా ఎదురు నిర్ధారించిన ఆరేయం. ఈ పట్టిక నిర్ధారించిన పరిస్థితులపై చెప్పిన విషయాలు, ఇచ్చిన మరుగు శిఖరం అనేక సంఖ్యలు దీర్ఘముఖం, నిర్మాణ సమయం క్రిందిత్వం కనుగొని ఉంచబడిన మూలాలు. సమాధానం పంచాలం దురితం, విషయం ప్రకృతి కలాయితో ఉపయోగించే హుందే హుందే వందులు పంచబడాలి, ఆ దిద్ది మరుగు శిఖరం ప్రభుత్వం ముందు ప్రభుత్వం కలిగి ఉంది.
B-తెలుగు: అంచలు: ఎదురు అంచనా మొదలుగా ఎదురు నిర్ధారించిన ఆరేయం. ఈ పట్టిక నిర్ధారించిన పరిస్థితులపై చెప్పిన విషయాలు, ఇచ్చిన మరుగు శిఖరం అనేక సంఖ్యలు దీర్ఘముఖం, నిర్మాణ సమయం క్రిందిత్వం కనుగొని ఉంచబడిన మూలాలు. సమాధానం పంచాలం దురితం, విషయం ప్రకృతి కలాయితో ఉపయోగించే హుందే హుందే వందులు పంచబడాలి, ఆ దిద్ది మరుగు శిఖరం ప్రభుత్వం ముందు ప్రభుత్వం కలిగి ఉంది.
C-తెలుగు: అంచలు: ఎదురు అంచనా మొదలుగా ఎదురు నిర్ధారించిన ఆరేయం. ఈ పట్టిక నిర్ధారించిన పరిస్థితులపై చెప్పిన విషయాలు, ఇచ్చిన మరుగు శిఖరం అనేక సంఖ్యలు దీర్ఘముఖం, నిర్మాణ సమయం క్రిందిత్వం కనుగొని ఉంచబడిన మూలాలు. సమాధానం పంచాలం దురితం, విషయం ప్రకృతి కలాయితో ఉపయోగించే హుందే హుందే వందులు పంచబడాలి, ఆ దిద్ది మరుగు శిఖరం ప్రభుత్వం ముందు ప్రభుత్వం కలిగి ఉంది.
D-తెలుగు: అంచలు: ఎదురు అంచనా మొదలుగా ఎదురు నిర్ధారించిన ఆరేయం. ఈ పట్టిక నిర్ధారించిన పరిస్థితులపై చెప్పిన విషయాలు, ఇచ్చిన మరుగు శిఖరం అనేక సంఖ్యలు దీర్ఘముఖం, నిర్మాణ సమయం క్రిందిత్వం కనుగొని ఉంచబడిన మూలాలు. సమాధానం పంచాలం దురితం, విషయం ప్రకృతి కలాయితో ఉపయోగించే హుందే హుందే వందులు పంచబడాలి, ఆ దిద్ది మరుగు శిఖరం ప్రభుత్వం ముందు ప్రభుత్వం కలిగి ఉంది.
E-తెలుగు: అంచలు: ఎదురు అంచనా మొదలుగా ఎదురు నిర్ధారించిన ఆరేయం. ఈ పట్టిక నిర్ధారించిన పరిస్థితులపై చెప్పిన విషయాలు, ఇచ్చిన మరుగు శిఖరం అనేక సంఖ్యలు దీర్ఘముఖం, నిర్మాణ సమయం క్రిందిత్వం కనుగొని ఉంచబడిన మూలాలు. సమాధానం పంచాలం దురితం, విషయం ప్రకృతి కలాయితో ఉపయోగించే హుందే హుందే వందులు పంచబడాలి, ఆ దిద్ది మరుగు శిఖరం ప్రభుత్వం ముందు ప్రభుత్వం కలిగి ఉంది.
Types of Ultrasound scans (Additional information)
A-Scan: Here depth is taken along x-axis and intensity of ultrasonic wave is taken along y-axis. If there are no defects in the material, then the separation between the peaks, due to the top surface reflected echo and that due to the bottom surface reflected echo, gives the thickness of the material. If there are multiple peaks, one can predict the location of defect and also the size of defect. This gives the image of a line along the direction of scan as a function of depth.

B-Scan: Here the width of the specimen is taken along x-axis, depth is taken along y-axis and intensity of echo at each point on the plane is plotted as a dot. Thus it is also called 2D-echo of the system. This gives the image of a vertical plane in the material or body under scan.

B-flow scan: Here a 2D echo is taken at consecutive time intervals and intensities are subtracted. The resultant is a 2D image of moving particles like blood cells. This is a more accurate scan method than 2D Doppler echo as Doppler echo may also record the movement of tissues and muscles. This B-flow scan can clearly isolate the blood flow. This is a more preferred and accurate scan for liver imaging compared to the colour Doppler scan.

C-scan: This is a combination of both A-mode and B-mode. Here the scanner is fixed at a particular depth level and scanner head is moved on the entire surface of the body to get a 2D scan of the tissue or object at the selected depth level along a horizontal plane. In this method a 10cmx10cm area can be scanned in 10 seconds.

M-scan: Here the image is taken either in A-mode or in B-mode as a function of time to create a moving image or video of the system. Hence it is called motion-mode scan or M-scan.

Doppler scan: This uses the Doppler effect to depict the moving organs and blood flow in the vessels. There are different types of Doppler scan as detailed below.

Continuous Doppler scan: In continuous mode Doppler scan, multiple waves are sent and received at a time. So the resolution of the produced image may not be very good and accurate for deep scans.
8.7 Applications of Ultrasonics

Applications of Ultrasonics

B - Corner

https://www.ajronline.org/doi/10.2214/AJR.06.1161

https://www.wikilectures.eu/w/Doppler_sonography/types_and_outputs

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4923974/
Pulsed wave Doppler scan: Here the ultrasonic waves are sent and received in pulses at regular intervals. Thus the accuracy for deep tissue scan will be improved in this type. But high velocities can not be recorded accurately, as compared to the continuous wave Doppler, where multiple waves scan the same situation at a time. This limitation on measuring high velocities in pulsed mode Doppler scan is called aliasing. This is not found in B-flow scan and power Doppler scan.

Colour Doppler scan: This assigns colours to different speed signals. Larger speed is assigned with red and lower speed with blue. Thus it can be used to study variations in speed of blood cells and moving tissues and muscles in the body.

Power Doppler scan: This records the intensity information of the reflected wave rather than the speed information. It, therefore, gives more morphological (structural) details of the system rather than velocities. Thus, it is useful in studying the inflammation status of tissues and quality analysis of nephrons in kidneys.

Spectral Doppler: Here a B-scan is combined with pulsed wave Doppler as a function of time. A plot of velocities at a location as a function of time is plotted to get spectral Doppler. Since it is combined with B-scan, the time plot is made for entire scan area. This is more useful to study blood pumping abnormalities in heart as a function of time.

Duplex Doppler: Here a B-scan is combined with colour Doppler scan. Thus both morphological and velocity information of tissues and blood vessels is obtained at a time.

Triplex Doppler: Here a colour Doppler and a duplex scan (altogether 3 scans) are implemented together. Thus it gives a time graph of entire 2D plane combined with B-scan and colour Doppler scan of the plane. This is useful for studying blood clots (thrombosis) in deep veins.

3D and 4D colour Doppler scan: Here a colour Doppler with C-scan is performed to generate a 3D image of the fetus or any other body part. If 3D Doppler scan is performed at regular time intervals to generate a 3D video, then it is called 4D colour Doppler scan.
8.7 Applications of Ultrasonics

\[ f = \frac{(c \pm v_0)}{(c \pm v_s)} f_0 \]

where \( f_0 \) is the frequency of the wave in the medium without any disturbance, \( c \) is the speed of sound in the medium, and \( v_0 \) and \( v_s \) are the velocities of the particle and the medium, respectively.

\[ f = (1 \pm \frac{v_0}{c}) (1 \pm \frac{v_s}{c})^{-1} f_0 = (1 \pm \frac{\Delta v}{c}) f_0 \]

where \( \Delta v = (v_0 - v_s) \) is the velocity difference. This equation gives the change in frequency due to the velocity difference.

\[ \Delta f = f - f_0 = \frac{2v \cos \theta}{c} f_0 \]

This relation holds for a wave propagating at an angle \( \theta \) with respect to the medium.

\[ \Delta f = (2v \cos \theta)/c f_0 \]

This relation shows how the frequency changes with the angle of propagation.
Theory of Doppler scan:

In 1843 Christian Doppler studied and explained the change in frequency of sound waves (known as Doppler shift), when there is a relative motion between the source and observer. The Doppler shift is given by

\[ f = \frac{c \pm v_0}{c \pm v_s} f_0. \]

Here \( f_0 \) is the original frequency of the source and \( f \) is the observed frequency, \( c \) is the original velocity of sound in the given medium. \( v_s, v_0 \) are the velocities of source and observer respectively.

If \( c \) is very large compared to \( v_0 \) and \( v_s \) one can construct the binomial expansion and consider the dominant terms.

\[ f = \left(1 \pm \frac{v_0}{c}\right) \left(1 \pm \frac{v_s}{c}\right)^{-1} f_0 = \left(1 \pm \frac{v_0}{c}\right) \left(1 \mp \frac{v_s}{c}\right) f_0 \]

\[ \approx \left(1 + \frac{v_0 - v_s}{c}\right) f_0 = \left(1 - \frac{\Delta v}{c}\right) f_0 \]

Thus, change in frequency is given by

\[ \Delta f = f - f_0 = \frac{\Delta v}{c} f_0 \]

Consider an ultrasonic wave incident at an angle \( \theta \) on a blood vessel. Let \( v \) be the velocity of blood. Then the component of the
NDT (NDE) అంశాలు

1. వీ-ట్యూబుల మాత్రము / వీ-క్రింది సంఖ్య అధీనంలో, అధికంగా సంఖ్య మాత్రముతో ప్రతిసామయంలో ప్రత్యేకించిన అవసరం. అందుకే, అనువిధిత విదేశి కాలు సందర్భం (y-మాత్రము సందర్భం) లేదా సంఖ్య మాత్రము సందర్భం (x-మాత్రము సందర్భం) జీవిత సమాధానం. సమాధానానికి మరియు ఈ సమాధానానికి లభించిన అనువిధిత అవసరం మనకు మాత్రము ఉపయోగించే కమలం ప్రకారం మనకు మాత్రము అవసరం. మనకు మాత్రము కనుకుని, వీ-ట్యూబుల మాత్రము సంఖ్య మాత్రము రెండు అదనం మేం విధానంలోను తమ మనకు మాత్రము అవసరం.

2. వీ-ట్యూబుల అంశాలు తోడు ఈంటే లేదా మాత్రము సంఖ్య అంశాలు కంటే మాత్రము సంఖ్య నియంత్రణ కార్యాచరణ యొక్క అదనం లేదా అనువిధిత అవసరం యొక్క అదనం నియంత్రణ కార్యాచరణ యొక్క అదనం. ఒక భాగాను అదనం మాత్రము నియంత్రించడం అయితే ఒక భాగాను అదనం నియంత్రించడం అయితే తమ మనకు మాత్రము అవసరం. కంటే మాత్రము సంఖ్య అదనం నియంత్రణ కార్యాచరణ యొక్క అదనం చేయడం అయితే తమ మనకు మాత్రము అవసరం.

3. సన్నిది అంశాలు ఉపయోగాలు: అందువల్ల అంశాలు కంటే మాత్రము సంఖ్య అదనం నియంత్రణ కార్యాచరణ యొక్క అదనం నియంత్రణ కార్యాచరణ యొక్క అదనం. ఇక్కడ, అదనం నియంత్రణ కార్యాచరణ యొక్క అదనం నియంత్రణ కార్యాచరణ యొక్క అదనం. అదనం నియంత్రణ కార్యాచరణ యొక్క అదనం నియంత్రణ కార్యాచరణ యొక్క అదనం. సన్నిది అంశాలు ఉపయోగాలు అదనం నియంత్రణ కార్యాచరణ యొక్క అదనం

పండులు

SONAR అంశాలు ఉపయోగం వచ్చాయి ఫియం సా. అందువల్ల సాంప్రదాయ అంశాలు కంటే మాత్రము సంఖ్య నియంత్రణ కార్యాచరణ యొక్క అదనం నియంత్రణ కార్యాచరణ యొక్క అదనం. కంటే మాత్రము సంఖ్య నియంత్రణ కార్యాచరణ యొక్క అదనం నియంత్రణ కార్యాచరణ యొక్క అదనం నియంత్రణ కార్యాచరణ యొక్క అదనం. సన్నిది అంశాలు ఉపయోగం వచ్చాయి ఫియం సా. 1912 సంవత్సరం వచ్చాయి ఫియం సా. 1912 సంవత్సరం వచ్చాయి ఫియం సా. 1940 సంవత్సరానికి అదనం నియంత్రణ కార్యాచరణ యొక్క అదనం.
blood velocity vector along the direction of the incident beam is given by $v \cos \theta$.

Since the component of velocity is opposite to the source wave, the velocity of source wave decreases by $v \cos \theta$. This decrement is considered as source velocity. Thus $v_s = -v \cos \theta$.

Since the object wave is along the component of particle velocity, observer wave velocity increases by $v \cos \theta$. This increment can be considered as observer velocity $\Delta v_0 = v \cos \theta$, giving $\Delta v = 2v \cos \theta$. Thus, for Doppler scan the change in frequency recorded is given by

$$\Delta f = \frac{2v \cos \theta}{c} f_0$$

This relation implies that the Doppler shift in frequency is maximum when the ultrasonic wave is incident normal to the blood flow.

**NDT (NDE) Applications**

1. **In non-destructive evaluation / non-destructive testing** applications, ultrasonic waves are passed onto the material to be tested. Here, a graph of intensity of the ultrasonic wave (taken along y-axis) is plotted as a function of depth (taken along x-axis). If there are no defects in the material, the top surface reflected echo peak and the bottom surface reflected echo peak separation gives the thickness of the material. If there are multiple peaks, one can predict the location of defect and also the size of defect.
మాత్రమే మద్దత చేసేవారు అంటే యది ఇంటినే నిష్టానికి నా వ్యవస్థ ఉంటే మనం చేసేవారు అంటే యొక్క కొరకు యొక్క కొన్ని సమయానికి నా వ్యవస్థ ఉంటే మనం చేసేవారు అంటే నా వ్యవస్థ ఉంటే మనం చేసేవారు అంటే యొక్క కొరకు యొక్క కొన్ని సమయానికి నా వ్యవస్థ ఉంటే మనం చేసేవారు అంటే నా వ్యవస్థ ఉంటే మనం చేసేవారు అంటే యొక్క కొరకు యొక్క కొన్ని సమయానికి నా వ్యవస్థ ఉంటే మనం చేసేవారు అంటే 

<table>
<thead>
<tr>
<th>శాసనం (SL)</th>
<th>నిర్ధారణ శాసనం (NL)</th>
<th>నిర్ధారణ నిర్ధారణ శాసనం (DI)</th>
<th>నిర్ధారణ నిర్ధారణ శాసనం (DL)</th>
<th>నిర్ధారణ నిర్ధారణ శాసనం (DT)</th>
<th>నిర్ధారణ నిర్ధారణ శాసనం (RL)</th>
<th>నిర్ధారణ నిర్ధారణ శాసనం (TS)</th>
<th>నిర్ధారణ నిర్ధారణ శాసనం (NL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>మన సంఖ్య (TS)</td>
<td>ఆప్యాస వాటాటి శాసనం (SL)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ఇస్తు ఎందరూ ఎందుకంటే ప్రత్యేక సంబంధం ఉంది. స్పష్టమైనంతే నా వ్యవస్థ ఉంటే మనం చేసేవారు అంటే యొక్క కొరకు యొక్క కొన్ని సమయానికి నా వ్యవస్థ ఉంటే మనం చేసేవారు అంటే 

\[ SNR = (SL - TL) - (NL - DI) = SL + DI - TL - NL \]
2. **Picosecond ultrasonics**: They use acousto-optic transducers for generation and detection of ultrasound waves. Here the material to be tested is coated with a thin substrate. Ultrasonic waves are focused onto the substrate using acoustic lens to generate a shock wave. The shock wave reaches the other end of the test sample and reflects back. These are used to identify the cracks and fractures in nanostructures and thin films.

![Picosecond ultrasonics diagram](image)

**Fig. Picosecond ultrasonics**

3. **Phased array ultrasonic sensors**: Here a sequence of ultrasonic waves with suitable phase difference are transmitted into the test medium to create constructive interference of ultrasonic waves. These waves, after reflection also create similar interference pattern with symmetry. If there exists any defect, the interference pattern changes. These are used for detecting defects in long pipe lines and large sheets of test samples in one shot.

![Phased array ultrasonic sensor diagram](image)

**Fig: Phased array ultrasonic sensor.**
చాలా ఎలక్ట్రానిస్ట్ విద్యార్థులు, ఎలక్ట్రానిక్ విద్యార్థులు యొక్క కాళ్ళిక ఉపయోగంలో విద్యార్థులు ఎలక్ట్రానిక్ పాఠశాలలో ఉపయోగం చేసిన ప్రపంచను ఎలక్ట్రానిక్ పాఠశాలలో ఉపయోగం చేసిన ప్రపంచను ఎలక్ట్రానిక్ పాఠశాలలో ఉపయోగం చేసిన ప్రపంచను ఎలక్ట్రానిక్ పాఠశాలలో ఉపయోగం చేసిన ప్రపంచను ఎలక్ట్రానిక్ పాఠశాలలో ఉపయోగం చేసిన ప్రపంచను ఎలక్ట్రానిక్ పాఠశాలలో ఉపయోగం చేసిన ప్రపంచను ఎలక్ట్రానిక్ 

$$\text{SNR}=(\text{SL}-\text{TL})+(\text{TS}-\text{TL})-(\text{NL}-\text{DI})=\text{SL}+\text{TS}+\text{DI}-2\text{TL}-\text{NL}$$

$$\text{SNR}>\text{DT}$$

$$\text{SNR}>\text{DT}+\text{RL}+\text{AG}$$

$$\text{SPL}=20 \log(p/1\mu Pa)$$

$$\text{SNR}>\text{DT}$$

$$\text{SNR}>\text{DT}+\text{RL}+\text{AG}$$

$$\text{SPL}=20 \log(p/1\mu Pa)$$

$$\text{SNR}>\text{DT}$$

$$\text{SNR}>\text{DT}+\text{RL}+\text{AG}$$

$$\text{SPL}=20 \log(p/1\mu Pa)$$
8.8 SONAR

SONAR stands for sound navigation and ranging. The basic property of ultrasonic waves used in this technique is that they can be reflected. So by sending Ultrasonic waves and receiving the echo using ultrasonic detector, one can detect submarines, big fishes, obstacles and objects during boat accidents.

Using this technique, moving objects under water can be detected on the basis of doppler effect. By comparing frequencies of reflected and transmitted ultrasonic signals one can find the velocity and hardness of the target.

We know velocity of sound through sea water. By calculating time interval between reflected and transmitted ultrasonic signals we can find the depth of the sea.

Did You Know?

SONAR was developed to detect underwater obstacles during navigation. This was developed within 1 month after the famous TITANIC ship mishap in 1912. The technique was patented by Richardson in 1912 and device was developed by Fessenden in 1914. Full pledged SONAR usage came in 1940.

SONAR are of two types namely active and passive sonars. Active sonars have both transmitter and receiver components within them. Passive sonars have only receiver components. Active sonars are used to detect ice bergs, study fish schools, morphology of sea bed etc. Passive sonars are used for communication and to detect other sonar signals.

The parameters that can be used to describe the active and passive sonar systems are as follows.

Parameters determined by equipment:
Projector source level. (SL)
8.8 SONAR

Self noise level (NL)
Receiving Direction Index (DI)
Detector threshold (DT)
Array gain (AG)

Parameters determined by the medium
Transmission loss (TL)
Reverberation level (RL)
Ambient noise level (NL)

Parameters determined by the target
Target strength (TS)
Target source level (SL).

Here same symbols are given to similar parameters irrespective of their location.

In the case of passive sonar, the source is external with strength SL. The signal received is reduced by a factor of TL due to transmission losses. Further the signal received is effected by the ambient noise (NL). Signal strength is enhanced by the Direction index (DI) of the receiver. i.e.; if the receiver is in the same direction as the source, the signal quality improves, otherwise not. With these, the signal to noise ratio of passive receiver is defined as

\[
SNR = (SL - TL) - (NL - DI) = SL + DI - TL - NL
\]

In the case of active sonar, initial source signal is produced by the sonar itself and it is further reflected by the target. After reflection from the target, the signal strength is termed as the target strength (TS). That also will get effected by transmission losses (TL). Thus the signal to noise ratio for Active sonar is given by

\[
SNR = (SL - TL) + (TS - TL) - (NL - DI)
\]
\[
= SL + TS + DI - 2TL - NL
\]

If SNR is greater than the detector threshold, then it will get detected. Otherwise it will not get detected. i.e.;

\[
SNR > DT
\]
If reverberation levels and Array gain of hydrophone are also considered, then

$$SNR > DT + RL + AG$$

Here all sound levels are measured in the units of sound pressure level recorded at the hydrophone. This can be defined as

$$SPL = 20 \log \left( \frac{p}{1 \mu Pa} \right)$$

Here $1 \mu Pa = 1 \text{ micro Pascal}$ is the reference level. Thus though all the above equations look like additions, they all contain logarithms of ratios of pressure levels.

Sonars have frequency range classified into 3 levels.

1. Low frequency range (1Hz-1KHz)
2. Mid frequency range (1KHz-10KHz)
3. High frequency range (10KHz-1MHz and above)

As the frequency of the signal increases, it scatters more in the water. Thus the range is lower for high frequency signals. But the quality of information received is more for high frequency signals. Thus there will be a trade off between the quality of the signal and the range. In majority of the cases, mid frequency signals are used.

The ecological effects of these sonar systems is that they effect the communication system and echo location mechanism of the dolphins and whales. Fish may become deaf temporarily in the presence of ultrasonic sounds. It was observed that the dolphins and whales show mass stranding in the mid frequency range.

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**E - Corner**

https://dosits.org/animals/effects-of-sound/anthropogenic-sources/sonar/
https://en.wikipedia.org/wiki/Sonar

Solved problems and Exercises

**Problem 1:** If an ultrasound generator generates 100 number of cycles with pulse of duration 1msec then find the time period and frequency of an ultrasound wave.

**Solution:**

Given \( N=100 \)

and \( t=1 \text{ msec} = 1 \times 10^{-3} \text{ sec} \)

we know that \( t = NT \)

\[ T = \frac{t}{N} = \frac{10^{-3}}{100} = 10^{-5} \text{ sec} \]

Frequency \( (f) = \frac{1}{T} \)

\[ f = \frac{10^{5}}{10^{-5}} = 10^5 \text{ Hz} \]

**Ans:** \( T=10^{-5} \text{ sec} \) & \( f=10^5 \text{ Hz} \)

**Problem 2:** In a surgical operation an ultrasound of intensity of \( 10\text{W/cm}^2 \) is used to incident on a tissue of thickness \( 0.5\text{cm} \) and observe the output intensity of \( 0.05\text{W/cm}^2 \). Find the absorption coefficient of the tissue.

**Solution:**

Given \( I_0=10\text{W/cm}^2 \)

\( I = 0.05\text{W/cm}^2 \)

\( t = 0.5\text{cm} \)

We know that \( \alpha = \frac{10}{t} \log \frac{I}{I_0} \text{ dB/cm} \)

\[ \alpha = \frac{10}{0.5} \log \frac{0.05}{10} = 20 \times (-2.3) = -46 \text{ dB/cm} \]

**Ans:** \( \alpha = -46 \text{ dB/cm} \)

**Problem 3:** Find the percent of reduction in intensity for a 1 MHz ultrasound beam traversing 1 cm of material having an attenuation of 10 dB/cm.

**Solution:**
Given thickness of material \((t) = 1\text{cm}\)
Attenuation per unit length \((\alpha) = -10 \text{ dB/cm}\)

The reduction of intensity is \(\alpha t\)
\[
\alpha = 10 \log \left( \frac{l}{l_0} \right)
\]
\[-10 = 10 \log \left( \frac{l}{l_0} \right)\]
\[
\frac{l_0}{l} = 10^1
\]
\[
l = \frac{l_0}{10}
\]
\[
l = \frac{l_0}{100} \times 10
\]

Therefore, reflectivity is 10%
And the reduction of intensity is \( = 100 - 10\) 
\(= 90\%\)

**Ans:** 90%

**Problem 4:** Find the percent reduction in intensity for a 1 MHz ultrasound beam traversing 10 cm of material having an attenuation of 1 dB/cm.

**Ans:** 90%

**Problem 4:** If an ultrasound having 1MHz frequency is reduced to 90%. Determine the intensity reduction if the ultrasound frequency were increased to 2MHz.

**Solution:**
Reduction to 90% means
Reflection intensity \(= (100-90) = 10\%\) only

\[
I = Io \times 10/100
\]
\[
I = Io/10
\]
Attenuation \(\alpha = 10 \log \frac{l}{l_0}\) dB
\[
\alpha = 10 \log \frac{1}{10}
\]
\[
\alpha = -10 \text{ dB reduction in intensity}\]
Solved problems and Exercises

But we know that the reduction is directly proportional to the frequency of wave
Therefore, for 1MHz reduction is -10dB
Similarly, for 2MHz reduction is -20dB

\[-20 = 10 \log \frac{I}{I_0}\]

\[-2 = \log \frac{I}{I_0}\]

\[\frac{I_0}{I} = 10^2\]

\[I = \frac{I_0}{100} \times 1\]

Therefore, Reflectivity 1 % only
Reduction in intensity is 99%
Ans: -20dB (99%)

Problem 5: Find the attenuation co-efficient of tissue when an ultrasound wave of 1MHz frequency having the intensity of 5W/cm$^2$ is incident and observed the output intensity from the tissue is 0.5mW/cm$^2$.

Ans: -30dB.

Problem 6: In a surgical operation an ultrasound of intensity 10W/cm$^2$ is used to incident on a tissue of thickness 0.5cm and observed the output intensity of 0.05W/cm$^2$. Find the absorption coefficient of the tissue.

Problem 7: Find the sound intensity level in decibels of 2x10$^{-2}$ W/m$^2$ ultrasound used in medical diagnostics.

Solution:

We know that \( \alpha = 10 \log \frac{I}{I_0} dB \)

\( I = 2 \times 10^{-2} \) W/m$^2$

\( I_o = 10^{-12} \) W/m$^2$

\[ \alpha = 10 \log \frac{2 \times 10^{-2}}{10^{-12}} \]

\[ \alpha = 10 \log \log (2 \times 10^{10}) \]

\[ \alpha = 10[\log \log (2) + \log \log (10^{10})] \]

\[ \alpha = 10[0.3 + 10] \]
\[ \alpha = 103 \text{dB} \]

Ans: 103 dB. Note: \( \text{dB} = 10 \log \left[ \frac{I}{I_0} \right] \), \( I_0 = 10^{-12} \text{W/m}^2 \)

**Problem 8:** In the clinical use of ultrasound, transducers are always coupled to the skin by a thin layer of gel or oil, replacing the air that would otherwise exist between the transducer and the skin. (a) Using the values of acoustic impedance given in Table 1 calculate the intensity reflection coefficient between transducer material and air. (b) Calculate the intensity reflection coefficient between transducer material and gel (assuming for this problem that its acoustic impedance is identical to that of water). (c) Based on the results of your calculations, explain why the gel is used.

| Table 1. The Ultrasound Properties of Various Media, Including Soft Tissue Found in the Body |
|-----------------|-----------------|-----------------|-----------------|
| Medium          | Density (kg/m³) | Speed of Ultrasound (m/s) | Acoustic Impedance (kg/(m² · s)) |
| Air             | 1.3             | 330              | 429             |
| Water           | 1000            | 1500             | 1.5 \times 10^6 |
| Barium titanate (transducer material) | 5600           | 5500             | 30.8 \times 10^6 |

**Solution:**
(a) We know that

\[ \text{Reflection coefficient (r)} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \]

In this case \( Z_1 = 429 \)
\( Z_2 = 30.8 \times 10^6 \)

Therefore
\[ r_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1} \]
Since \( Z_2 \gg Z_1 \),

\[ r_1 = \frac{30.8 \times 10^6 - 429}{30.8 \times 10^6 + 429} \]

(b) In this case \( Z_1 = 1.5 \times 10^6 \)

\[ Z_2 = 30.8 \times 10^6 \]

Therefore

\[ r_2 = \frac{Z_2 - Z_1}{Z_2 + Z_1} \]

\[ r_2 = \frac{30.8 \times 10^6 - 1.5 \times 10^6}{30.8 \times 10^6 + 1.5 \times 10^6} \]

\[ r_2 = \frac{30.8 - 1.5}{30.8 + 1.5} \]

\[ r_2 = \frac{29.3}{32.3} \]

\[ r_2 = 0.89 \]

In case (a), reflectivity \( (R) = r_1^2 = 1 \), therefore transmissivity \( T = 1 - r_1^2 = 0 \). Whereas in case (b), Reflectivity \( r_2^2 = 0.791 \) and transmissivity \( T = 1 - r_2^2 = 0.2079 \).

Therefore, gel has the more transmissivity than the air.

**Ans:** Gel has more transmissivity than air.

**Problem 9:** The time delay between transmission and the arrival of the reflected wave of a signal using ultrasound traveling through a piece of fat tissue was 0.13 ms. At what depth did this reflection occur?

**Ans:** 0.09425m

**Problem 10.** Ultrasound that has a frequency of 3 MHz is sent toward blood in an artery that is moving toward the source at 25.0 cm/s. Use the speed of sound in human tissue as 1540 m/s.

1. What frequency does the blood receive?
2. What frequency returns to the source?
3. What beat frequency is produced if the source and returning frequencies are mixed?
**Solution:**

Case i: Transducer Transmit ultrasound waves at freq $f_s$ towards blood flow.

Blood is moving with velocity $v$ at angle $\theta=0$

So the frequency it receives is given by

$$f_0 = \frac{(c + v \cos \theta)}{c} f_s$$

$c$ is the ultra sound velocity in the medium =1540 m/sec

and the plus sign is chosen because the motion is toward the source.

Enter the given values into the equation.

$$f_0 = \left(3,000,000 \right) \frac{\left(1540 + 0.25\right)}{1540}$$

$$f_0 = \left(3,000,000 \right) \frac{\left(1540 + 0.25\right)}{1540} \text{ m/s}$$

$$f_0 = 3,000,487 \text{ Hz}.$$  

Case ii: The blood acts as a moving source.

The receiver in transducer acts as a stationary observer.

The frequency leaving the blood is $f_o = 3,000,487 \text{ Hz}$, but it is moving upward.

Transducer receives this wave with freq $f_T = f_o \left(\frac{c}{c-v \cos \theta}\right)$ The minus sign is used because the motion is toward the observer.

Enter the given values into the equation:

$$f_T = \left(3,000,487 \text{ Hz} \right) \frac{1540}{1540 - 0.25}$$

$$f_T = 3,000,974 \text{ Hz}.$$

Case iii: The beat frequency is simply the absolute value of the difference between $f_s$ and $f_T$, as stated in:

$$f_B = |f_T - f_s|.$$ 

Substitute known values:

$$|3,000,974 \text{ Hz} - 3,000,000 \text{ Hz}|$$

the beat frequency: 974 Hz.

**Problem 11:** Ultrasound reflected from an oncoming bloodstream that is moving at 40.0 cm/s is mixed with the original frequency of
Solved problems and Exercises

3.50 MHz to produce beats. What is the beat frequency? (The velocity of ultrasound in tissue is 1540 m/sec.)

**Solution:** Given that

Blood stream velocity \( v \) = 40 cm/sec = 40 x 10^2 m/sec

Velocity of sound in blood \( c \) = 1540 m/sec

Initial frequency \( f_0 \) = 2.5 MHz = 3.5 x 10^6 Hz

Usually prefers the angle of incidence \( \theta \) = 0

We know that, Beat frequency (or) Doppler frequency shift is

\[
\Delta f = \frac{2v \cos \theta}{c} f_0
\]

\[
\Delta f = \frac{2 \times 40 \times 10^{-2} \times \cos \cos (0)}{1540} \times 3.5 \times 10^6
\]

\[
\Delta f = 1818 Hz
\]

**Problem 12:** A diagnostic ultrasound echo is reflected from moving blood and returns with a frequency 500 Hz higher than its original 2 MHz. What is the velocity of the blood? (The velocity of ultrasound in tissue is 1540 m/sec.)

**Solution:**

Given that

Blood stream velocity \( v \) = ?

Velocity of sound in blood \( c \) = 1540 m/sec

Initial frequency \( f_0 \) = 2 MHz = 2 x 10^6 Hz

Change in frequency \( \Delta f \) = 500 Hz

Usually, angle of incidence \( \theta \) = 0

\[
\Delta f = \frac{2v \cos \theta}{c} f_0
\]

\[
v = \frac{\Delta f c}{2 \cos \theta f_0}
\]

\[
v = \frac{500 \times 1540}{2(0) \times 2 \times 10^6}
\]

\[
v = 19.25 \text{ cm/sec}
\]

**Ans:** 19.25 cm/sec Hint: DFS = \([2v\cos(\theta)/C]f_0\)

**Problem 13:** a) Find the size of the smallest detail observable in human tissue with 20.0-MHz ultrasound. (b) Is its effective penetration depth great enough to examine the entire eye (about 3.00 cm is needed)? (c) What is the wavelength of such ultrasound in 0°C air? (The velocity of ultrasound in tissue is 1540 m/sec.)
Solution:
(a) Given frequency of ultrasound \( f = 20 \text{ MHz} = 20 \times 10^6 \text{ Hz} \)

The velocity of ultrasound in tissue \( v = 1540 \text{ m/sec} \)

We know that \( v = f \lambda \)

Therefore, \( \lambda = \frac{v}{f} \)

\[
\lambda = \frac{1540}{20 \times 10^6} = 77 \times 10^{-6} = 0.077 \text{ mm}
\]

We know that the smallest size of the object can be find using the ultrasound scan is equal to the wavelength of the ultrasound wave. Therefore, from the above result, the smallest size is 0.077 mm.

(b) As rule of thumb, effective scan can be done to a depth of about \( 500\lambda \) into tissue.

Therefore, the penetration depth limit = \( 500\lambda \)

\[
= 500 \times 0.077 = 0.0385 \text{ m}
\]

As penetration limit is 3.85 cm, so given frequency is enough to examine the entire eye.

(c) We know that \( \lambda = \frac{v}{f} \)

\[
\lambda = \frac{330}{20 \times 10^6} = 16.5 \text{ mm}
\]

Ans: (a) Wavelength = 77 \( \mu \)m, (b) penetration limit is 3.85 cm so given frequency is enough to examine the entire eye
(c) Wavelength = 16.5 \( \mu \)m

Problem 14: (a) A bat uses ultrasound to find its way among trees. If this bat can detect echoes 1.00 ms apart, what minimum distance between objects can it detect? (b) Could this distance explain the difficulty that bats have finding an open door when they accidentally get into a house?

Ans: 77 cm, yes

Problem 15: A dolphin is able to tell in the dark that the ultrasound echoes received from two sharks come from two different objects
only if the sharks are separated by 3.50 m, one being that much farther away than the other. (a) If the ultrasound has a frequency of 100 kHz, show this ability is not limited by its wavelength. (b) If this ability is due to the dolphin’s ability to detect the arrival times of echoes, what is the minimum time difference the dolphin can perceive? (given ultrasound velocity = 1540 m/sec)

**Solution:**

(a) Given that

Ultrasound frequency = 100 kHz

Ultrasound velocity = 1540 m/sec

Then wavelength \( \lambda = \frac{v}{f} = \frac{1540}{100 \times 10^3} \)

\[ \lambda = 0.0154 \text{ m} \]

Wavelength = 0.0154 m < 3.5 m, so the ability of dolphin to identify distinct objects separated by 3.5 m is not limited by wavelength.

(b) Given distance between the two sharks is 3.5 m

The difference between the two echoes received from the sharks separated by 3.5 m, \( \Delta t = ? \)

\[ \Delta t = 2(\Delta x/v) \]

\[ \Delta t = 2(3.5/1540) \]

\[ = 4.545 \text{ msec} \]

**Problem 16:** Write your observations from the figure given below

**Solution:**

Findings from the figure are

1. Pulse echo method is used for detecting flaws in the testing material.
2. Figure (a) tells transmitted pulse at starting position and reflected echo pulse at other end of the testing material.
3. Figure (b) tells that there is a defect as extra peak observed due to reflection at this defect before the echo pulse peak.
4. Figure (c) tells that due to large size of the defect not able to observe echo pulse peak.
**Problem 17:** Given frequency of ultrasonic waves is 5MHz, velocity of sound in the steel medium is 1018m/sec and diameter of transducer is 10mm. Calculate near field distance for ultrasonic waves?

Solution: Near field distance \( N = \frac{D^2 f}{4v} \)

Here \( N = \) near field distance
\( D = \) diameter of the transducer = 10mm = 0.01m
\( v = \) velocity of sound in steel = 1018m/sec
\( f = \) frequency of ultrasonics = 5MHz

\[ N = \frac{0.01^2 \times 5000000}{4 \times 1018} = 12\text{mm} \]

**Problem 18:** Find the velocity of 2MHz ultrasonic wave through water using ultrasonic interferometer. Distance between the two successive maxima of current is 0.375mm.

**Solution:** Ultrasound velocity in a medium using interferometer \( v = \frac{f \lambda}{2} \)

Here \( \lambda \) found from interferometer is \( \lambda = 2d = 2 \times 0.375 = 0.75\text{mm} \)

So velocity of ultrasound waves in water \( v = 2 \times 10^6 \times 0.75 \times 10^{-3} \)
\[ = 1500\text{m/sec} \]

**Problem 19:** Ultrasonic wave velocity in acetone using ultrasonic interferometer found 2700m/sec. Find the wavelength of ultrasonics waves in acetone.? (Given frequency of ultrasonics is 5MHz).

Ans: 540µm

**Problem 20:** Deduce acoustic impedance \( Z = \rho v \) from \( Z = \frac{p}{v} \)

Here \( \rho \) is density of the medium
\( v \) is velocity of sound in the medium
\( p \) is pressure applied on the medium
Grade your Understanding

1. Ultrasonic waves have short wavelengths than that of audible sound
   [ √ ]
2. Velocity of Ultrasonic waves is greater than that of audible sound
   [ √ ]
3. Sonoluminescence is a concept that shows conversion of sound energy to Light energy
   [ √ ]
4. Natural logarithm of ratio of two amplitudes of ultrasonic waves is measured in decibel
   [ ]
5. Ultrasonic waves does not obey Snell’s law
   [ ]
6. The frequency of oscillations of the ferromagnetic bar will be twice the frequency of the applied field
   [ √ ]
7. Piezoelectric generators whose fundamental frequency is in the range of 40KHz
   [ ]
8. In Y Cut crystal ,the frequency of longitudinal oscillations generated will be half of the frequency of the crystal
   [ ]
9. Kundt’s tube method, Piezo-electric detector and MEMS can also detect audible sound
   [ ]
10. A drill that oscillates with ultrasonic frequency can bore any hard material
   [ √ ]


Glossary

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<tr>
<th>Glossary: Ultrasonics</th>
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<tbody>
<tr>
<td><strong>Cavitation</strong></td>
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<tr>
<td><strong>Ferromagnetic</strong></td>
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<td>Seismic wave</td>
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Accessibility and affordable Higher Education ensuring Accountability
Perspective plan for effective governance in Higher Education
Strengthening institutional networking and global linkages
Curricular restructuring and Technology Enabled Learning
Human Resource potential enrichment
Enhancing quality and accelerating research

Significance of the Emblem

The Emblem Symbolizes Three Components:

- **Flames**: Dissolving Ignorance
- **Blossoms**: Germination of Ideas
- **Books**: Knowledge

First Year B.Sc.

**PHYSICS**

Mechanics, Waves & Oscillations

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